# Intrinsic Ratings Compendium

(WORKING DRAFT)

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**Abstract**

This paper keeps a running compendium of Intrinsic Performance Ratings (IPR's) for selected events and player performances in the recorded history of chess. The IPR methodology is based on [RH11, RMH11] and updated here.

## 1 Introduction

The idea of Intrinsic Performance Ratings (IPR's) is to judge skill based on the quality of decisions made rather than the outcomes of contests. Aside from the issue that the outcome depends on the skill of opponents and on factors variously called "luck," there is a simple sample-size motivation. A chess professional may play 50 games in a given year and call that a lot, but as a statistical sample this is scant. However, those games may average 30 important move decisions, yielding a healthy sample of 1,500 *moves*. Analysis of those moves by computer programs to sufficient depth to be stronger than the player can then provide both an objective measure of skill, and reasonably informative confidence intervals on the assessment.

A common feature of chess magazines or columns, one long called "Solitaire Chess" in magazines of the US Chess Federation, involves pausing before each move (usually those by the winning side) of a selected game, and choosing from several plausible alternatives. A strong player composing the puzzle has provided point values for each choice. At the end the reader adds up the points for all of his/her choices, and there is a table giving corresponding skill levels. The levels are often given as ratings on the international Elo scale, where for instance 2200 is commonly the threshold for "master," or it may give prose names master, expert, amateur, etc. for those levels. We do not know of any attempt to make this correspondence scientific.

The IPR model is basically "Solitaire Chess" done scientifically, using suitably-scaled differences in values given to moves by authoritative chess programs as the "points." Although the differences are negative, the model would be unchanged if we declared that the best move is always worth 5 points and differences in the usual pawn/centipawn units of chess engines were subtracted

from it. The correspondence between points and Elo rating is first established by training the model on large sets of games by players with established Elo ratings. The model generates projections of how many points a player with a given Elo rating would score on a standardized "Solitaire Set."

To generate an IPR for a player's performance in an event, or for a whole event, or for any set of games, we run the training process in reverse: First we train the model on that set of games. Then we take the parameter values that were fitted in the training, and use them to generate a projected points value on the Solitaire Set. The corresponding Elo value is then read off. We do not go directly from the parameters to Elo because there is more than one dependent parameter in the model, and the tradeoff between the two parameters called $s$ and $c$ in the current simple form already seems difficult to assess. The "Solitaire" step also affords a reasonable way to project confidence intervals that currently seem to be no worse than about 30% too narrow—i.e., modeling error requires no more than a $1.4$ multiplier on them.

Full details are in the papers [RH11, RMH11], after earlier work [DHR09, HRD10] that built on [Haw03, Haw07]. The main differences from work by Guid and Bratko [GB06, GPB08, GB11] are the use of Multi-PV analysis to obtain authoritative values for all reasonable options, not just the top move(s) and the move played, and the discovery that human players behave as though relative values are scaled in proportion to the overall value of a position. The latter means that a value difference of, say, 20 centipawns between moves $m_1$ and $m_2$ as judged by an engine yields a greater incidence of human players selecting the better move $m_1$ when the position is (say) within 20 centipawns of being equal, as when one side is (say) 100 centipawns ahead. Indeed a "marginal centipawn" seems to have $5x$ impact when the engine's evaluation is +20 to one side as when it is +100. This is like the idea that price movements in stocks or bonds should be plotted in proportion to the current price, i.e. on log-log paper rather than standard axes.

To reprise some details from [RMH11, RMH11], the defining equation of the particular model used there and here is the following, which relates the probability $p_i$ of the $i$-th alternative move to $p_0$ for the best move and its difference in value:

$$\frac{\log(1/p_i)}{\log(1/p_0)} = e^{-\left(\frac{\delta}{s}\right)^c}, \qquad \text{where} \qquad \delta_i = \int_{v_i}^{v_0} \frac{1}{1+|z|}dz. \tag{1}$$

Here when the value $v_0$ of the best move and $v_i$ of the $i$-th move have the same sign, the integral giving the scaled difference simplifies to $|\log(1+v_0) - \log(1+v_i)|$. This employs the empirically-determined logarithmic scaling law.

The skill parameters are called $s$ for "sensitivity" and $c$ for "consistency" because $s$ when small can enlarge small differences in value, while $c$ when large sharply cuts down the probability of poor moves. The equation solved directly for $p_i$ becomes

$$p_i = p_0^{\alpha} \qquad \text{where} \qquad \alpha = e^{-\left(\frac{\delta}{s}\right)^c}. \tag{2}$$

The constraint $\sum_i p_i = 1$ thus determines all values. By fitting these derived probabilities to actual frequencies of move choice in training data, we can find values of $s$ and $c$ corresponding to the training set.

Once we have $s$ and $c$, these equations give us *projected probabilities* $p_{i,t}$ for every legal move $m_i$ in the position at every relevant game turn $t$. Per arbitrary choice we *omit*: game turns 1–8, turns involved in repetitions of the position, and turns where the program judges an advantage

greater than 300 centipawns for either side. These and some other modeling decisions are given detail and justification in [RH11].

To set up the correspondence to Elo rating, we define the *expected average error* statistic $AE_e$. For comparison we also define the *expected move-matching* statistic $MM_e$, which is the projected number of agreement's with the computer program's first choice of move. The following definitions tacitly assume that all move decisions are independent, which although violated by the idea of multi-move "plans," is arguably close enough when one has a large set of moves in various games. Note that they also give projected standard deviations for these two quantities.

$$\begin{array}{rclcrcl} MM_e &=& \sum_{t=1}^{T} p_{0,t}, & & \sigma_{MM_e} &=& \sqrt{\sum_{t=1}^{T} p_{0,t}(1-p_{0,t})} \\ AE_e &=& \frac{1}{T}\sum_{t=1}^{T}\sum_{i\geq 1} p_{i,t}\delta_{i,t}, & & \sigma_{AE_e} &=& \sqrt{\frac{1}{T}\sum_{t=1}^{T}\sum_{i\geq 1} p_{i,t}(1-p_{i,t})\delta_{i,t}}. \end{array} \tag{3}$$

Table 1 gives the values of $AE_e$ that were obtained by first fitting the training data for 2006–09, to obtain $s, c$, then computing the expectation for the union of the training sets. It was found that a smaller set $S$ of moves comprising the games of the 2005 and 2007 world championship tournaments and the 2006 world championship match gave identical results to the fourth decimal place, so $S$ was used as the fixed "Solitaire Set."

| Elo | 2700 | 2600 | 2500 | 2400 | 2300 | 2200 |
|---|---|---|---|---|---|---|
| $AE_e$ | .0572 | .0624 | .0689 | .0749 | .0843 | .0883 |

Table 1: Correspondence between Elo rating from 2006–2009 and projected Average Error.

