CSE462/562: Database Systems (Spring 23)

Lecture 16: External sorting

4/11/2023

University at Buffalo
Department of Computer Science and Engineering
School of Engineering and Applied Sciences

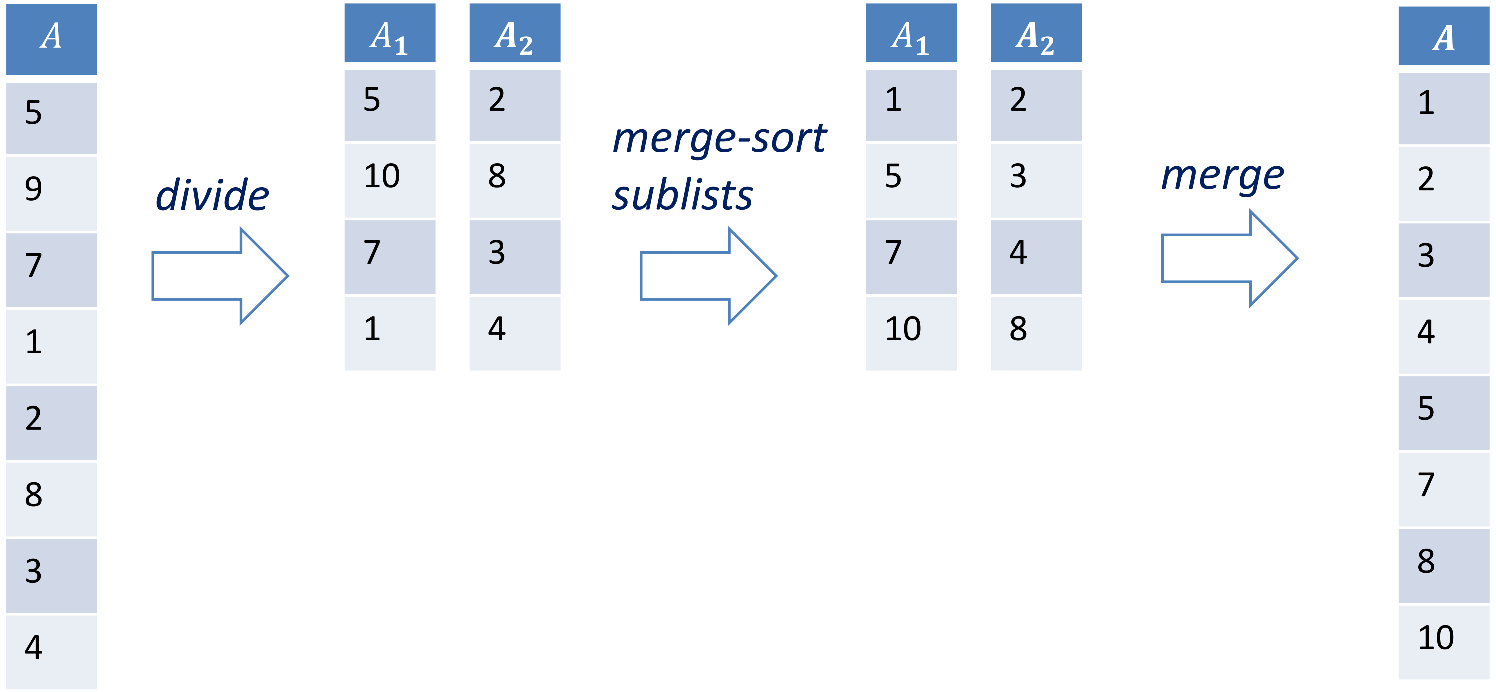
What is external sorting/hashing?

- Problem: sort or hashing 1TB of data over 1GB of RAM
  - Why not virtual memory?
    - Swaps involve expensive random I/Os
  - Why not using B-Tree/extendible hashing/linear hashing?
    - Dynamic structures carry additional overhead for maintenance (not needed in QP)
    - Missing optimization opportunities with hybrid approach (see later)

- General wisdom:
  - I/O cost dominates the total cost
  - Design algorithms to reduce the number of I/Os

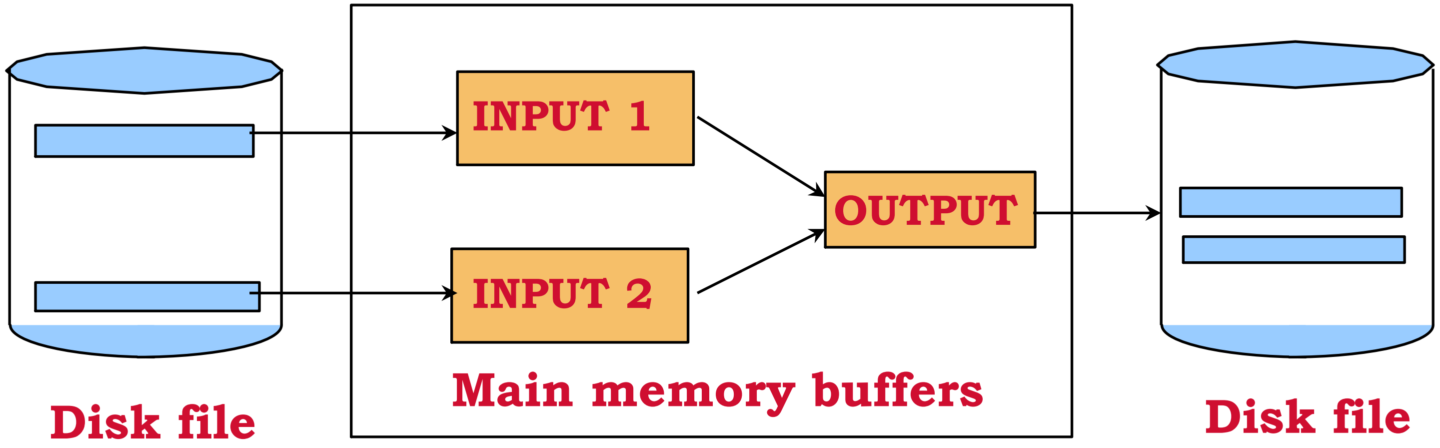
Two-way merge-sort: a starting point

- Recall the two-way merge-sort
  - given a list of items in $A[0..n-1]$
  - recursively divide and conquer the problem
    - divide the list into two halves $A_1\left[0..\left\lfloor\frac{n}{2}\right\rfloor\right], A_2\left[\left\lfloor\frac{n}{2}\right\rfloor+1, n-1\right]$
    - merge-sort A_1 and A_2 individually
    - merge the two sorted list A_1, A_2



