Statistical Privacy For Privacy Preserving Information Sharing

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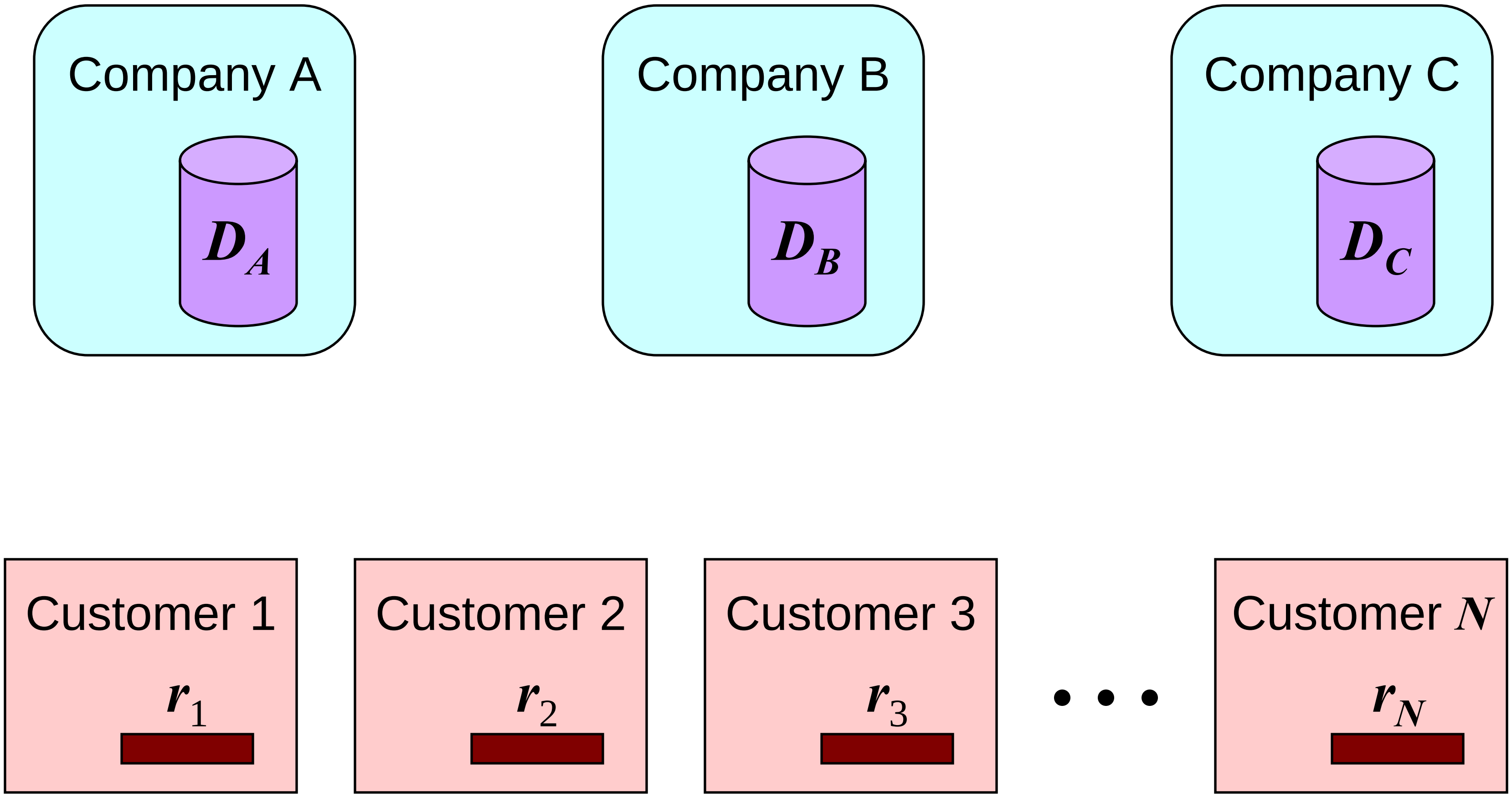
Introduction

- To operate, an e-business needs to query data owned by clients or other businesses
- The owners are concerned about privacy of their data, they will not ship all data to one server
- We want algorithms that efficiently evaluate multi-party queries while disclosing as little extra information as possible.

Introduction



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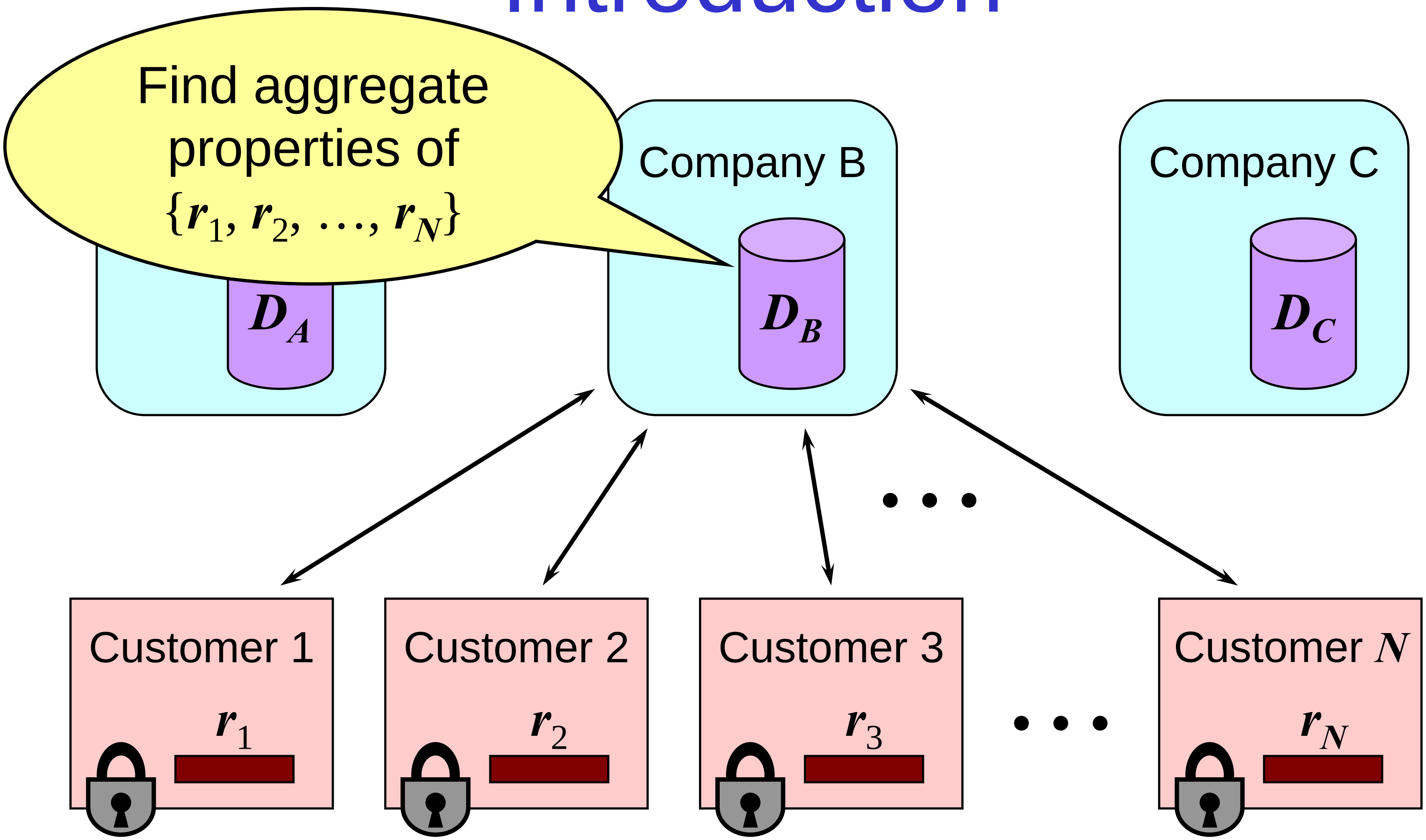