A simple linear fit then yields the rule to produce the Elo rating for any $(s, c)$, which we call an "Intrinsic Performance Rating" (IPR) when the $(s, c)$ are obtained by analyzing the games of a particular event and player(s).

$$\text{IPR} = 3571 - 15413 \cdot AE_e. \tag{4}$$

This expresses, incidentally, that at least from the vantage of RYBKA 3 run to reported depth 13, perfect play has a rating under 3600. This is reasonable when one considers that if a 2800 player such as Vladimir Kramnik is able to draw one game in fifty, the opponent can never have a higher rating than that. The fitted $s, c$ values obtained in [RH11], including those forming a "central artery" of values $s_{fit}, c_{fit}$ in a single fitted line, became the following table of Elo values in [RMH11]:

2006–2009

| Elo | $s$ | $c$ | IPR | $2\sigma_e$ range | $2\sigma_a$ range | #moves | $c_{fit}$ | $s_{fit}$ | IPR$_{fit}$ |
|---|---|---|---|---|---|---|---|---|---|
| 2700 | .078 | .502 | 2690 | 2648–2731 | 2632–2748 | 7,032 | .513 | .080 | 2698 |
| 2600 | .092 | .523 | 2611 | 2570–2652 | 2553–2668 | 7,807 | .506 | .089 | 2589 |
| 2500 | .092 | .491 | 2510 | 2480–2541 | 2468–2553 | 16,773 | .499 | .093 | 2528 |
| 2400 | .098 | .483 | 2422 | 2393–2452 | 2381–2464 | 20,277 | .492 | .100 | 2435 |
| 2300 | .108 | .475 | 2293 | 2257–2328 | 2243–2342 | 17,632 | .485 | .111 | 2304 |
| 2200 | .123 | .490 | 2213 | 2170–2257 | 2153–2274 | 11,386 | .478 | .120 | 2192 |
| 2100 | .134 | .486 | 2099 | 2048–2150 | 2028–2170 | 9,728 | .471 | .130 | 2072 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 2000 | .139 | .454 | 1909 | 1853–1966 | 1830–1989 | 9,471 | 464 | .143 | 1922 |
| 1900 | .159 | .474 | 1834 | 1790–1878 | 1769–1893 | 16,195 | .457 | .153 | 1802 |
| 1800 | .146 | .442 | 1785 | 1741–1830 | 1723–1848 | 15,930 | .450 | .149 | 1801 |
| 1700 | .153 | .439 | 1707 | 1642–1772 | 1616–1798 | 8,429 | .443 | .155 | 1712 |
| 1600 | .165 | .431 | 1561 | 1496–1625 | 1470–1651 | 9,050 | .436 | .168 | 1565 |
| 1991–1994 | | | | | | | | | |
| 2700 | .079 | .487 | 2630 | 2576–2683 | 2555–2704 | 4,954 | .513 | .084 | 2659 |
| 2600 | .092 | .533 | 2639 | 2608–2670 | 2596–2682 | 13,425 | .506 | .087 | 2609 |
| 2500 | .098 | .500 | 2482 | 2453–2512 | 2441–2524 | 18,124 | .499 | .092 | 2537 |
| 2400 | .101 | .484 | 2396 | 2365–2426 | 2353–2438 | 19,968 | .492 | .103 | 2406 |
| 2300 | .116 | .480 | 2237 | 2204–2270 | 2191–2284 | 20,717 | .485 | .117 | 2248 |
| 2200 | .122 | .477 | 2169 | 2136–2202 | 2123–2215 | 21,637 | .478 | .122 | 2173 |
| 1976–1979 | | | | | | | | | |
| 2600 | .094 | .543 | 2647 | 2615–2678 | 2602–2691 | 11,457 | .506 | .087 | 2609 |
| 2500 | .094 | .512 | 2559 | 2524–2594 | 2509–2609 | 11,220 | .499 | .091 | 2547 |
| 2400 | .099 | .479 | 2397 | 2363–2431 | 2350–2444 | 16,635 | .492 | .103 | 2406 |
| 2300 | .121 | .502 | 2277 | 2240–2313 | 2226–2328 | 15,284 | .485 | .116 | 2257 |

Table 2: Elo correspondence in three four-year intervals.

Nlote that the fitted Elo values don't say exactly 2700, 2600, 2500, etc. This is the natural result of doing a linear fit. Some points in the 1600–2100 range are anomalous, and this may owe to various factors pertaining to the quality of the games and gamescores. Only the Elo 2200 through 2700 data for 2006–2009 were used in the linear fit for the ratings.

The procedure for generating an IPR for a given set $T$ of positions and chosen moves is the following—where $S$ is the fixed "Solitaire Set" defined above.

1. Do a regression on the test set $T$ to fit $s_T, c_T$.
2. Use $s_T, c_T$ to project $AE_e$ on the reference set $S$ (not on $T$).
3. Derive IPR from $AE_e$ via equation (4).
4. Use $s_T, c_T$ on the test set $T$ (not on $S$) only to project $\sigma_T = \sigma_{AE_e}$.
5. Output $[\text{IPR} - 2\sigma_T, \text{IPR} + 2\sigma_T]$ as the proposed "95%" confidence interval.

As noted toward the start of this section, early testing suggests replacing $\sigma_T$ by $\sigma_a = 1.4\sigma_T$ to get an "actual" 95% confidence interval given the model as it stands. Hence we show both ranges in the tables. These fits and confidence intervals underlie teh following results.