External two-way merge sort

- Needs 3 buffers
- Instead of recursion
  - works bottom up from the input



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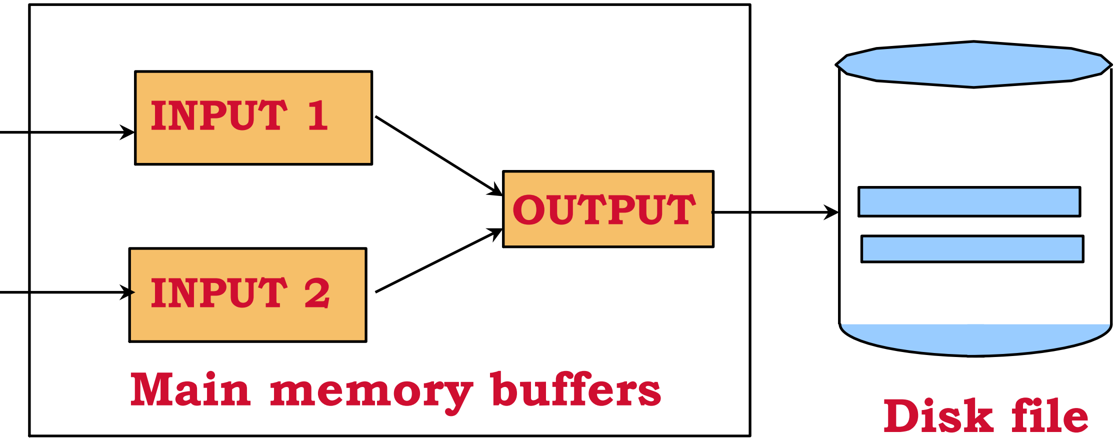
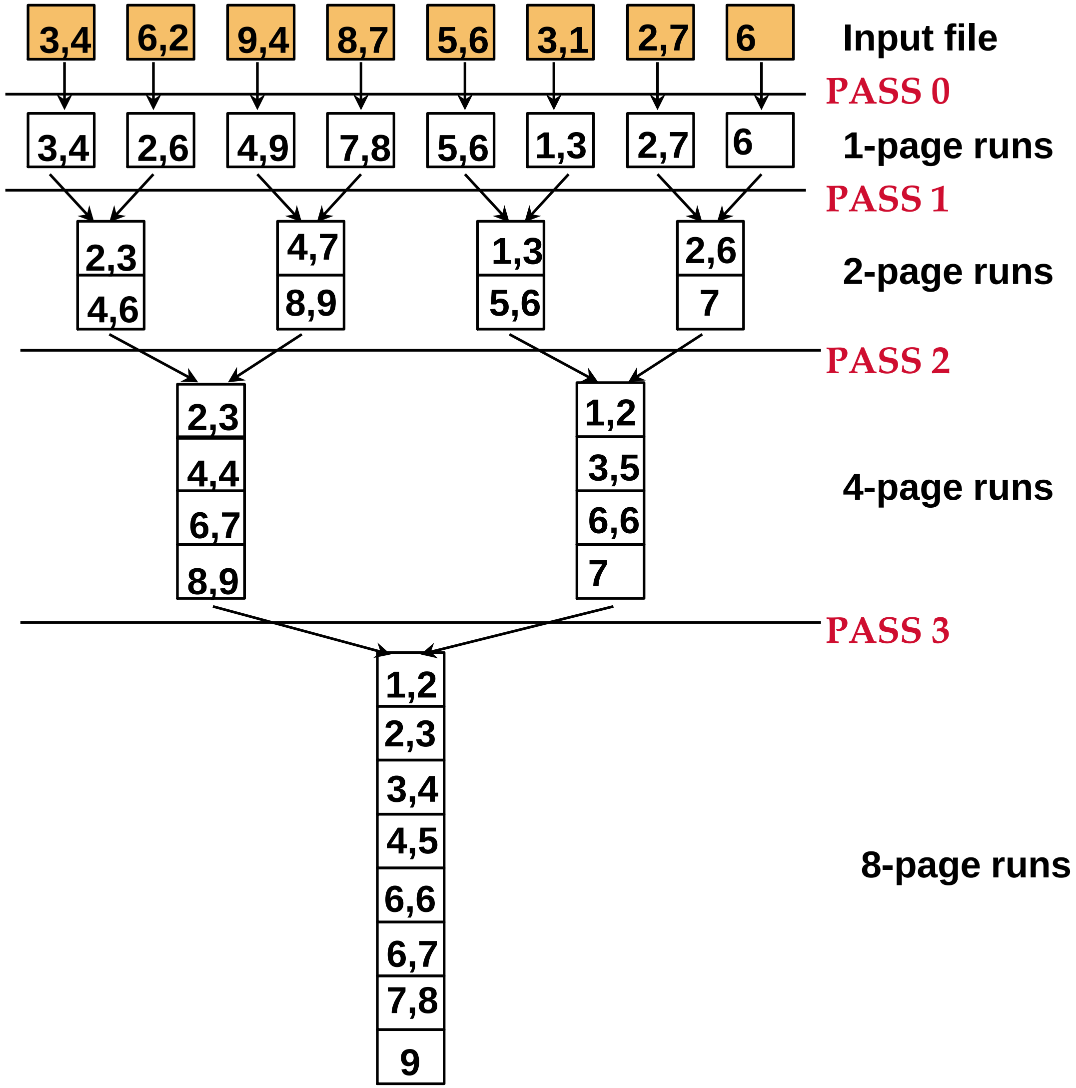
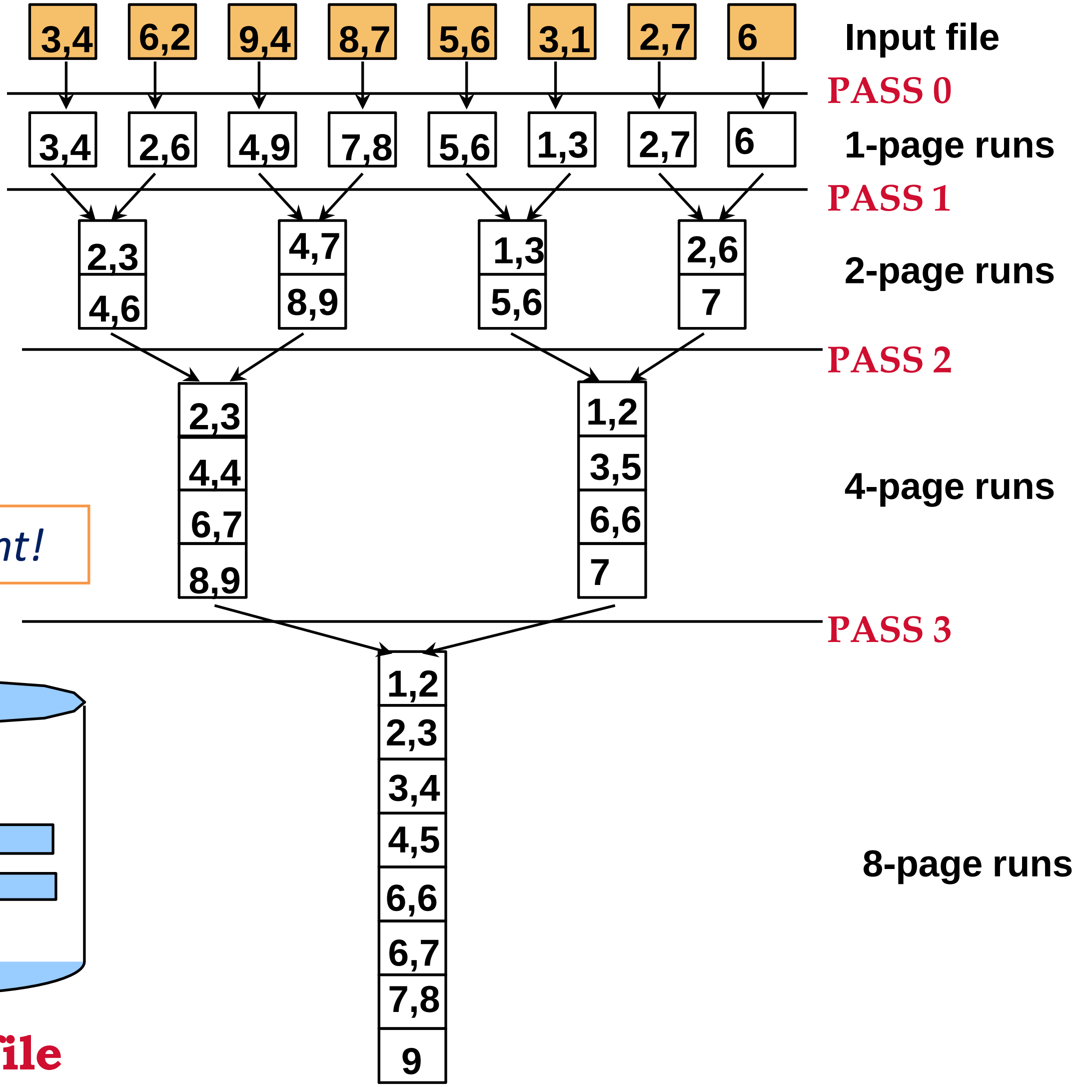