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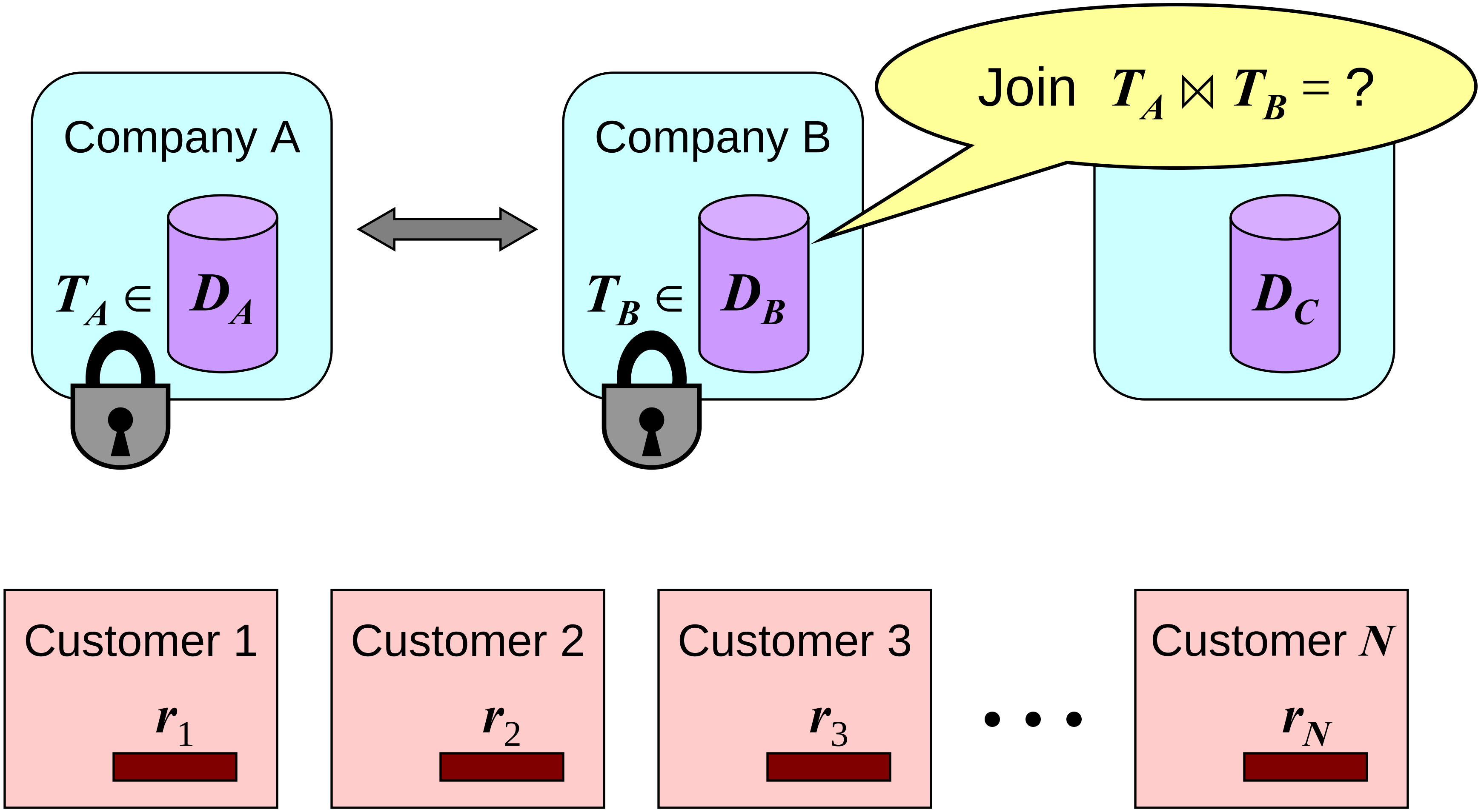
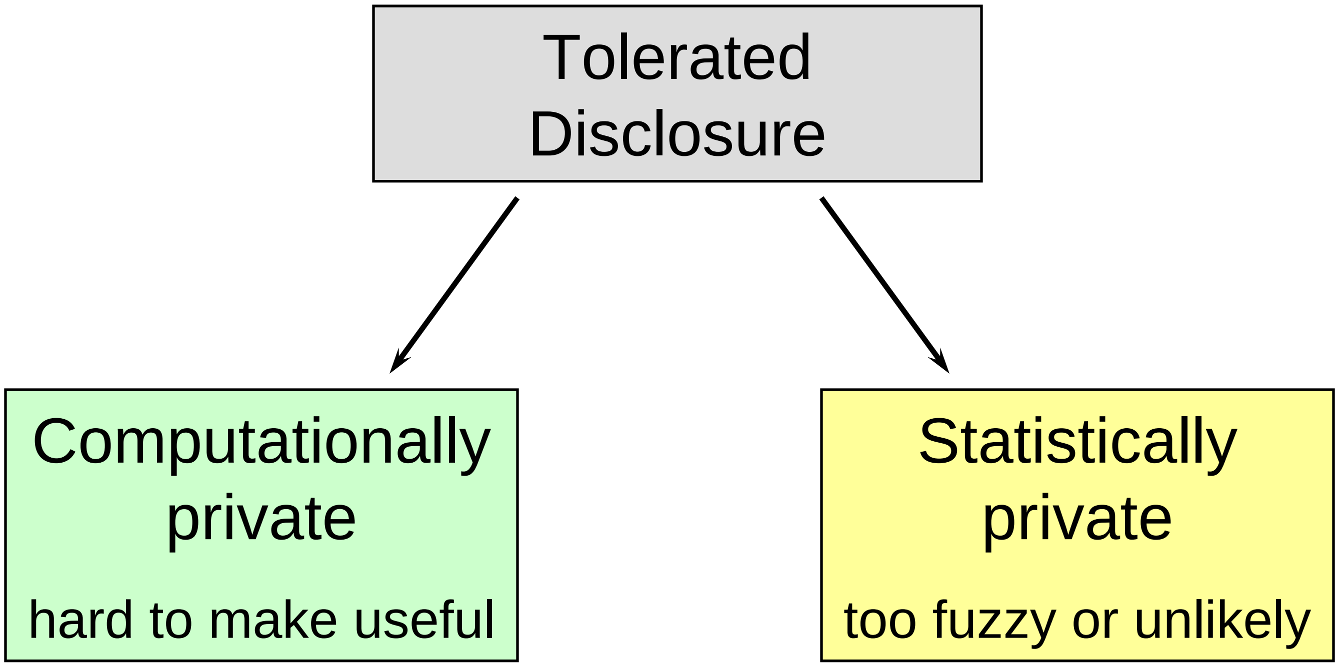


Privacy and Disclosure



Privacy and Disclosure

Tolerated Disclosure

Most studied

Computationally private

hard to make useful

Statistically private

too fuzzy or unlikely

- Yao [1986]: Any two-party data operation can be made computationally private, if the operation is converted into a Boolean circuit.

