Lecture Notes 03
CSE 350 Spring 2026

Quiz 1/29/26: For complexity analysis on two functions $f(n)$ and $g(n)$, we suppose we denote two functions having the same trend as $f(n) \equiv g(n)$. From your earlier algorithm courses, you might remember that polynomials of the same dgree as having the same trend. But how can we define the notion of $\equiv$ formally. Please indicate whether it is okay to define $f(n) \equiv g(n)$ as $\lim_{n\to+\infty} |f(n) - g(n)| \leq c$ for some constant c, and explain why using the example on lecture slide 03, page 7.

Answer: No. Taking $f_3(n) = 3n + 2$ and $f_4(n) = 5n - 4$ as an example, which we consider as having the same trend since they both are polynomials of degree 1.

However, $\lim_{n\to+\infty} |f_3(n) - f_4(n)|$ does not exist. To show this, we need to prove that $\exists\varepsilon > 0, \forall n_0 \in \mathcal{N}, \exists n \geq n_0, |f_3(n) - f_4(n)| > \varepsilon$.

Now take an arbitrary $\varepsilon = 100 > 0$. Solving for n based on the inequality $|f_3(n) - f_4(n)| > \varepsilon$ gives us $n > 103$. Therefore, by taking $\varepsilon = 100$, for any $n_0 \in \mathcal{N}$, we simply take $n = \max\{n_0, 104\}$, we will have $|f_3(n) - f_4(n)| > \varepsilon$. $\square$