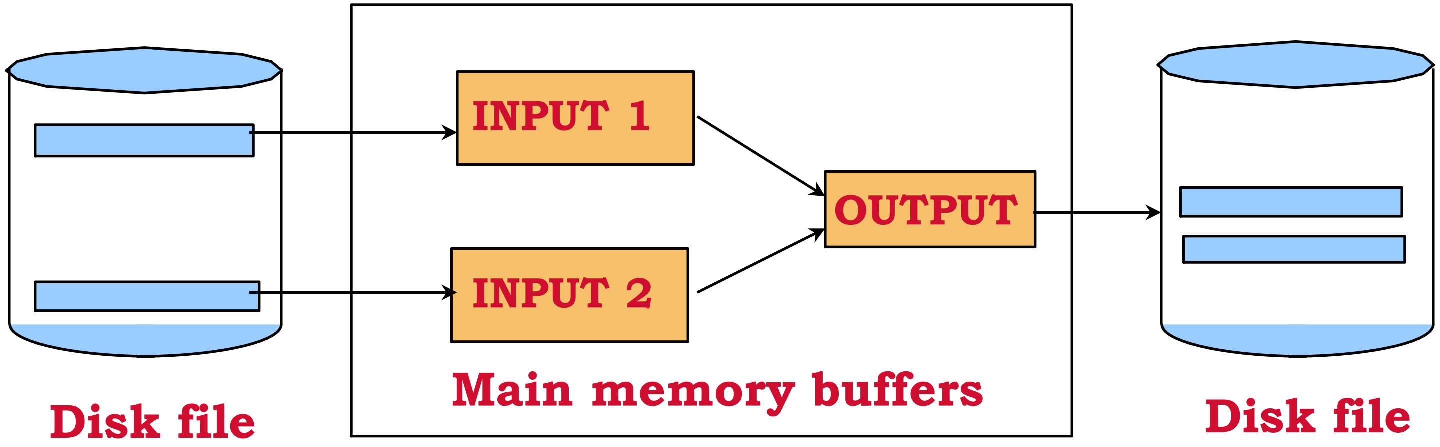


External two-way merge sort

- Input: N pages
- Cost for a pass: reading & writing N pages once
- # of passes: height of the tree = $\lceil \log_2 N \rceil + 1$
- Total cost: $2N(\lceil \log_2 N \rceil + 1)$ I/Os
  - Transfer cost: $2t_T N(\lceil \log_2 N \rceil + 1)$
  - *Seek cost:* $2t_S N(\lceil log_2 N \rceil + 1)$
  - *total* = $2(t_T + t_S)N(\lceil \log_2 N \rceil + 1)$



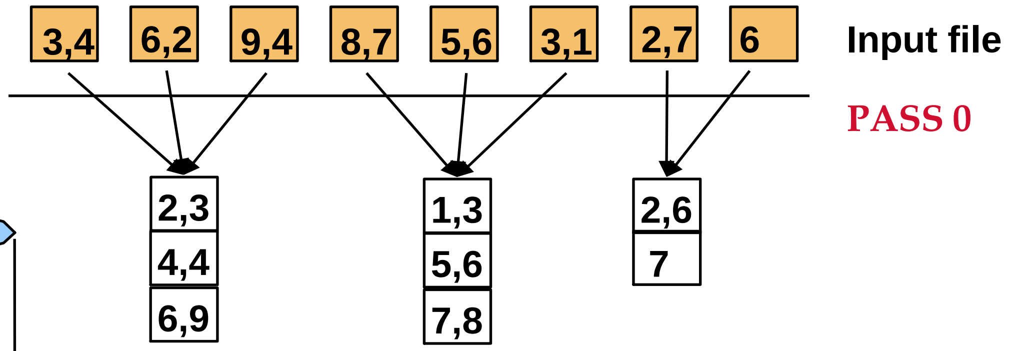
Not so efficient!