Privacy and Disclosure

Tolerated Disclosure

Today's Talk

Computationally private

hard to make useful

Statistically private

too fuzzy or unlikely

Talk Outline

- Introduction
- Randomization and privacy for association rules
- Amplification: upper bound on breaches
- Experimental results
- Conclusion

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Privacy Preserving Associations

- We have one server and many clients
- Each client has a private transaction (a set of items)
  - Example: product preferences
- The server wants to find frequent subsets of items
  - (aggregate statistical information)
- Each client wants to hide its transaction from the server

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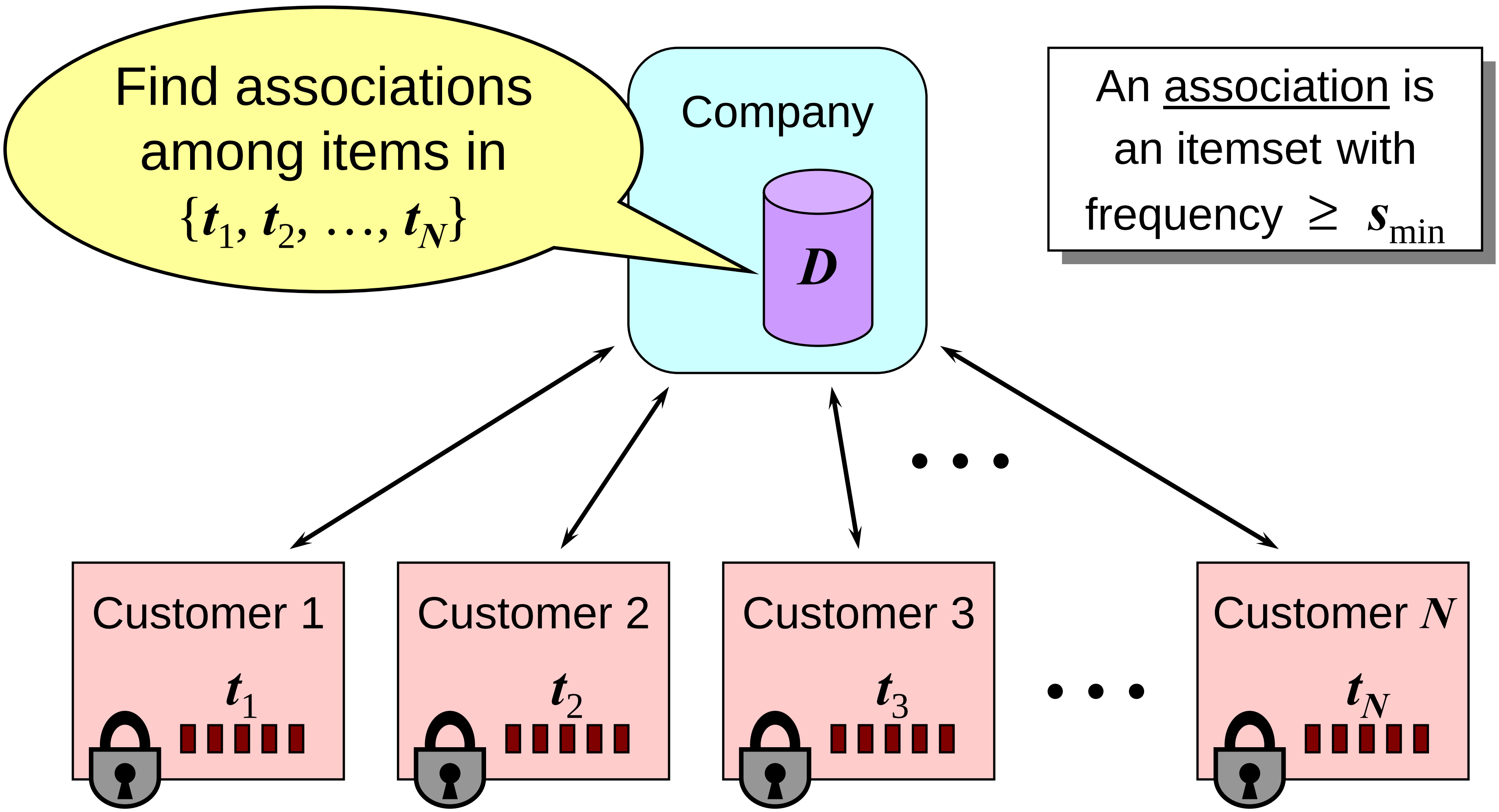
Privacy Preserving Associations



Privacy Preserving Associations

- Let $\boldsymbol{T}$ be the set of all transactions, and $\boldsymbol{t} \in \boldsymbol{T}$ be a transaction
- Any itemset $\boldsymbol{A}$ has support (frequency) $\boldsymbol{s}$ in $\boldsymbol{T}$ if

$$s = \text{supp}\,(A) = \frac{\#\{t \in T \mid A \subseteq t\}}{|T|}$$

- Itemset $\boldsymbol{A}$ is frequent if $\boldsymbol{s} \geq \boldsymbol{s}_{\min}$
- Antimonotonicity: if $\boldsymbol{A} \subseteq \boldsymbol{B}$, then $\text{supp}\,(\boldsymbol{A}) \geq \text{supp}\,(\boldsymbol{B})$.
- Association rule: $\boldsymbol{A} \Rightarrow \boldsymbol{B}$ holds when the union $\boldsymbol{A} \cup \boldsymbol{B}$ is frequent and: $\text{supp}\,(\boldsymbol{A} \cup \boldsymbol{B}) \geq \text{supp}\,(\boldsymbol{A}) \cdot \boldsymbol{conf}_{\min}$

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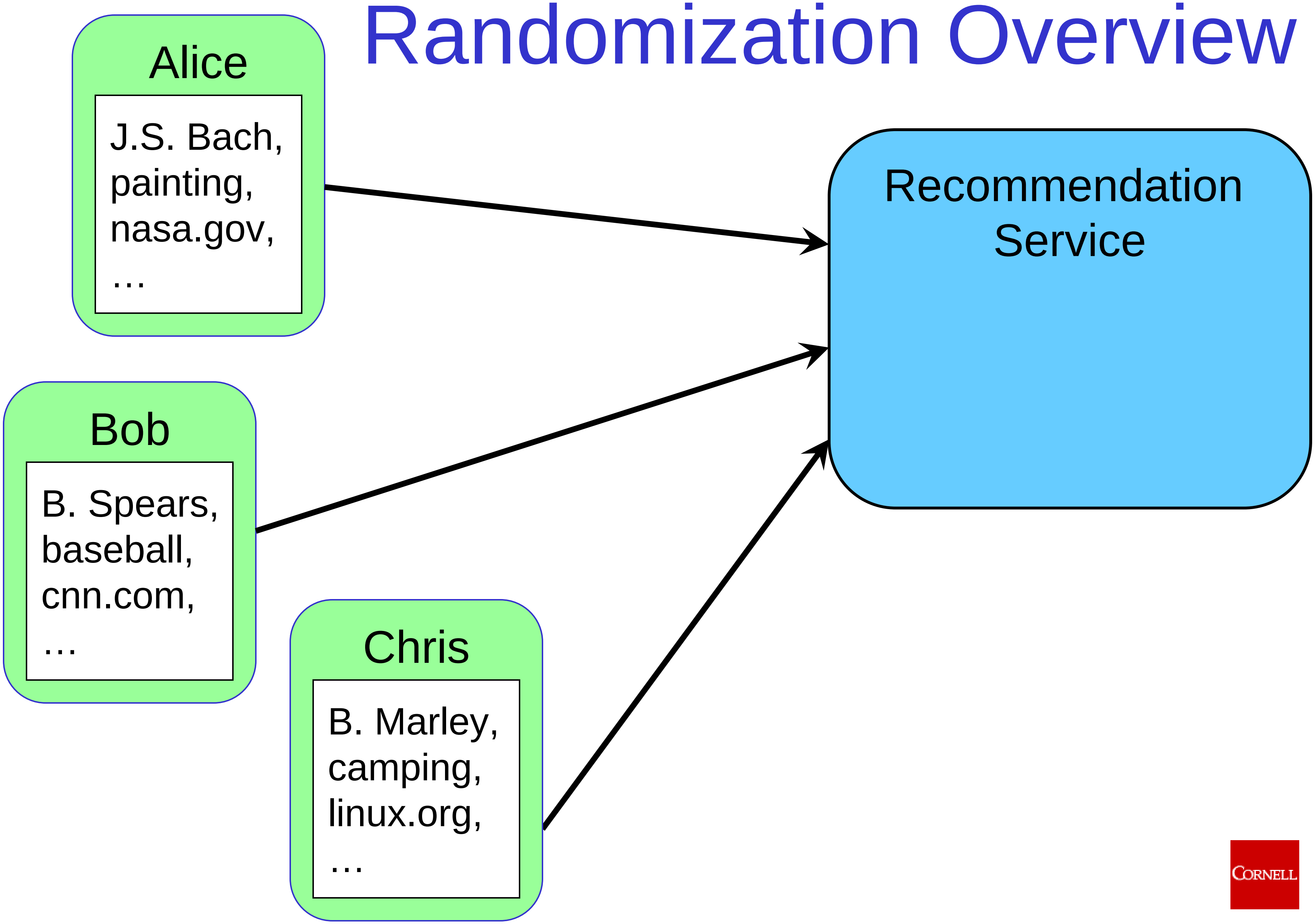
Randomization Overview