# 2 IPRs of Tournaments

Since the IPR is based only on game analysis and has no functional component from Elo, it extends before the adoption of Elo to the beginning of chess. Hence we include some tournaments before 1971, while noting the correspondence bewteen IPR and tournament category after that.

| Event | IPR | $2\sigma_e$ range | $2\sigma_a$ range |
|---|---|---|---|
| St. Petersburg 1896 quadrangular | 2390 | 2342–2438 | 2323–2458 |
| Cambridge Springs 1904, top 9 | 2432 | 2385–2479 | 2366–2497 |
| St. Petersburg 1914 prelims | 2332 | 2281–2382 | 2261–2402 |
| St. Petersburg 1914 finals | 2575 | 2534–2617 | 2517–2633 |
| New York 1927 | 2579 | 2536–2622 | 2518–2639 |
| AVRO 1938 | 2605 | 2564–2646 | 2547–2663 |
| The Hague 1948 | 2510 | 2444–2576 | 2417–2602 |
| Curacao 1962 Candidates' | 2538 | 2494–2582 | 2476–2600 |

Table 3: Intrinsic Ratings of some pre-1971 events

| Player, player in event, or entire event IPR | |
|---|---|
| Howard Staunton, versus P. de Saint-Amant | 1899 |
| Staunton, all major matches | 1940 |
| Adolf Anderssen, London 1851 | 2004 |
| Anderssen, versus Paul Morphy | 2112 |
| Morphy, versus Anderssen | 2124 |
| Morphy, 59 most important games overall | 2344 |
| Anderssen, 1860 onward | 2100 |
| Wilhelm Steinitz, up to 1870 | 1937 |
| Steinitz, 1871–1882 | 2320 |
| Steinitz, London 1883 | 2486 |
| Steinitz, all games versus Zukertort | 2352 |
| Steinitz, all games versus Chigorin | 2146 |
| Steinitz, all games versus Gunsberg | 2495 |
| Steinitz, all games versus Lasker | 2334 |
| Johannes Zukertort, all games | 2188 |
| Zukertort, London 1883 | 2445 |
| Zukertort, all games with Steinitz | 2199 |
| Emanuel Lasker, all games with Steinitz | 2471 |
| Jose Raúl Capablanca at New York, 1927 | 2936 |
| Capablanca, AVRO 1938 | 2680 |
| Capablanca, Buenos Aires Olympiad finals, 1939 | 2709 |
| Paul Keres at The Hague 1948 | 2657 |

Table 4: Some historical player IPR's

| Event | cat: Elo | IPR | $2\sigma_e$ range | $2\sigma_a$ range | IPR-Elo | #moves |
|---|---|---|---|---|---|---|
| Las Palmas 1996 | 21: 2756 | 2697 | 2612–2781 | 2579–2815 | -59 | 1,760 |
| Linares 1998 | 21: 2752 | 2715 | 2651–2780 | 2625–2805 | -37 | 2,717 |
| Linares 2000 | 21: 2751 | 2728 | 2645–2810 | 2612–2843 | -23 | 1,636 |
| Dortmund 2001 | 21: 2755 | 2752 | 2760–2834 | 2637–2866 | -3 | 1,593 |

| | | | | | | |
|---|---|---|---|---|---|---|
| Mexico 2007 | 21: 2751 | 2708 | 2647–2769 | 2623–2793 | -43 | 3,213 |
| Morelia-Linares 2008 | 21: 2755 | 2855 | 2808–2903 | 2789–2922 | +100 | 3,453 |
| Nanjing 2008 | 21: 2751 | 2766 | 2691–2842 | 2660–2873 | +15 | 1,936 |
| Bilbao GSF 2008 | 21: 2768 | 2801 | 2731–2872 | 2702–2900 | +33 | 2,013 |
| Linares 2009 | 21: 2755 | 2750 | 2696–2803 | 2675–2825 | -5 | 3,830 |
| Sofia M-Tel 2009 | 21: 2754 | 2711 | 2626–2795 | 2592–2829 | -51 | 1,937 |
| Nanjing 2009 | 21: 2763 | 2715 | 2644–2785 | 2616–2814 | -48 | 2,192 |
| Moscow Tal Mem. 2009 | 21: 2763 | 2731 | 2663–2800 | 2635–2827 | -32 | 2,706 |
| Linares 2010 | 21: 2757 | 2681 | 2607–2756 | 2577–2786 | -76 | 2,135 |
| Nanjing 2010 | 21: 2766 | 2748 | 2674–2821 | 2645–2850 | -18 | 1,988 |
| Shanghai 2010 | 21: 2759 | 2829 | 2727–2931 | 2686–2972 | +70 | 920 |
| Bilbao 2010 | 22: 2789 | 2904 | 2822–2987 | 2788–3020 | +115 | 1,060 |
| Moscow Tal Mem. 2010 | 21: 2757 | 2690 | 2629–2750 | 2604–2775 | -67 | 3,493 |
| Bazna 2011 | 21: 2757 | 2750 | 2675–2825 | 2645–2855 | -7 | 1,885 |
| Sao Paulo/Bilbao 2011 | 22: 2780 | 2626 | 2539–2713 | 2504–2748 | -154 | 1,998 |
| Moscow Tal Mem. 2011 | 22: 2776 | 2807 | 2755–2860 | 2734–2881 | +31 | 3,401 |
| Wijk aan Zee Tata A 2012 | 21: 2755 | 2723 | 2681–2765 | 2664–2782 | -32 | 6,092 |
| Averages | 21: 2760 | 2747 | | | -13 | 2,474 |
| Weighted by moves | 21: 2760 | 2742 | | | -17.2 | |
| Aggregate run, all moves | 21: 2760 | 2742 | 2727–2756 | 2721–2762 | -18 | 51,962 |

Table 5: Intrinsic Ratings of Category 21 and higher standard tournaments, through January 2012 (Tata 2012 not yet in averages).

| Event | cat: Elo | IPR | $2\sigma_e$ range | $2\sigma_a$ range | IPR-Elo | #moves |
|---|---|---|---|---|---|---|
| Linares 1999 | 20: 2735 | 2717 | 2652–2782 | 2627–2808 | -18 | 3,134 |
| Astana 2001 | 20: 2733 | 2771 | 2691–2850 | 2660–2882 | +38 | 1,713 |
| Linares 2002 | 20: 2732 | 2702 | 2631–2773 | 2603–2801 | -30 | 2,270 |
| Dortmund 2002 B | 20: 2727 | 2669 | 2539–2800 | 2487–2852 | -58 | 780 |
| Linares 2003 | 20: 2733 | 2695 | 2628–2762 | 2601–2789 | -38 | 2,549 |
| Linares 2004 | 20: 2731 | 2739 | 2673–2805 | 2647–2831 | +8 | 2,251 |
| Linares 2005 | 20: 2743 | 2699 | 2628–2771 | 2599–2800 | -44 | 2,418 |
| San Luis 2005 | 20: 2738 | 2657 | 2597–2716 | 2574–2740 | -81 | 3,694 |
| Morelia-Linares 2006 | 20: 2732 | 2628 | 2563–2692 | 2538–2718 | -104 | 3,621 |
| Sofia M-Tel 2006 | 20: 2744 | 2744 | 2678–2810 | 2651–2836 | 0 | 2,197 |
| Hoogeveen Essent 2006 | 20:2730 | 2485 | 2343–2628 | 2286–2685 | -245 | 844 |
| Moscow Tal Memorial 2006 | 20: 2727 | 2732 | 2667–2796 | 2642–2822 | +5 | 2,767 |
| Morelia-Linares 2007 | 20: 2746 | 2717 | 2659–2775 | 2636–2798 | -29 | 3,284 |
| Dortmund 2007 | 20: 2727 | 2815 | 2744–2885 | 2715–2914 | +88 | 1,812 |
| Moscow Tal Memorial 2007 | 20: 2741 | 2748 | 2685–2811 | 2660–2836 | +7 | 2,579 |
| Wijk aan Zee Corus A 2008 | 20: 2742 | 2730 | 2687–2773 | 2670–2790 | -12 | 5,774 |
| Sofia M-Tel 2008 | 20: 2737 | 2690 | 2605–2775 | 2571–2809 | -47 | 1,869 |
| Moscow Tal Memorial 2008 | 20: 2745 | 2664 | 2587–2741 | 2556–2772 | -81 | 2,764 |