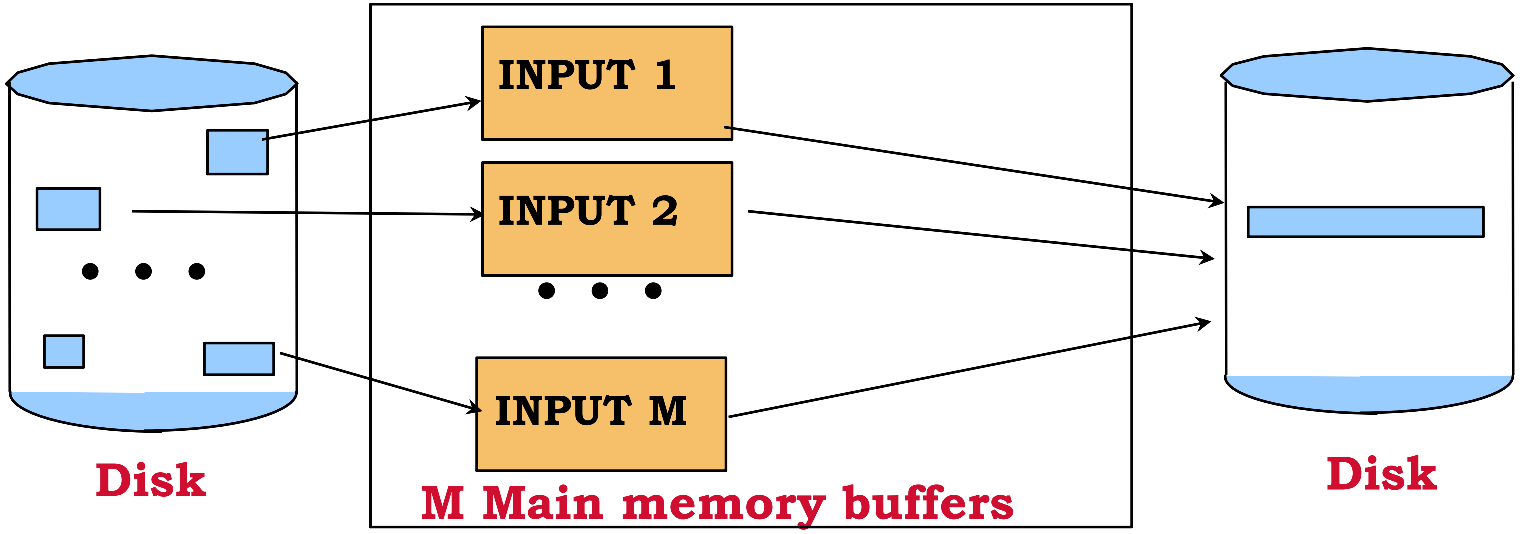


External multi-way merge sort

- How do we fully utilize all the M buffers?
  - Solution: (M-1)-way merge-sort
- Pass 0: internal sort to produce initial runs
  - read every M pages into memory
  - use some internal sorting algorithm (e.g., quick sort)
    - *can produce even larger runs (later)*
  - write all the M pages as a run

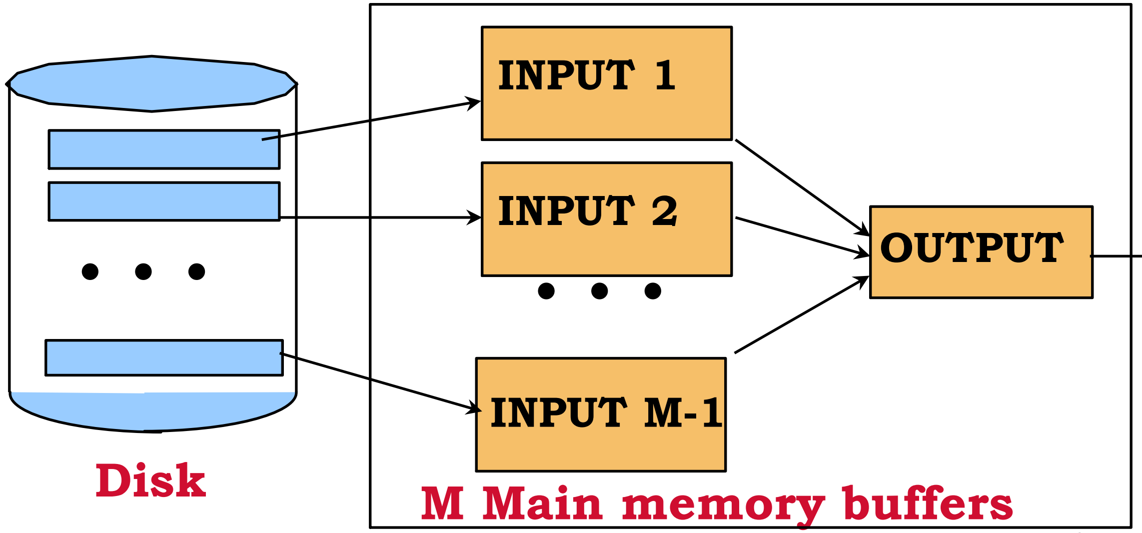
N pages in input
$\lceil \frac{N}{M} \rceil$ runs after pass 0
Cost:
$2N$ pages read/written +
$2\left\lceil \frac{N}{M} \right\rceil$ seeks
i.e. $2Nt_T + 2\left\lceil \frac{N}{M} \right\rceil t_S$

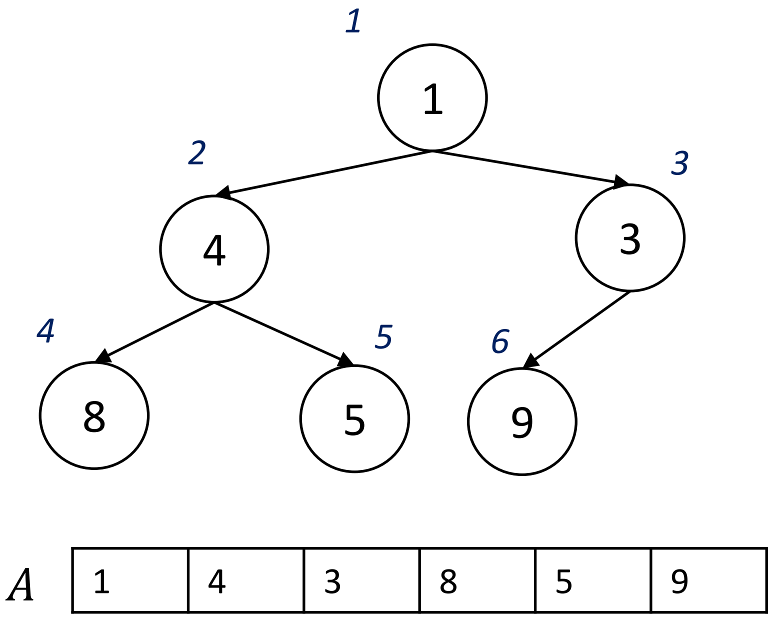




General multi-way merge sort

- Pass 1, 2, …: merge as many runs as possible from previous pass into a sorted run
  - maintain *a min-heap/max-heap (aka priority queue)*
    - supports $O(logM)$ time insertion of any item and deletion of the smallest/largest item
    - a complete binary tree where parent is smaller/larger than both children
    - how to implement
      - numbering nodes level by level sequentially from 1, store in an array $A[1..n]$
        - (how to translate 1-based index to 0-based in C/C++?)
      - parent of $A[i]$ is $A[i/2]$, left child of $A[i]$ is $A[i*2]$, right child of $A[i]$ is $A[i*2+1]$
      - push-down or push-up to maintain the variant