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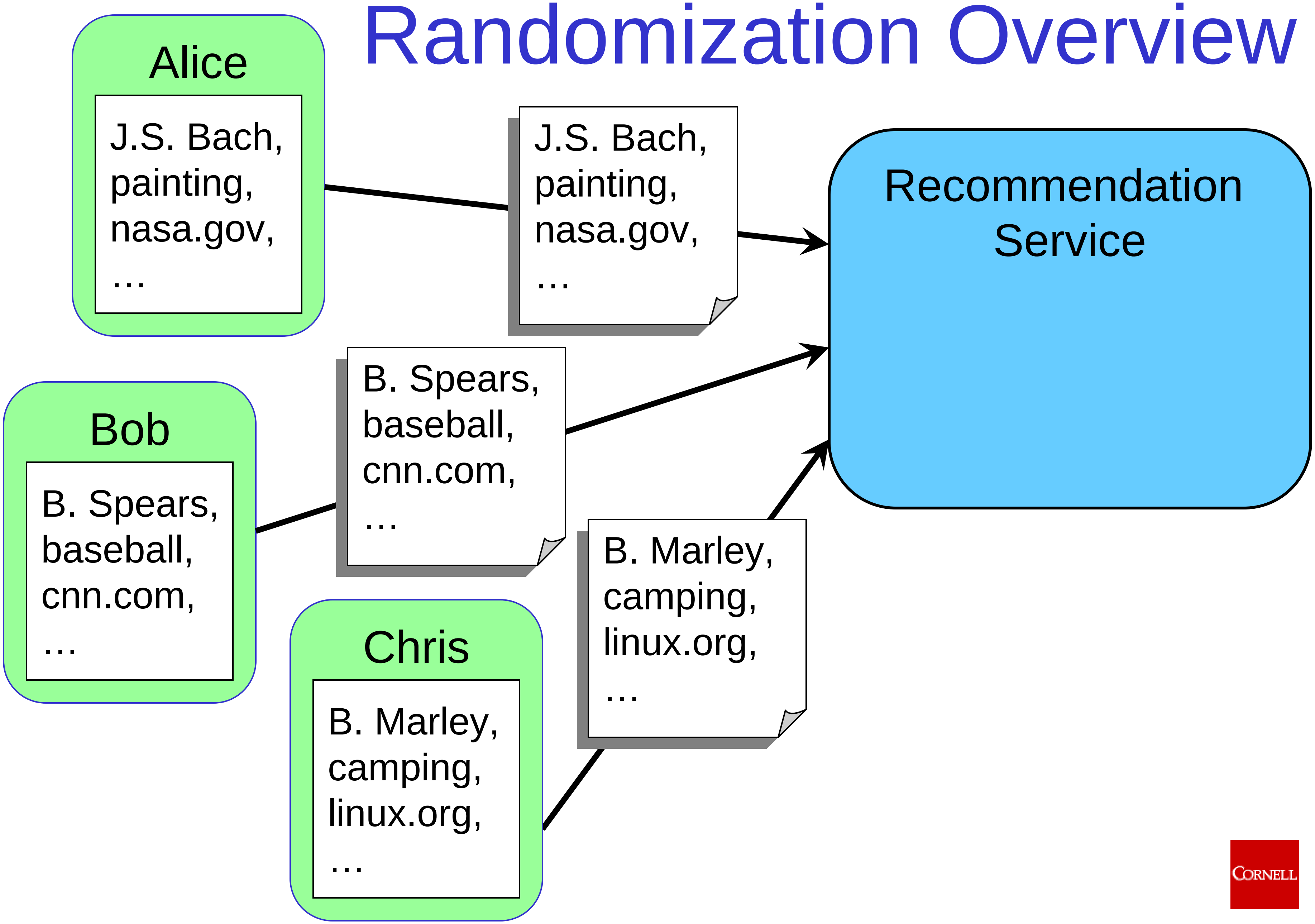