| | | | | | | |
|---|---|---|---|---|---|---|
| Bazna Kings 2009 | 20: 2729 | 2664 | 2577–2751 | 2542–2785 | -65 | 1,897 |
| Dortmund 2009 | 20: 2744 | 2803 | 2728–2879 | 2697–2909 | +59 | 1, 597 |
| Bilbao GSF 2009 | 20: 2739 | 2613 | 2474–2752 | 2418–2807 | -126 | 806 |
| Astrakhan GP 2010 | 20: 2730 | 2796 | 2759–2833 | 2744–2848 | +66 | 6,090 |
| Bazna Kings 2010 | 20: 2742 | 2718 | 2642–2793 | 2612–2823 | -24 | 1,904 |
| London Classic 2010 | 20: 2725 | 2668 | 2594–2742 | 2565–2771 | -57 | 2,312 |
| Wijk aan Zee Tata 2011 | 20: 2740 | 2751 | 2707–2795 | 2690–2812 | 11 | 5,576 |
| Dortmund 2011 | 20: 2731 | 2704 | 2638–2770 | 2612–2796 | -27 | 2,521 |
| Hoogeveen Unive 2011 | 20: 2732 | 2662 | 2533–2791 | 2482–2843 | -70 | 829 |
| London Classic 2011 | 20: 2748 | 2709 | 2650–2768 | 2626–2792 | -39 | 2,594 |
| Reggio Emilia 2011-12 | 20: 2744 | 2554 | 2463–2645 | 2426–2681 | -190 | 1,834 |
| Averages | 20: 2736 | 2698 | | | -38 | 2,561 |
| Weighted by moves | 20: 2736 | 2711 | | | -25.3 | |
| Aggregate run | 20: 2736 | 2712 | 2700–2725 | 2695–2730 | -24 | 74,280 |

Table 6: IPR's of all Category 20 events through January 2012

| Event | cat: Elo | IPR | $2\sigma_e$ range | $2\sigma_a$ range | IPR-Elo | #moves |
|---|---|---|---|---|---|---|
| Montreal 1979 | 15: 2622 | 2588 | 2534–2642 | 2513–2663 | -34 | 4,732 |
| Linares 1993 | 18: 2676 | 2522 | 2469–2574 | 2449–2595 | -154 | 6,129 |
| Linares 1994 | 18: 2685 | 2517 | 2461–2574 | 2438–2596 | -168 | 5,536 |
| Dortmund 1995 | 17: 2657 | 2680 | 2615–2744 | 2589–2770 | +23 | 2,459 |
| Dortmund 1996 | 18: 2676 | 2593 | 2518–2667 | 2489–2697 | -83 | 2,796 |
| Dortmund 1997 | 18: 2699 | 2639 | 2569–2709 | 2541–2737 | -60 | 2,583 |
| Dortmund 1998 | 18: 2699 | 2655 | 2579–2732 | 2548–2762 | -44 | 2,284 |
| Dortmund 1999 | 19: 2705 | 2749 | 2655–2844 | 2617–2882 | +44 | 1,364 |
| Sarajevo 1999 | 19: 2703 | 2664 | 2592–2737 | 2563–2766 | +19 | 2,755 |
| Corus 2006 | 19: 2715 | 2736 | 2693–2779 | 2676–2797 | +21 | 5,800 |
| Corus 2007 | 19: 2717 | 2763 | 2716–2811 | 2697–2829 | +46 | 5,095 |
| Sofia M-Tel 2007 | 19: 2725 | 2576 | 2482–2670 | 2445–2708 | -149 | 2,184 |
| London 2009 | 18: 2696 | 2700 | 2630–2770 | 2602–2798 | +4 | 2,360 |

Table 7: Some other events, for comparison to Tables 5 and 6.

The IPR's are on-balance below the tournament average ratings, but the latter's aggregate is just within the narrower confidence interval of the aggregate IPR. The regressions are *not* linear, so the parity of the aggregate run with the weighted average is notable. The comparison events are selective but still show no inflationary trend.

# 3 IPR's of Matches

Again we begin with some historical matches and their results. We distinguish the following under a more-liberal definition of world championship besides the standard one: Andressen earned the title by unofficial but fairly general understanding from London 1851, and then lost it in 1858 to Paul Morphy, who vacated it back to Anderssen before 1866. See `http://www.worldchesslinks.net/ezc08.html` for our inclusion of the 1866–1876 Steinitz matches, while we consider Karpov-Korchnoi 1974 to have been known as a likely championship match at the time of its playing.

| Event/Player | | IPR | $2\sigma_e$ range | $2\sigma_a$ range | | #moves |
|---|---|---|---|---|---|---|
| Morphy, New York 1857 | | 2289 | 2077–2502 | 1992–2587 | | 368 |
| Opponents, all games | | 1859 | 1589–2129 | 1481–2237 | | 384 |
| Combined | | 2065 | 1891–2240 | 1821–2310 | | 752 |
| Morphy-Paulsen final | | 2196 | 1984–2408 | 1900–2492 | | 397 |
| Morphy | | 2265 | 1978–2663 | 1863–2668 | | 195 |
| Paulsen | | 2116 | 1805–2427 | 1681–2551 | | 202 |
| Morphy-Loewenthal 1858 | | 2252 | 2074–2429 | 2003–2500 | | 802 |
| Loewenthal | | 1965 | 1675–2255 | 1559–2371 | | 403 |
| Morphy | | 2561 | 2365–2758 | 2286–2837 | | 399 |
| Morphy-Harrwitz 1858 | | 2443 | 2247–2640 | 2168–2719 | | 472 |
| Harrwitz | | 2496 | 2253–2739 | 2156–2836 | | 240 |
| Morphy | | 2433 | 2130–2735 | 2009–2856 | | 232 |
| Harrwitz from game 3 | | 2433 | 2167–2698 | 2061–2804 | | 173 |
| Morphy from game 3 | | 2582 | 2259–2905 | 2129–3035 | | 165 |
| Morphy-Mongredien 1859 | | 1825 | 1457–2193 | 1310–2340 | | 262 |
| Mongredien | | 1194 | 557–1831 | 302–2086 | | 133 |
| Morphy | | 2304 | 1931–2677 | 1782–2827 | | 129 |