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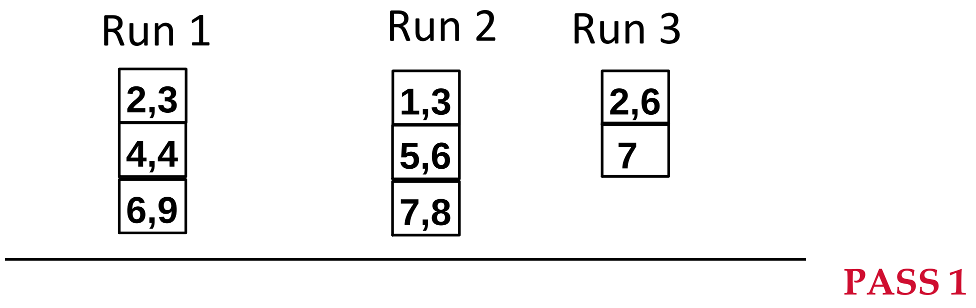
  - load one page from each of the $M-1$ runs
    - and maintain pointers of next page to read
  - for each loaded page
    - insert the first key into the min-heap
    - maintain next slot ids for each page
  - Repeatedly remove the smallest item from the min heap
    - and replace it with the next key in its run
    - write out the output page once it's full

For illustration, let's now assume M = 4 instead of 3 from now on.



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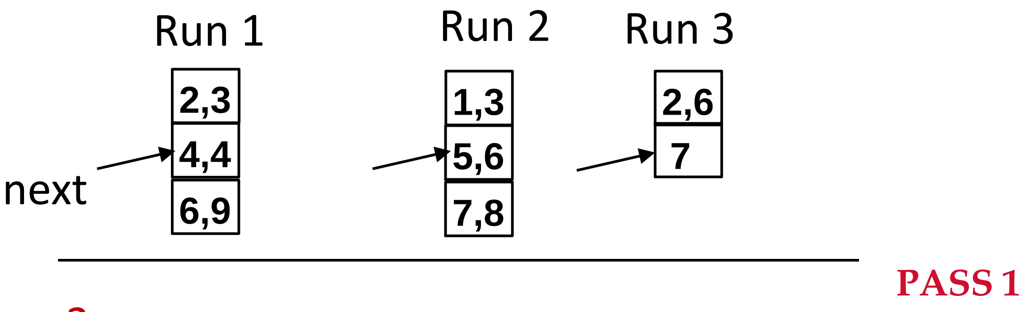

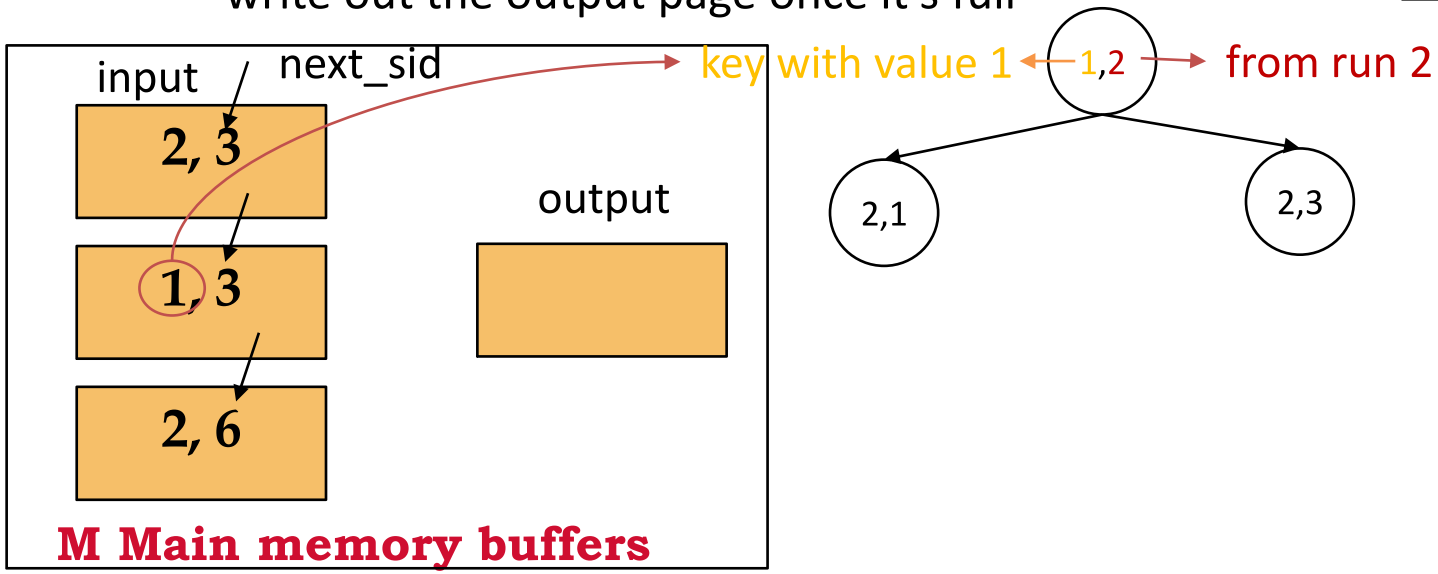

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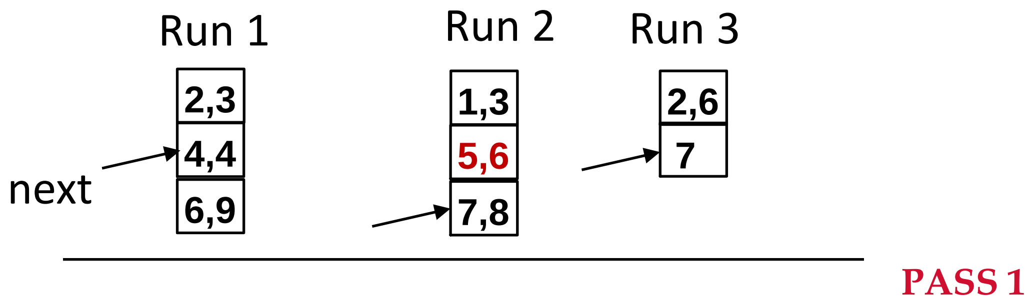