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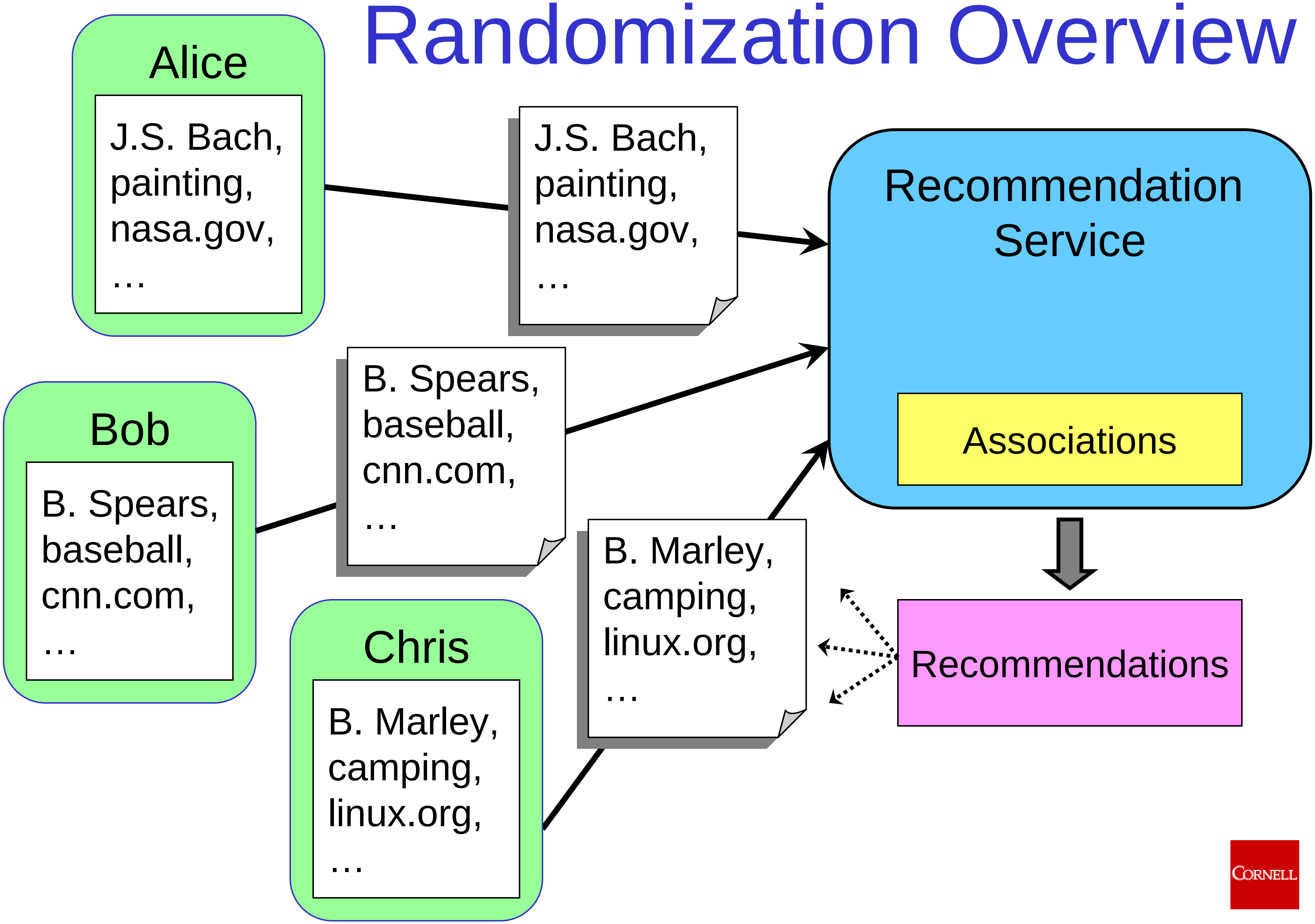

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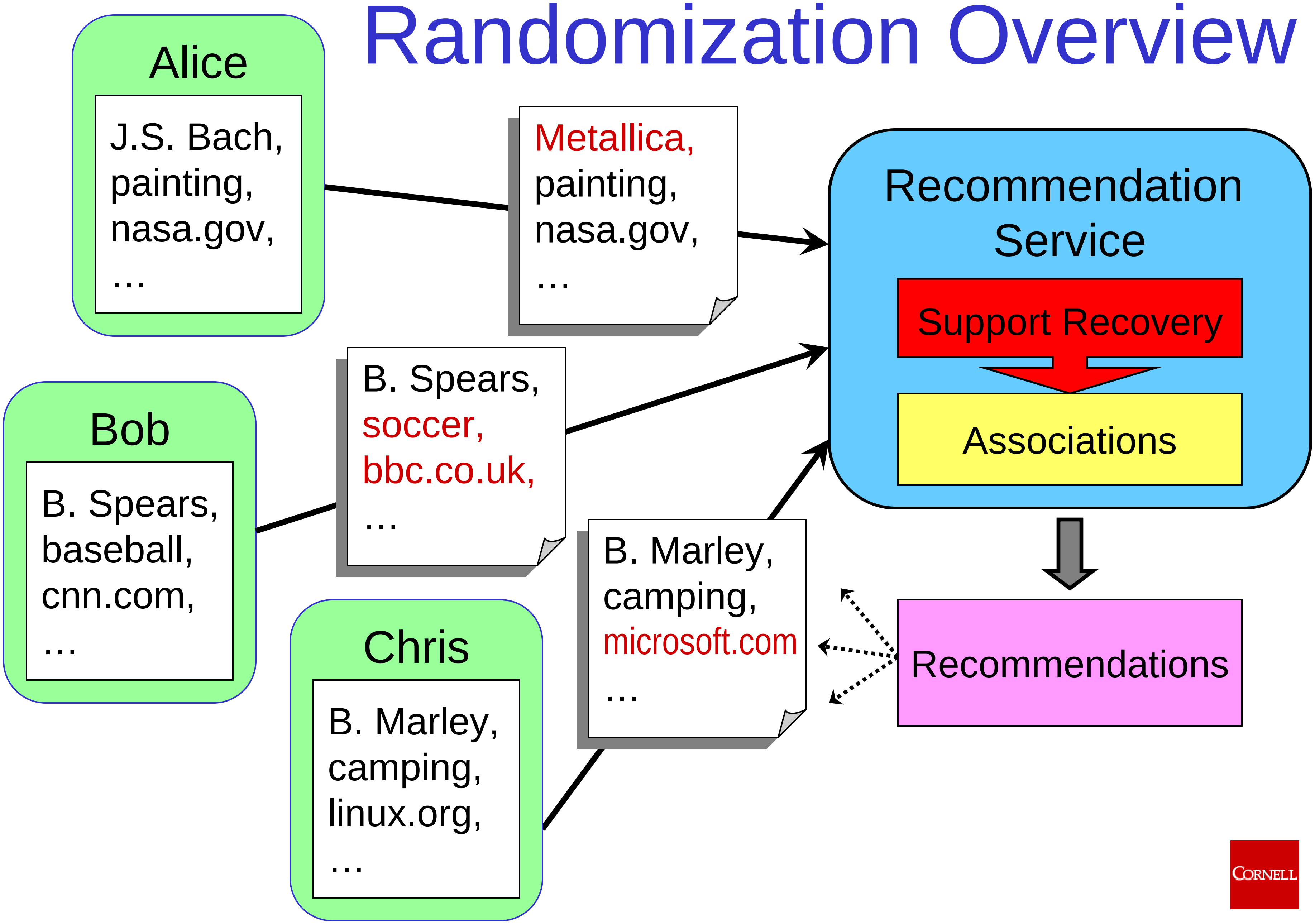


The Problem

- How to randomize transactions so that
  - we can find frequent itemsets
  - while preserving privacy at transaction level?

Randomization Example

A randomization may "look strong" but sometimes fail to hide some items of an individual transaction.

- Randomization example: given a transaction,
  - keep item with 20% probability,
  - replace with a new random item with 80% probability.