Table 8: 19th-Century early matches.

| Event/Player | | IPR | $2\sigma_e$ range | $2\sigma_a$ range | | #moves |
|---|---|---|---|---|---|---|
| Wch 1858 | | 2121 | 1903–2339 | 1816–2426 | | 633 |
| Anderssen | | 2122 | 1811–2433 | 1686–2557 | | 318 |
| Morphy | | 2124 | 1820–2428 | 1698–2550 | | 315 |
| Wch 1866 | | 2091 | 1908–2274 | 1834–2347 | | 797 |
| Anderssen | | 2137 | 1886–2387 | 1785–2488 | | 402 |
| Steinitz | | 2063 | 1798–2328 | 1692–2433 | | 395 |
| Wch 1866B | | 2263 | 2077–2449 | 2002–2523 | | 666 |
| Steinitz | | 2306 | 2053–2559 | 1951–2660 | | 332 |
| Bird | | 2245 | 1981–2508 | 1876–2613 | | 334 |
| Wch 1872 | | 2191 | 1978–2403 | 1893–2489 | | 635 |
| Steinitz | | 2388 | 2139–2638 | 2039–2738 | | 315 |
| Zukertort | | 1911 | 1555–2266 | 1413–2408 | | 320 |

| | | | | | | |
|---|---|---|---|---|---|---|
| Wch 1876 | | 2088 | 1834–2341 | 1733–2443 | | 405 |
| Blackburne | | 1927 | 1547–2306 | 1396–2458 | | 206 |
| Steinitz | | 2266 | 1935–2596 | 1803–2728 | | 199 |
| Wch 1886 | | 2338 | 2206–2470 | 2154–2523 | | 1,189 |
| Steinitz | | 2352 | 2150–2553 | 2070–2634 | | 593 |
| Zukertort | | 2320 | 2148–2493 | 2079–2562 | | 596 |
| Wch 1889 | | 2257 | 2099–2414 | 2036–2477 | | 978 |
| Chigorin | | 2188 | 1957–2420 | 1864–2512 | | 493 |
| Steinitz | | 2322 | 2112–2532 | 2028–2616 | | 485 |
| Wch 1890 | | 2397 | 2269–2526 | 2218–2577 | | 1,020 |
| Gunsberg | | 2302 | 2106–2499 | 2027–2578 | | 509 |
| Steinitz | | 2493 | 2327–2660 | 2260–2727 | | 511 |
| Wch 1892 | | 2045 | 1868–2221 | 1798–2291 | | 1,000 |
| Chigorin | | 2073 | 1846–2300 | 1756–2390 | | 498 |
| Steinitz | | 2039 | 1769–2309 | 1661–2417 | | 502 |
| Wch 1894 | | 2424 | 2315–2533 | 2271–2576 | | 1,390 |
| Lasker | | 2570 | 2432–2708 | 2377–2763 | | 691 |
| Steinitz | | 2269 | 2100–2438 | 2032–2506 | | 699 |
| Wch 1896 | | 2361 | 2236–2485 | 2187–2535 | | 1,249 |
| Lasker | | 2317 | 2143–2490 | 2074–2560 | | 618 |
| Steinitz | | 2414 | 2238–2590 | 2167–2661 | | 631 |

Table 9: 19th-Century world championship matches.

| Event/Player | | IPR | $2\sigma_e$ range | $2\sigma_a$ range | | #moves |
|---|---|---|---|---|---|---|
| Wch 1907 | | 2676 | 2565–2786 | 2521–2830 | | 889 |
| Lasker | | 2869 | 2734–3004 | 2680–3058 | | 440 |
| Marshall | | 2438 | 2268–2607 | 2201–2675 | | 449 |
| Wch 1908 | | 2451 | 2337–2564 | 2292–2610 | | 1190 |
| Lasker | | 2603 | 2462–2743 | 2406–2799 | | 595 |
| Tarrasch | | 2300 | 2122–2479 | 2050–2551 | | 595 |
| Wch 1909 | | 2675 | 2561–2788 | 2516–2833 | | 640 |
| Janowski | | 2537 | 2379–2696 | 2315–2759 | | 322 |
| Lasker | | 2811 | 2660–2962 | 2599–3023 | | 318 |
| Wch 1910A | | 2701 | 2586–2816 | 2541–2862 | | 845 |
| Lasker | | 2735 | 2577–2892 | 2514–2955 | | 422 |
| Schlechter | | 2703 | 2544–2861 | 2481–2925 | | 423 |
| Wch 1910B | | 2411 | 2259–2563 | 2199–2624 | | 815 |
| Janowski | | 2357 | 2141–2573 | 2055–2660 | | 409 |
| Lasker | | 2467 | 2255–2680 | 2169–2765 | | 406 |
| Wch 1921 | | 2667 | 2566–2768 | 2525–2809 | | 869 |
| Capablanca | | 2808 | 2683–2932 | 2633–2982 | | 431 |
| Lasker | | 2525 | 2364–2686 | 2300–2750 | | 438 |

| | | | | | | |
|---|---|---|---|---|---|---|
| Wch 1927 | | 2770 | 2705–2835 | 2678–2861 | | 2,069 |
| Alekhine | | 2812 | 2731–2893 | 2699–2925 | | 1,035 |
| Capablanca | | 2730 | 2626–2834 | 2585–2876 | | 1,034 |
| Wch 1929 | | 2521 | 2440–2601 | 2408–2633 | | 1792 |
| Alekhine | | 2567 | 2460–2675 | 2418–2717 | | 896 |
| Bogolyubov | | 2462 | 2341–2582 | 2293–2631 | | 896 |
| Wch 1934 | | 2417 | 2346–2488 | 2317–2517 | | 1927 |
| Alekhine | | 2451 | 2352–2549 | 2313–2588 | | 964 |
| Bogolyubov | | 2364 | 2259–2469 | 2217–2511 | | 963 |
| Wch 1935 | | 2559 | 2481–2638 | 2449–2670 | | 1878 |
| Alekhine | | 2595 | 2486–2703 | 2443–2747 | | 936 |
| Euwe | | 2521 | 2407–2635 | 2361–2681 | | 942 |
| Wch 1937 | | 2400 | 2293–2508 | 2251–2550 | | 1626 |
| Alekhine | | 2427 | 2265–2588 | 2200–2653 | | 816 |
| Euwe | | 2359 | 2217–2502 | 2160–2559 | | 810 |