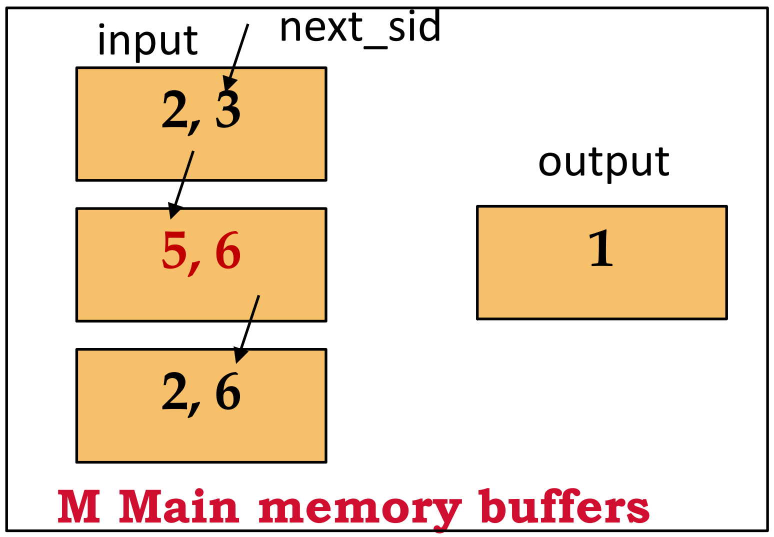

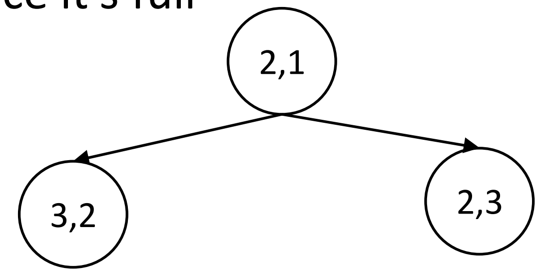

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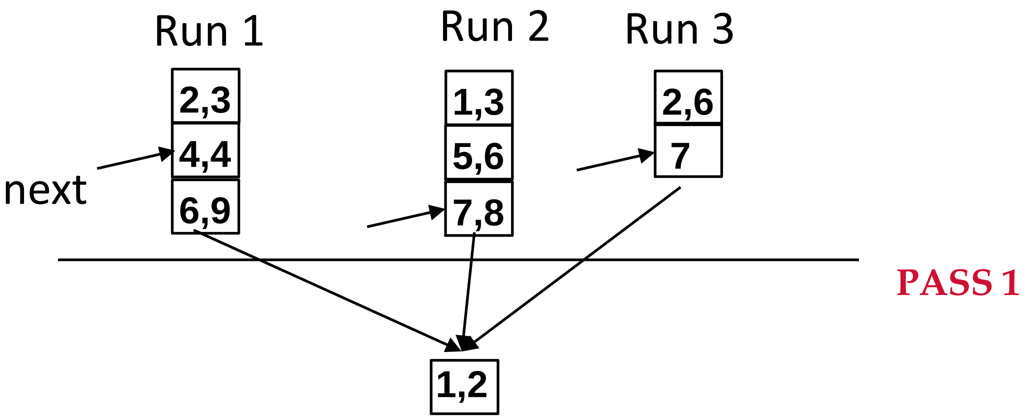