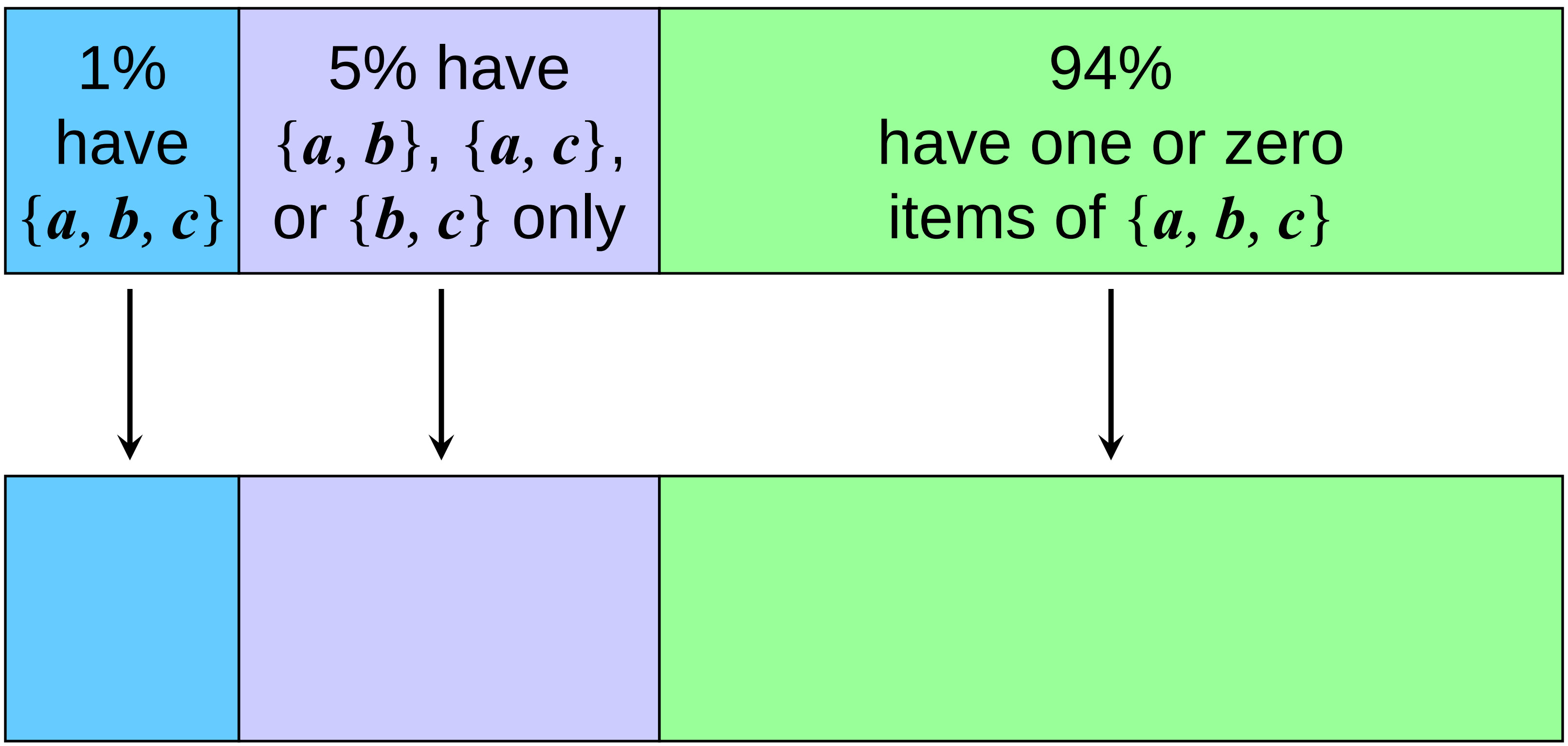
Example: $\{a, b, c\}$

10 M transactions of size 10 with 10 K items:

1% have $\{a, b, c\}$	5% have $\{a, b\}$, $\{a, c\}$, or $\{b, c\}$ only	94% have one or zero items of $\{a, b, c\}$

Example: $\{a, b, c\}$

10 M transactions of size 10 with 10 K items:



After randomization: How many have $\{a, b, c\}$?

Example: $\{a, b, c\}$

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$\downarrow \bullet\, 0.2^3$	$\downarrow \bullet\, 0.2^2 \bullet 8 \bullet 0.8/10{,}000$	$\downarrow$ at most $\bullet\, 0.2 \bullet (9 \bullet 0.8/10{,}000)^2$
0.008% 800 ts.	0.000128% 13 trans.	less than 0.00002% 2 transactions

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0.008% 800 ts. **98.2%**	0.000128% 13 trans. **1.6%**	less than 0.00002% 2 transactions **0.2%**

After randomization: How many have $\{a, b, c\}$?

Example: $\{a, b, c\}$

- Given nothing, we have only 1% probability that $\{a, b, c\}$ occurs in the original transaction
- Given $\{a, b, c\}$ in the randomized transaction, we have about 98% certainty of $\{a, b, c\}$ in the original one.
- This is what we call a privacy breach.
- The example randomization preserves privacy "on average," but not "in the worst case."

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Privacy Breaches

- Suppose the "adversary" wants to know if $z \in t$, where
  - t is an original transaction;
  - t' is the corresponding randomized transaction;
  - A is a (frequent) itemset, $z \in A$
- Itemset A causes a privacy breach of level β (e.g. 50%) if:

$$\boxed{\text{Prob}\left[z \in t \mid A \subseteq t'\right] \geq \beta}$$

Knowledge of $A \subseteq t'$ makes a jump from $\text{Prob}\,[z \in t]$ to $\text{Prob}\,[z \in t \mid A \subseteq t']$ (in the adversary's viewpoint).

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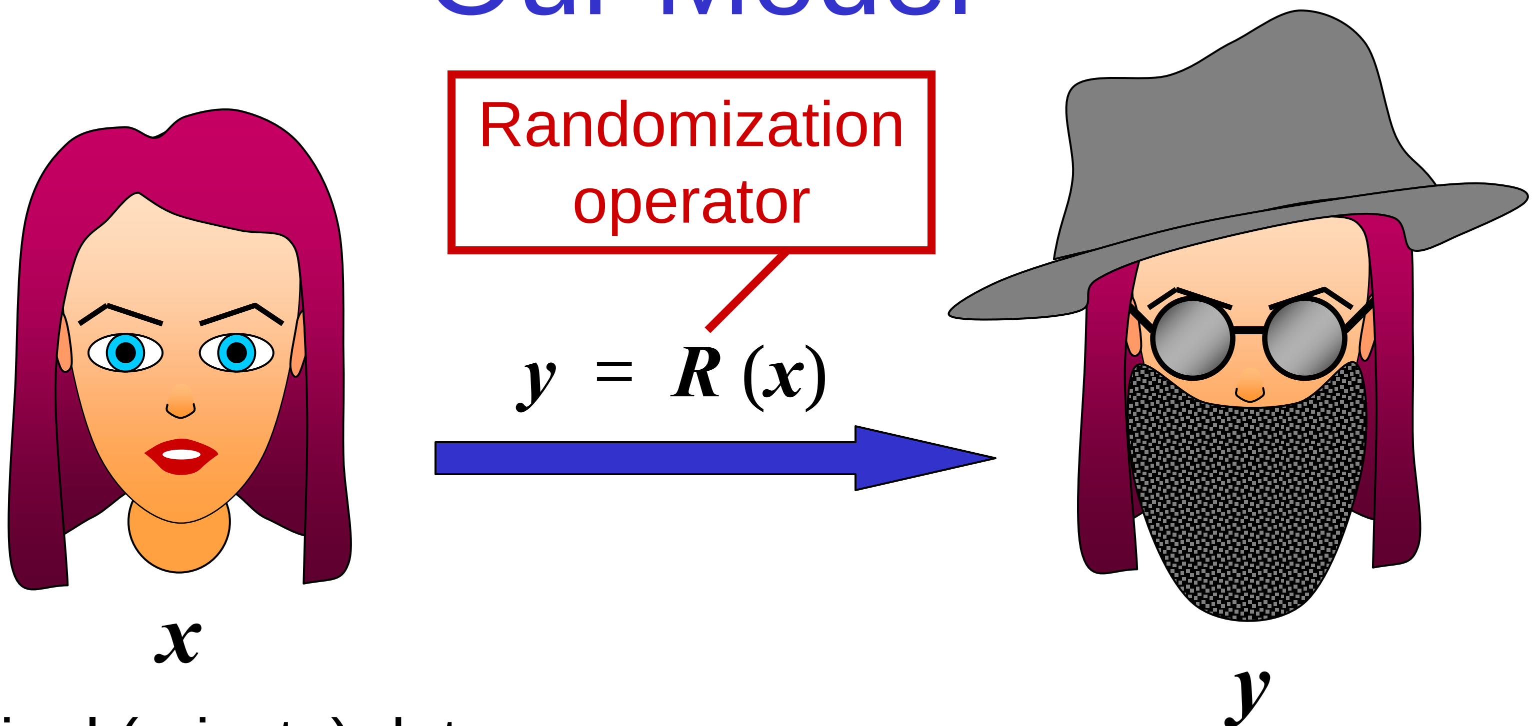
Generalized Approach

- We want a bound for all privacy breaches
  - not only for: item $\in t$ versus itemset $\subseteq t'$
- No knowledge of data distribution is required in advance
  - We don't have to know $\mathrm{Prob}\,[\text{item} \in t]$
- Applicable to numerical data as well
- Easy to work with, even for complex randomizations

Our Model



Original (private) data

Assumptions:

- Described by a random variable X.
- Each client is independent.