Table 10: 20th-Century world championship matches before WW II.

| Event/Player | | IPR | $2\sigma_e$ range | $2\sigma_a$ range | | #moves |
|---|---|---|---|---|---|---|
| Wch 1951 | | 2555 | 2498–2611 | 2476–2633 | | 1848 |
| Botvinnik | | 2517 | 2427–2606 | 2391–2642 | | 922 |
| Bronstein | | 2583 | 2511–2654 | 2483–2682 | | 926 |
| Wch 1954 | | 2715 | 2643–2786 | 2614–2815 | | 1516 |
| Botvinnik | | 2706 | 2597–2814 | 2554–2857 | | 757 |
| Smyslov | | 2717 | 2620–2813 | 2581–2852 | | 759 |
| Wch 1957 | | 2668 | 2601–2735 | 2575–2762 | | 1352 |
| Botvinnik | | 2713 | 2627–2799 | 2593–2833 | | 676 |
| Smyslov | | 2611 | 2504–2719 | 2461–2762 | | 676 |
| Wch 1958 | | 2626 | 2562–2690 | 2537–2716 | | 1596 |
| Botvinnik | | 2792 | 2714–2871 | 2683–2902 | | 799 |
| Smyslov | | 2438 | 2336–2540 | 2295–2581 | | 797 |
| Wch 1960 | | 2693 | 2614–2772 | 2582–2804 | | 1432 |
| Botvinnik | | 2690 | 2585–2795 | 2543–2837 | | 715 |
| Tal | | 2683 | 2561–2805 | 2512–2854 | | 717 |
| Wch 1961 | | 2570 | 2490–2649 | 2459–2681 | | 1726 |
| Botvinnik | | 2665 | 2559–2771 | 2517–2813 | | 864 |
| Tal | | 2466 | 2348–2584 | 2301–2631 | | 862 |
| Wch 1963 | | 2744 | 2686–2802 | 2663–2826 | | 1565 |
| Botvinnik | | 2730 | 2649–2811 | 2617–2844 | | 782 |
| Petrosian | | 2762 | 2679–2845 | 2645–2878 | | 783 |
| Wch 1966 | | 2681 | 2629–2733 | 2608–2754 | | 1645 |
| Petrosian | | 2631 | 2556–2706 | 2526–2736 | | 824 |

| | | | | | | |
|---|---|---|---|---|---|---|
| Spassky | | 2724 | 2652–2797 | 2623–2826 | | 821 |
| Wch 1969 | | 2547 | 2489–2604 | 2466–2628 | | 1517 |
| Petrosian | | 2552 | 2472–2631 | 2440–2663 | | 758 |
| Spassky | | 2530 | 2446–2615 | 2412–2649 | | 759 |

Table 11: World Championship Matches in Moscow before the Elo System. (**Note**: these use a revised methodology to which the whole paper will be converted.)

| Event/Player | Elo [avg] | IPR | $2\sigma_e$ range | $2\sigma_a$ range | IPR-Elo | #moves |
|---|---|---|---|---|---|---|
| Wch 1972 | 2723 | 2646 | 2546–2747 | 2505–2787 | -77 | 1,367 |
| Fischer | 2785 | 2650 | 2496–2805 | 2434–2867 | | 680 |
| Spassky | 2660 | 2643 | 2512–2775 | 2459–2827 | | 687 |
| Wch 1974 | 2685 | 2662 | 2575–2744 | 2541–2778 | -23 | 1,787 |
| Karpov | 2700 | 2652 | 2529–2775 | 2480–2824 | -48 | 889 |
| Korchnoi | 2670 | 2667 | 2551–2783 | 2505–2829 | -3 | 898 |
| Wch 1978 | 2695 | 2708 | 2640–2775 | 2613–2802 | +13 | 2,278 |
| Karpov | 2725 | 2722 | 2632–2812 | 2596–2848 | | 1,141 |
| Korchnoi | 2665 | 2697 | 2598–2796 | 2558–2836 | | 1,137 |
| Wch 1981 | 2698 | 2749 | 2659–2839 | 2623–2875 | +51 | 1,176 |
| Karpov | 2700 | 2852 | 2731–2972 | 2683–3020 | | 587 |
| Korchnoi | 2695 | 2651 | 2520–2782 | 2486–2835 | | 589 |
| Wch 1984 g1-9 | 2710 | 2588 | 2442–2735 | 2382–2793 | | 589 |
| Karpov | 2705 | 2669 | 2484–2854 | 2409–2928 | | 293 |
| Kasparov | 2715 | 2487 | 2251–2723 | 2157–2817 | | 296 |
| Wch 1984 g10-29 | 2710 | 3002 | 2924–3080 | 2892–3111 | | 828 |
| Karpov | 2705 | 2957 | 2826–3087 | 2774–3139 | | 411 |
| Kasparov | 2715 | 3042 | 2949–3135 | 2912–3172 | | 417 |
| Wch 1984 g30-48 | 2710 | 2771 | 2674–2868 | 2635–2907 | | 1,029 |
| Karpov | 2705 | 2678 | 2532–2824 | 2473–2883 | | 515 |
| Kasparov | 2715 | 2863 | 2736–2990 | 2686–3041 | | 514 |
| Wch 1984 all | 2710 | 2810 | 2751–2870 | 2727–2894 | +100 | 2,446 |
| Karpov | 2705 | 2761 | 2672–2851 | 2636–2887 | | 1,219 |
| Kasparov | 2715 | 2859 | 2781–2937 | 2750–2969 | | 1,227 |
| Wch 1985 | 2710 | 2720 | 2636–2805 | 2602–2838 | +10 | 1,420 |
| Karpov | 2720 | 2701 | 2583–2819 | 2536–2866 | | 712 |
| Kasparov | 2700 | 2734 | 2612–2856 | 2564–2905 | | 708 |
| Wch 1986 | 2723 | 2841 | 2763–2919 | 2732–2951 | +118 | 1,343 |
| Karpov | 2705 | 2807 | 2700–2914 | 2657–2957 | | 670 |
| Kasparov | 2740 | 2907 | 2801–3014 | 2758–3057 | | 673 |
| Wch 1987 | 2720 | 2742 | 2660–2824 | 2627–2857 | +22 | 1,490 |
| Karpov | 2700 | 2838 | 2730–2946 | 2687–2989 | | 742 |
| Kasparov | 2740 | 2659 | 2540–2779 | 2492–2826 | | 748 |
| Wch 1990 | 2765 | 2620 | 2527–2712 | 2491–2749 | -145 | 1,622 |