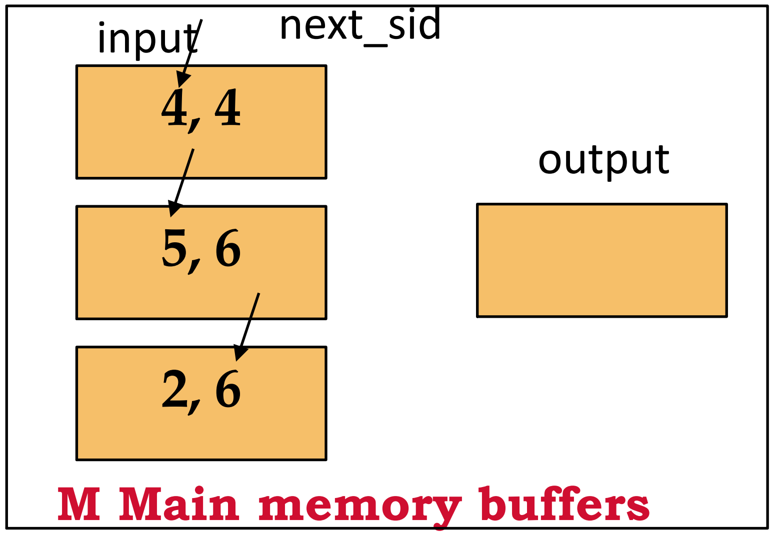

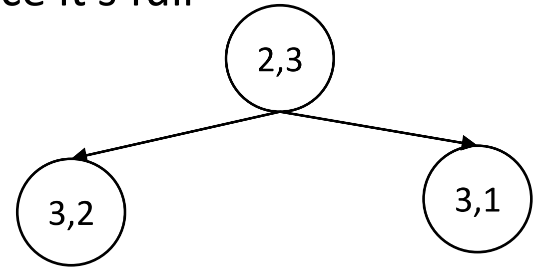

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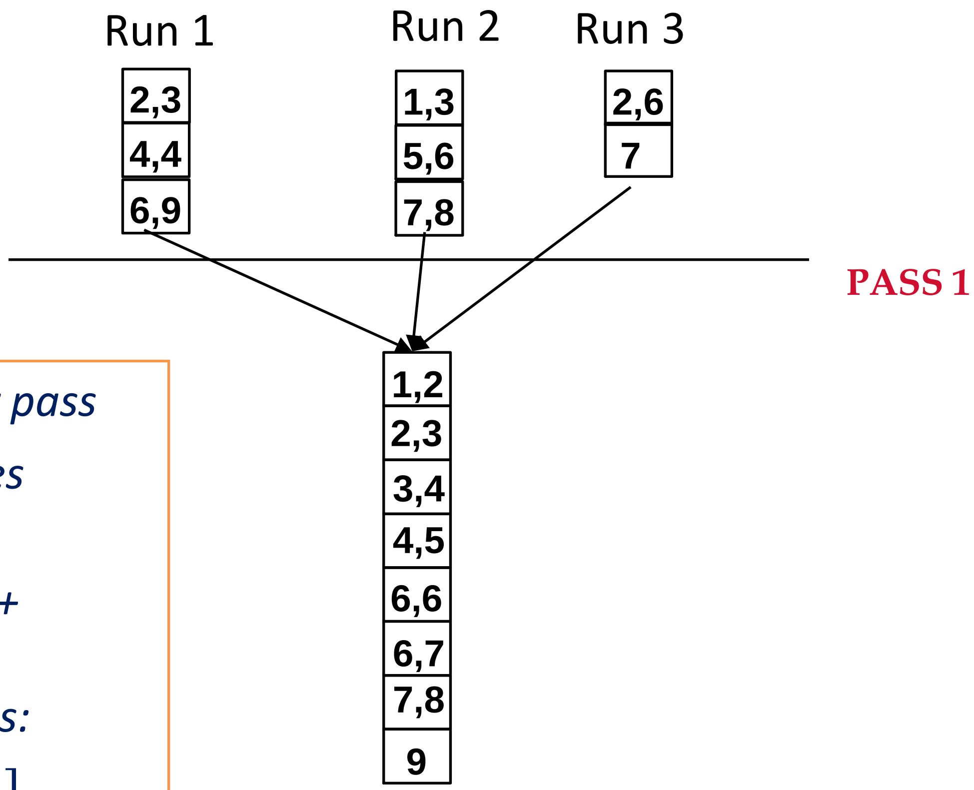
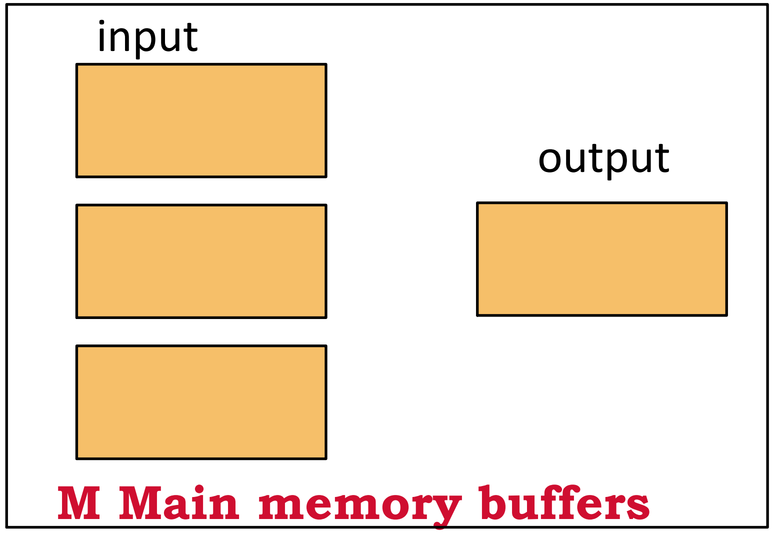




N pages to read/write per pass
$\left\lceil log_{M-1} \left\lceil \frac{N}{M} \right\rceil \right\rceil$ merge passes
Cost per merge pass:
$2N$ pages read/written +
$2N$ seeks
Total cost for merge passes:
$2(t_T + t_S)N\lceil \log_{M-1} \lceil \frac{N}{M} \rceil \rceil$

Cost analysis

- Cost analysis:
  - Pass 0: $2Nt_T + 2\left\lceil\frac{N}{M}\right\rceil t_S$
  - Pass 1, 2, … combined: $2(t_T + t_S)N\lceil\log_{M-1}\lceil\frac{N}{M}\rceil\rceil$
  - Total = $2t_T N\left(\left\lceil log_{M-1}\left\lceil\frac{N}{M}\right\rceil\right\rceil + 1\right) + 2t_S\left(\left\lceil\frac{N}{M}\right\rceil + N\lceil log_{M-1}\lceil\frac{N}{M}\rceil\rceil\right)$

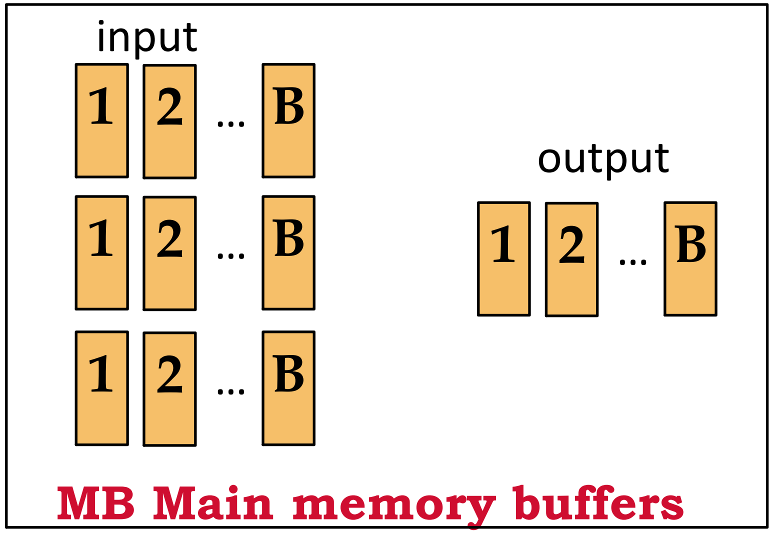
- *gain of utilizing all available buffers*
- *importance of a high fan-in during merging*

N	M=3	=5	=9	=17	=129	=257
100	7	4	3	2	1	1
1,000	10	5	4	3	2	2
10,000	13	7	5	4	2	2
100,000	17	9	6	5	3	3
1,000,000	20	10	7	5	3	3
10,000,000	23	12	8	6	4	3
100,000,000	26	14	9	7	4	4
1,000,000,000	30	15	10	8	5	4

  - Can we do it better?

Batching I/Os for merge sort

- Refinement 1
  - reducing random I/Os by reading/writing B pages per run during merge
  - using $(M-1)$-way merge sort
    - memory usage increases to MB pages
    - number of pages transferred do not change
    - but the number of random seeks per merge pass reduced to approximately $2\lceil\frac{N}{B}\rceil$
  - total cost reduced to $2t_T N\left(\left\lceil log_{M-1}\left\lceil\frac{N}{MB}\right\rceil\right\rceil + 1\right) + 2t_S\left(\left\lceil\frac{N}{MB}\right\rceil + \lceil\frac{N}{B}\rceil\lceil log_{M-1}\lceil\frac{N}{MB}\rceil\rceil\right)$

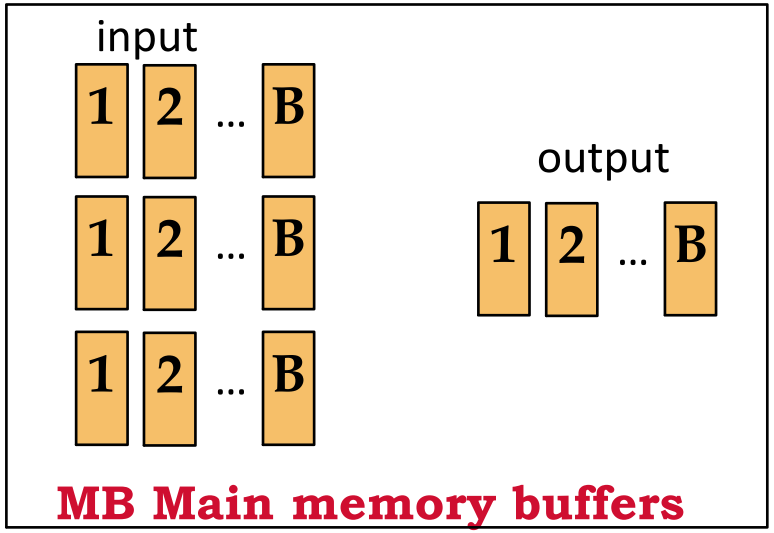


Exercise: what if we only have M pages instead of MB pages and still read/write pages in B-page batches?