Randomized data

Described by a random variable $Y = R(X)$.

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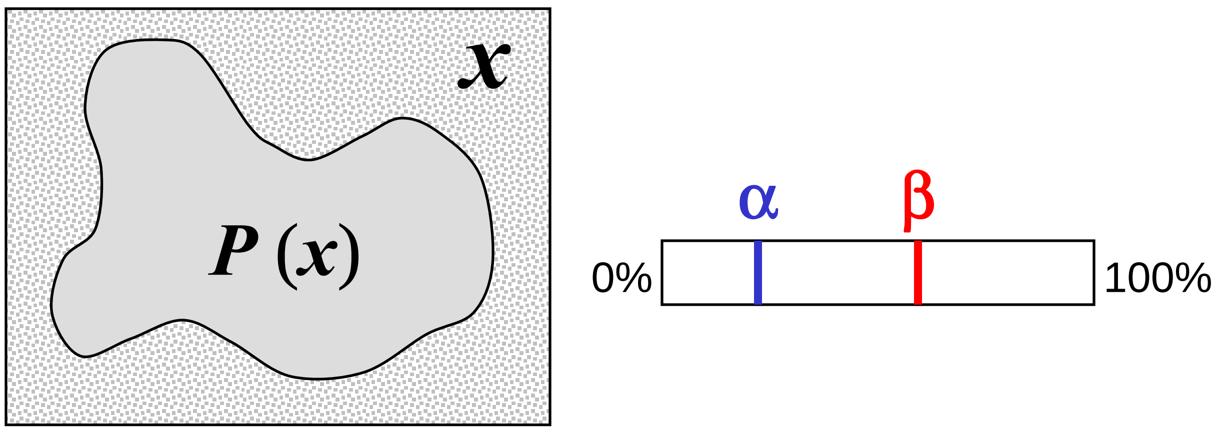
α-to-β Privacy Breach

Let $P(x)$ be any property of client's private data;

Let $0 < \alpha < \beta < 1$ be two probability thresholds.



Example:

$P(x)$ = "transaction x contains $\{a, b, c\}$"

α = 1% and β = 50%

α-to-β Privacy Breach

Let $\boldsymbol{P}(\boldsymbol{x})$ be any property of client's private data;

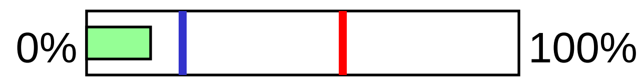
Let $0 < \alpha < \beta < 1$ be two probability thresholds.

Client

$$\boldsymbol{X} = \boldsymbol{x}$$

SERVER

$$\text{Prob}\,[\boldsymbol{P}(\boldsymbol{X})] \le \alpha$$



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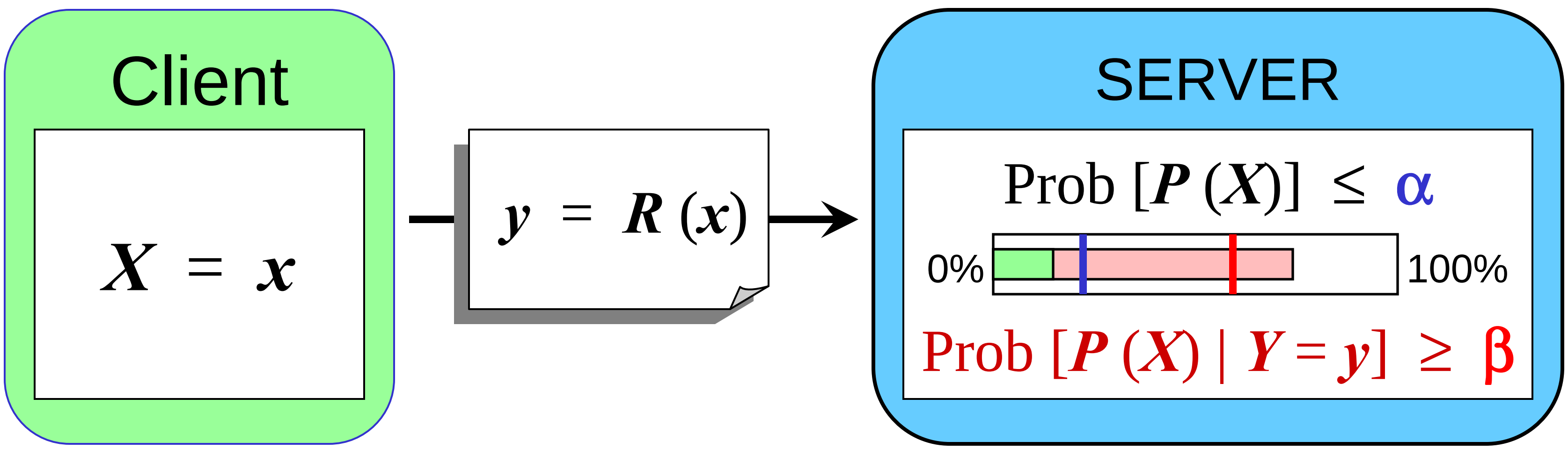

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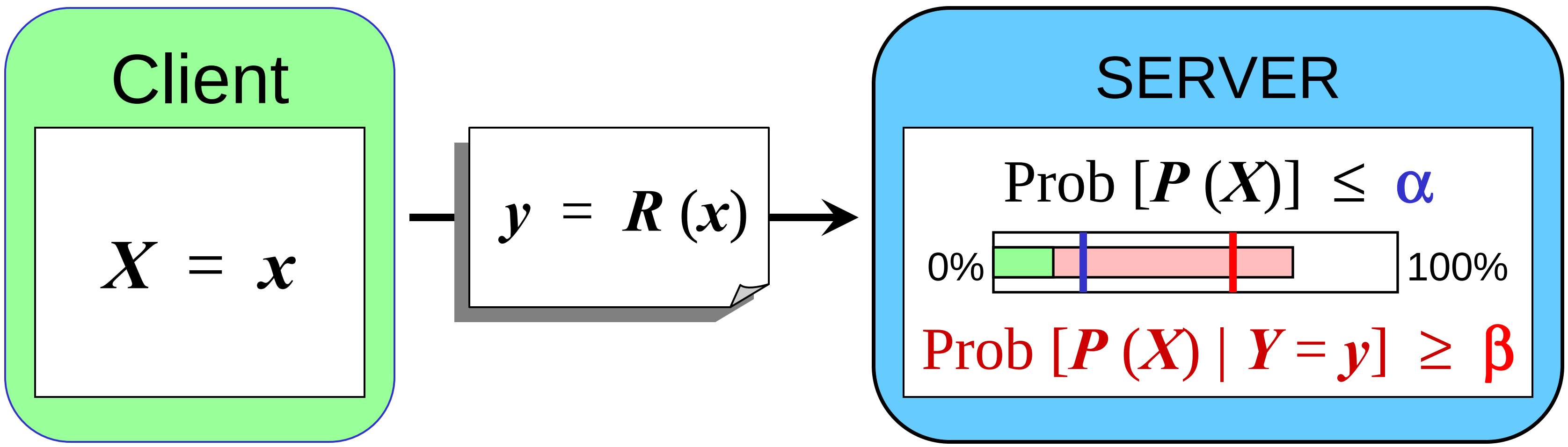

Disclosure of y causes an α-to-β privacy breach w.r.t. property $P(x)$.