| | | | | | | |
|---|---|---|---|---|---|---|
| Karpov | 2730 | 2547 | 2403–2692 | 2345–2749 | | 812 |
| Kasparov | 2800 | 2674 | 2554–2794 | 2507–2841 | | 810 |
| Wch 1993 | 2730 | 2683 | 2578–2787 | 2537–2829 | -47 | 1,187 |
| Kasparov | 2805 | 2631 | 2477–2786 | 2415–2847 | | 593 |
| Short | 2655 | 2734 | 2594–2875 | 2538–2931 | | 594 |
| Wch 1995 | 2760 | 2702 | 2557–2847 | 2499–2905 | -58 | 767 |
| Anand | 2725 | 2617 | 2400–2834 | 2313–2921 | | 382 |
| Kasparov | 2795 | 2807 | 2625–2989 | 2552–3062 | | 385 |
| Wch 2000 | 2810 | 2918 | 2829–3008 | 2794–3043 | +108 | 867 |
| Kasparov | 2849 | 2851 | 2706–2997 | 2648–3055 | | 435 |
| Kramnik | 2770 | 2969 | 2858–3080 | 2814–3125 | | 432 |
| Wch 2004 | 2756 | 2871 | 2763–2978 | 2721–3020 | +115 | 726 |
| Kramnik | 2770 | 2945 | 2815–3074 | 2764–3126 | | 363 |
| Leko | 2741 | 2785 | 2605–2695 | 2533–3037 | | 363 |
| Wch 2006 | 2778 | 2802 | 2709–2895 | 2672–2933 | +24 | 911 |
| Kramnik | 2743 | 2791 | 2660–2922 | 2608–2974 | | 453 |
| Topalov | 2813 | 2832 | 2705–2958 | 2655–3009 | | 458 |
| Wch 2008 | 2778 | 2679 | 2528–2830 | 2468–2890 | -99 | 562 |
| Anand | 2783 | 2723 | 2506–2940 | 2419–3026 | | 279 |
| Kramnik | 2772 | 2610 | 2393–2827 | 2306–2914 | | 283 |
| Wch 2010 | 2796 | 2720 | 2613–2826 | 2571–2869 | -76 | 985 |
| Anand | 2787 | 2737 | 2572–2903 | 2506–2969 | | 491 |
| Topalov | 2805 | 2703 | 2566–2839 | 2512–2894 | | 494 |
| Averages | 2740 | 2742 | | | +2.25 | 19,147 |
| Weighted by moves | 2730 | 2736 | | | +6.03 | |
| Aggregate run | 2734 | 2735 | 2712–2758 | 2703–2767 | +1 | 20,934 |

Table 12: World Championship matches since 1972.

| Event/Player | | IPR | $2\sigma_e$ range | $2\sigma_a$ range | | #moves |
|---|---|---|---|---|---|---|
| Capablanca-Kostic 1919 | | 2545 | 2355–2734 | 2280–2810 | | 372 |
| Capablanca | | 3001 | 2818–3184 | 2744–3257 | | 185 |
| Kostic | | 2005 | 1704–2305 | 1584–2425 | | 187 |
| Fischer-Taimanov CM 1971 | 2680 | 2765 | 2625–2905 | 2569–2961 | +85 | 528 |
| Fischer | 2740 | 2923 | 2752–3095 | 2683–3164 | +183 | 263 |
| Taimanov | 2620 | 2616 | 2402–2829 | 2317–2914 | -4 | 265 |
| Fischer-Larsen CM 1971 | 2710 | 2478 | 2183–2773 | 2066–2890 | -232 | 359 |
| Fischer | 2760 | 2830 | 2468–3192 | 2323–3336 | +70 | 178 |
| Larsen | 2660 | 2187 | 1772–2602 | 1606–2768 | -473 | 181 |
| Fischer-Petrosian CM 1971 | 2700 | 2814 | 2689–2939 | 2639–2989 | +114 | 496 |
| Fischer | 2760 | 2969 | 2828–3111 | 2771–3167 | +209 | 247 |
| Petrosian | 2640 | 2646 | 2435–2858 | 2350–2942 | +6 | 249 |
| Fischer, all 1971 matches | 2754 | 2921 | 2808–3034 | 2763–3079 | +157 | 688 |
| Karpov-Polugayevsky CM 1974 | 2665 | 2651 | 2505–2798 | 2446–2857 | -14 | 461 |

| | | | | | | |
|---|---|---|---|---|---|---|
| Karpov | 2700 | 2754 | 2580–2928 | 2511–2998 | +54 | 229 |
| Polugaevsky | 2630 | 2472 | 2210–2734 | 2105–2839 | -158 | 232 |
| Karpov-Spassky CM 1974 | 2675 | 2748 | 2642–2854 | 2600–2896 | +73 | 737 |
| Karpov | 2700 | 2692 | 2519–2866 | 2449–2935 | -8 | 368 |
| Spassky | 2650 | 2810 | 2685–2935 | 2635–2985 | +160 | 369 |
| Karpov-Portisch Milan 1975 final | 2670 | 2620 | 2448–2791 | 2380–2860 | -50 | 321 |
| Karpov | 2705 | 2731 | 2552–2909 | 2480–2981 | +26 | 159 |
| Portisch | 2635 | 2388 | 2066–2709 | 1937–2838 | -247 | 162 |
| Fischer-Spassky 1992 | 2673 | 2698 | 2626–2770 | 2597–2799 | +25 | 2,077 |
| Fischer | 2785 | 2724 | 2622–2825 | 2582–2866 | -61 | 1,037 |
| Spassky | 2560 | 2659 | 2554–2764 | 2512–2806 | +99 | 1,040 |
| WWch 1996 | 2540 | 2591 | 2474–2708 | 2427–2755 | +51 | 657 |
| Susan Polgar | 2550 | 2728 | 2558–2898 | 2490–2966 | +178 | 329 |
| Xie Jun | 2530 | 2478 | 2322–2634 | 2260–2697 | -52 | 328 |
| WWch 2011 | 2589 | 2828 | 2713–2943 | 2667–2989 | +239 | 570 |
| Hou Yifan | 2578 | 2971 | 2849–3093 | 2801–3142 | +393 | 283 |
| Humpy Koneru | 2600 | 2680 | 2479–2881 | 2398–2961 | +80 | 287 |

Table 13: Some other post-1900 matches.

