CLAUSE FORM

Notation: $X \rightarrow Y$ for: Rewrite all occurrences of X as Y.
Algorithm Clause-Form;
Input A wff α of FOL (or propositional logic);
Output A logically equivalent formula in clause form;
begin

1. Convert α to a logically equivalent formula in Prenex Normal Form:
   (a) $(\alpha \equiv \beta) \rightarrow ((\alpha \supset \beta) \wedge (\beta \supset \alpha))$
   (b) $(\alpha \supset \beta) \rightarrow (\neg\alpha \vee \beta)$
   (c) **repeat:**
      i. $\neg\neg\alpha \rightarrow \alpha$
      ii. $\neg(\alpha \wedge \beta) \rightarrow (\neg\alpha \vee \neg\beta)$
      iii. $\neg(\alpha \vee \beta) \rightarrow (\neg\alpha \wedge \neg\beta)$
      iv. $\neg\exists x[\alpha] \rightarrow \forall x[\neg\alpha]$
      v. $\neg\forall x[\alpha] \rightarrow \exists x[\neg\alpha]$

      until '$\neg$' only applies to atomic wffs;
   (d) **begin optional section:**
      i. $(\alpha \vee \alpha) \rightarrow \alpha$
      ii. $(\alpha \wedge \alpha) \rightarrow \alpha$

      end optional section;
   (e) Rename variables such that variables bound by different quantifiers have unique names
   (f) Move all quantifiers to the left, without changing their order
2. Convert PNF(α) to Skolem Normal Form:
   repeat:
   (a) $\forall x_1 \ldots \forall x_n \exists y[\alpha(y)] \rightarrow \forall x_1 \ldots \forall x_n[\alpha(f(x_1 \ldots x_n))]$ // f is a new Skolem function
   (b) $\exists y[\alpha(y)] \rightarrow \alpha(c)$ // c is a new Skolem constant

   until all existential quantifiers are eliminated
3. Convert SNF(α) to Conjunctive Normal Form:
   (a) $\forall x[\alpha(x)] \rightarrow \alpha(x)$
   (b) **repeat:**
      i. $(\alpha \vee (\beta \wedge \gamma)) \rightarrow ((\alpha \vee \beta) \wedge (\alpha \vee \gamma))$
      ii. **begin optional section:**
         A. $(\alpha \vee \alpha) \rightarrow \alpha$
         B. $(\alpha \wedge \alpha) \rightarrow \alpha$

         end optional section;

      until the formula is a conjunction of disjunctions of literals
4. (optionally:) Convert CNF(α) to Clause Form:
   (a) $(\alpha \vee \beta) \rightarrow \alpha\beta$ (or: $[\alpha, \beta]$) // $\alpha\beta$ (or: $[\alpha, \beta]$) is a "clause"
   (b) $(\alpha \wedge \beta) \rightarrow \{\alpha, \beta\}$ // these are sets of clauses
   (c) Rename variables again such that each clause has different variables

end.

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http://www.cse.buffalo.edu/~rapaport/563/clause-form.2003.03.25.pdf