$$2t_T N\left(\left\lceil log_{\lfloor\frac{M}{B}\rfloor-1}\left\lceil\frac{N}{M}\right\rceil\right\rceil + 1\right) + 2t_S\left(\left\lceil\frac{N}{M}\right\rceil + \lceil\frac{N}{B}\rceil\lceil log_{\lfloor\frac{M}{B}\rfloor-1}\lceil\frac{N}{M}\rceil\rceil\right)$$

Pipelining output

- Refinement 2
  - in most cases, do not need to write the final file
    - pipelining to the next operator
    - or output to user
  - Hence, no need to count the write of the final pass
  - total cost reduced to $t_T N \left(2 \left\lceil log_{\lfloor \frac{M}{B} \rfloor - 1} \left\lceil \frac{N}{M} \right\rceil \right\rceil + 1\right) + t_S \left(2 \left\lceil \frac{N}{M} \right\rceil + \lceil \frac{N}{B} \rceil (2 \lceil log_{\lfloor \frac{M}{B} \rfloor - 1} \lceil \frac{N}{M} \rceil \rceil - 1)\right)$



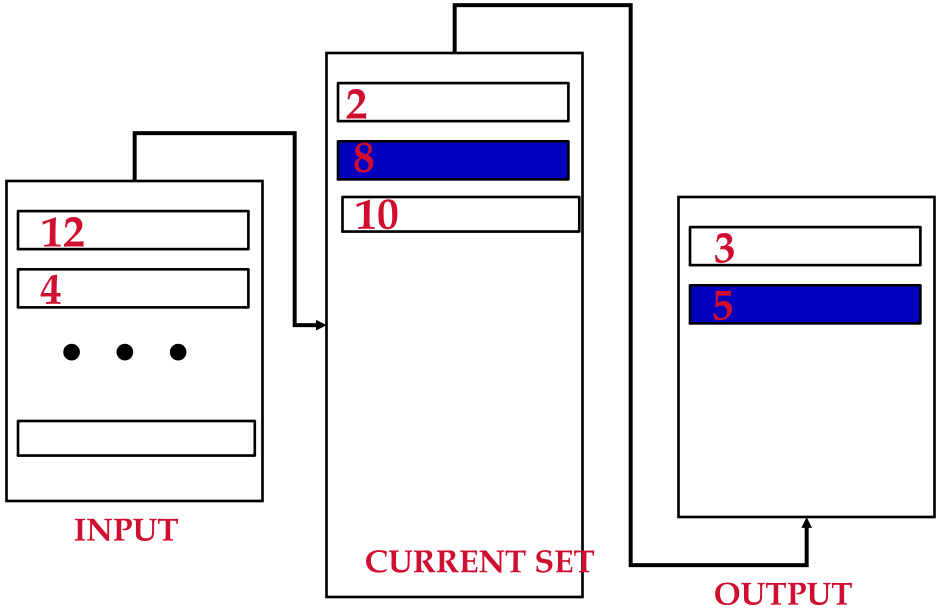
Tournament sort

- Refinement 3
  - producing initial runs as large as possible in pass 0
  - Alternative to quick-sort: “tournament sort” (a.k.a. “heapsort”, “replacement selection”)
- Keep two heaps in memory, H1 and H2, reserve an input buffer page and an output buffer page

```
read M-2 pages of records, inserting into H1;
while (records left) {
    m = H1.removemin();  put m in output buffer;
    if (H1 is empty)
        swap H1 and H2 (pointer swap only!); start new output run;
    else
        read in a new record r (use 1 buffer for input pages);
        if (r < m)  H2.insert(r);
        else        H1.insert(r);
}
H1.output();  start new run;  H2.output();
```

Tournament sort

- Tournament sort explained:



- 1 input, 1 output, M - 2 for current and next set (min heaps)
- Main idea: ensure the *smallest* key in the current set (H1) is *greater* than any key that has been written to this output run.
  - If it can't be satisfied, write to the next set (H2), which goes into the next run.
- Memory usage of the min-heaps combined never exceeds the M-2 pages

Tournament sort

- Fact: average length of a run is *2(M-2)*
- Total cost reduced to on average

$$t_T N\left(2\left\lceil log_{\lfloor\frac{M}{B}\rfloor-1}\left\lceil\frac{N}{2M-4}\right\rceil\right\rceil+1\right)+t_S\left(2\left\lceil\frac{N}{2M-4}\right\rceil+\lceil\frac{N}{B}\rceil(2\lceil log_{\lfloor\frac{M}{B}\rfloor-1}\lceil\frac{N}{2M-4}\rceil\rceil-1)\right)$$

- Worst-Case:
  - What is min length of a run?
  - How does this arise?
- Best-Case:
  - What is max length of a run?
  - How does this arise?
- Quicksort is faster, but … longer runs often means fewer passes!

Using B+ Trees for Sorting

- Scenario: Table to be sorted has B+ tree index on sorting column(s).
- Idea: Can retrieve records in order by traversing leaf pages.
- *Is this a good idea?*
- Cases to consider:
  - B+ tree is clustered — ***Good idea since it's already available!***
  - B+ tree is not clustered — ***Could be a very bad idea! (Random I/O) unless all columns are included in the key***

Certain basic operator implementation w/ sorting

- Some basic operators can be implemented on top of sorting
  - Can use pipelining over the sort results
- Examples
  - deduplication (projection in standard RA)
    - maintain the last key
    - for each output from the sort
      - emit it if it is different from the last key
      - otherwise, discard it
  - aggregation
    - maintain the aggregation state
    - for each output from the sort
      - emit the finalized aggregation value if it is different from the last key (unless this is the first)
      - otherwise, accumulate it to the state
  - exercise: work out the details of $\cup, \cap, -$
- No additional I/O due to pipelining
  - can support rewinding (why?)

This lecture

- Summary:
  - External sorting (multi-way merge-sort)
  - Certain operator implementation using sorting
- Next lecture
  - Join algorithms
    - nested loop
    - index nested loop
    - sort-merge join
    - hash join and hybrid hashing
- Homework assignment 5 released today
  - Covers topics in QP & QO
  - Solution will be released on Apr 27