# 4 IPRs of Big Swiss Event(s)

We analyzed all 624 available games from 647 played at the 2011 Canadian Open, including all by players with FIDE ratings 2400 and above, which form an unbiased sample. Table 14 shows the IPR's and compares them to Chess Federation of Canada ratings before and after the event, FIDE ratings before, and the tournament performance ratings (TPR's) based on the CFC ratings. The final two columns are the confidence intervals for the IPR alone. The final rows summarize the sample, the whole event (152 players minus 3 early withdrawals leaving 149), and the whole event weighted by number of games played and number of analyzed moves. The bottom-right restricts to the 115 players who had FIDE ratings before the event. All players with 2400+ *Canadian* ratings are named. While the IPR's for tournaments with average ratings over 2700 are by-and-large low, these are markedly high especially relative to thse players' FIDE ratings. The time control was 40 moves in 90 minutes plus 30 second increment, then G/30 plus the same increment.

| Name | Can R | FIDE R | TPR | IPR | IPR-TPR | $2\sigma_e$ range | $2\sigma_a$ range | #moves |
|---|---|---|---|---|---|---|---|---|
| Arencibia | 2537 | 2476 | 2745 | 2723 | -22 | 2491–2956 | 2398–3049 | 273 |
| Benjamin | 2641 | 2553 | 2688 | 2412 | -276 | 2196–2629 | 2110–2715 | 373 |
| Bluvshtein | 2634 | 2611 | 2622 | 2533 | -89 | 2323–2744 | 2239–2828 | 316 |
| Bojkov | 2544 | 2544 | 2595 | 2154 | -441 | 1765–2543 | 1610–2698 | 219 |
| Calugar | 2437 | 2247 | 2144 | 2301 | +157 | 2091–2512 | 2007–2596 | 327 |
| Cheng | 2500 | 2385 | 2661 | 2728 | +67 | 2502–2954 | 2411–3044 | 297 |
| Cummings | 2459 | 2350 | 2473 | 2833 | +360 | 2683–2983 | 2623–3043 | 322 |
| Fedorowicz | 2508 | 2454 | 2422 | 2390 | -32 | 2088–2692 | 1967–2813 | 199 |
| Gerzhoy | 2647 | 2483 | 2622 | 2963 | +341 | 2802–3124 | 2738–3189 | 211 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Golod | 2576 | 2582 | 2582 | 2638 | +56 | 2376–2899 | 2272–3003 | 218 |
| Hebert | 2486 | 2414 | 2519 | 2789 | +270 | 2598–2979 | 2522–3055 | 285 |
| Krnan | 2470 | 2390 | 2651 | 2694 | +43 | 2488–2900 | 2405–2982 | 266 |
| Krush | 2578 | 2487 | 2539 | 2497 | -42 | 2217–2717 | 2189–2805 | 316 |
| Meszaros | 2409 | 2418 | 2278 | 2413 | +133 | 2219–2607 | 2141–2684 | 337 |
| Mikhalevski | 2664 | 2569 | 2519 | 2616 | +96 | 2412–2820 | 2330–2902 | 248 |
| Milicevic | 2400 | 2288 | 2352 | 2113 | -240 | 1799–2426 | 1674–2552 | 214 |
| Mulyar | 2422 | 2410 | 2412 | 2636 | +224 | 2483–2788 | 2422–2849 | 378 |
| Noritsyn | 2597 | 2425 | 2563 | 2394 | -171 | 2166–2621 | 2075–2713 | 286 |
| Pechenkin | 2408 | 2297 | 2309 | 2648 | +339 | 2439–2857 | 2355–2940 | 311 |
| Perelshteyn | 2532 | 2534 | 2650 | 2629 | -21 | 2425–2833 | 2343–2915 | 258 |
| Perez Rod'z | 2467 | 2467 | 2676 | 2627 | -49 | 2321–2933 | 2198–3056 | 195 |
| Plotkin | 2411 | 2243 | 2260 | 2715 | +455 | 2570–2861 | 2512–2919 | 330 |
| Regan | 2422 | 2409 | 2268 | 2525 | +257 | 2323–2728 | 2242–2809 | 356 |
| Rozentalis | 2614 | 2571 | 2666 | 2721 | +55 | 2528–2913 | 2452–2990 | 291 |
| Sambuev | 2739 | 2528 | 2571 | 2677 | +106 | 2499–2855 | 2428–2926 | 400 |
| Samsonkin | 2532 | 2378 | 2707 | 2535 | -172 | 2267–2802 | 2159–2910 | 233 |
| Sapozhnikov | 2424 | 2295 | 2480 | 2404 | -76 | 2203–2605 | 2122–2685 | 341 |
| Shabalov | 2618 | 2577 | 2549 | 2639 | +90 | 2417–2861 | 2328–2590 | 262 |
| Thavandiran | 2447 | 2320 | 2607 | 2622 | +15 | 2360–2884 | 2255–2989 | 254 |
| Yoos | 2439 | 2373 | 2289 | 1939 | -350 | 1607–2271 | 1474–2404 | 268 |
| Zenyuk | 2429 | 2222 | 2342 | 2790 | +448 | 2606–2975 | 2532–3049 | 229 |
| Averages | 2516 | 2429 | 2508 | 2558 | +50 | | | |
| Std. Dev. | 92 | | 157 | 218 | | | | |
| Whole event: | 149 | | | | | Restricted to | FIDE-rated | players: 115 |
| Average | 2144 | | 2142 | 2117 | | 2203 | 2211 | 2139 |
| Std. Dev. | 258 | | 261 | 379 | | 345 | 229 | 220 |
| Wtd. avgs. | | | | | | IPR | CanR | FIDE R |
| By games | 2156 | | 2154 | 2134 | | 2219 | 2221 | 2147 |
| By moves | 2173 | | 2172 | 2161 | | 2242 | 2236 | 2161 |

Table 14: Comparison of FIDE and CFC ratings, TPR's, and IPR's for 2011 Canadian Open

1. The IPR's have similar overall average to the Canadian ratings, especially under weighting by games or moves.

2. FIDE ratings of Canadian players are deflated relative to apparent skill. This is commonly believed to be due to a lack of playing opportunities in FIDE-rated events.

3. The IPR's have higher deviations from their own mean than the TPR's.

4. The IPR's have large deviation, and yet several TPR's fall outside even the 2.8-sigma range. This may constrain the usefulness of the IPR as an estimator of the TPR.

| Event/Player | Elo | IPR | $2\sigma_e$ range | $2\sigma_a$ range | IPR-Elo | #moves |
|---|---|---|---|---|---|---|

Table 15: Other results by 2012 Canadian Open high performers.

# 5 Conclusions

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