α-to-β Privacy Breach

Checking for α-to-β privacy breaches:

- There are exponentially many properties $\boldsymbol{P}(\boldsymbol{x})$;
- We have to know the data distribution in order to check whether $\text{Prob}\,[\boldsymbol{P}(\boldsymbol{X})] \leq \alpha$ and $\text{Prob}\,[\boldsymbol{P}(\boldsymbol{X}) \mid \boldsymbol{Y} = \boldsymbol{y}] \geq \beta$.

Is there a simple property of randomization operator $\boldsymbol{R}$ that limits privacy breaches?

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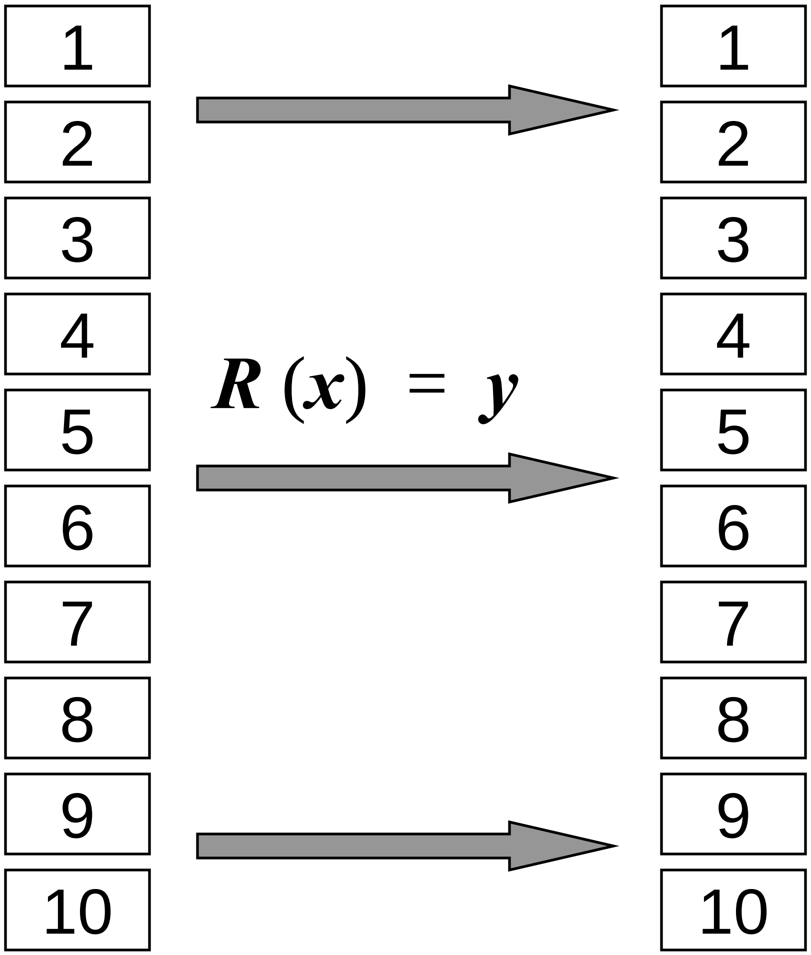
Amplification Condition



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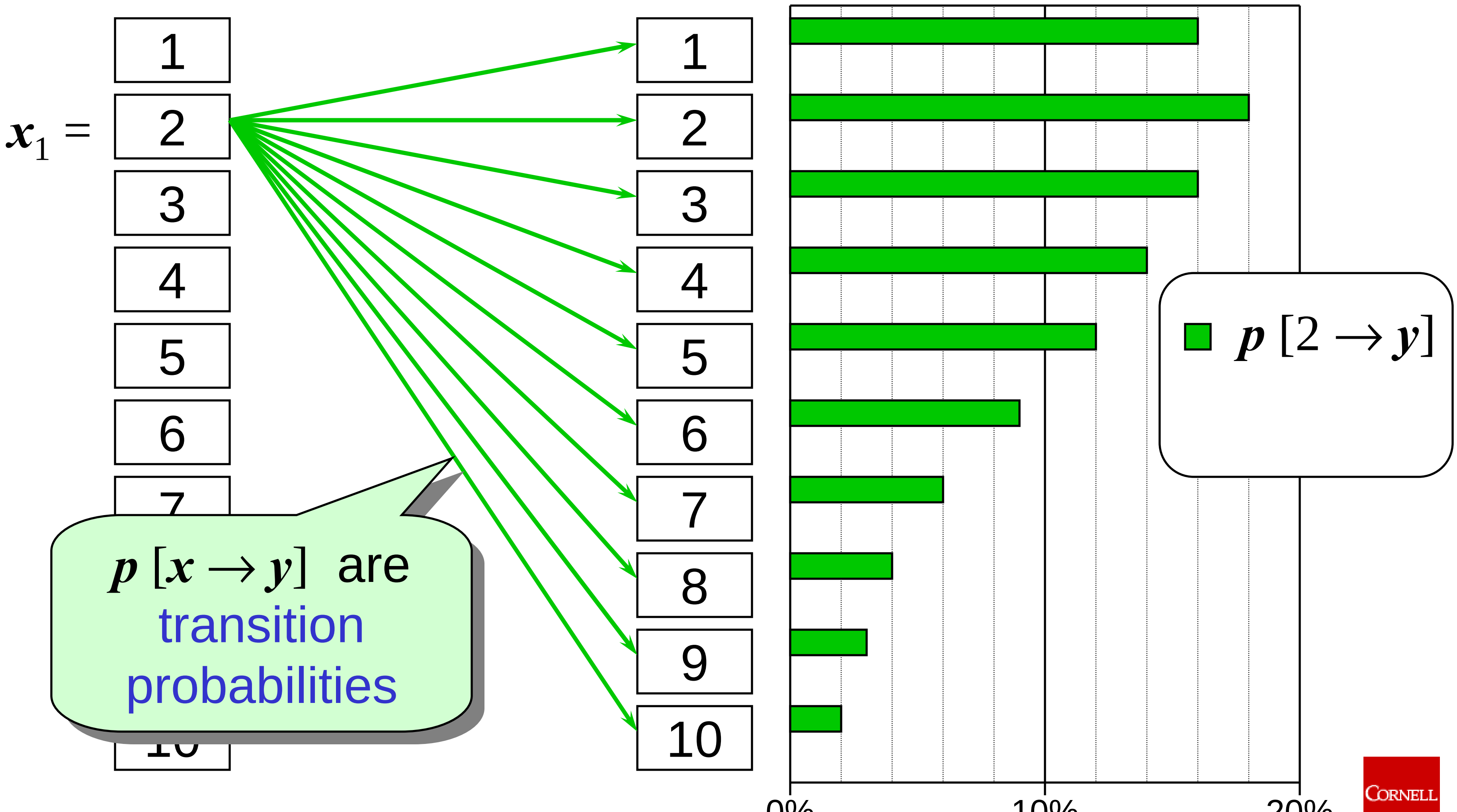

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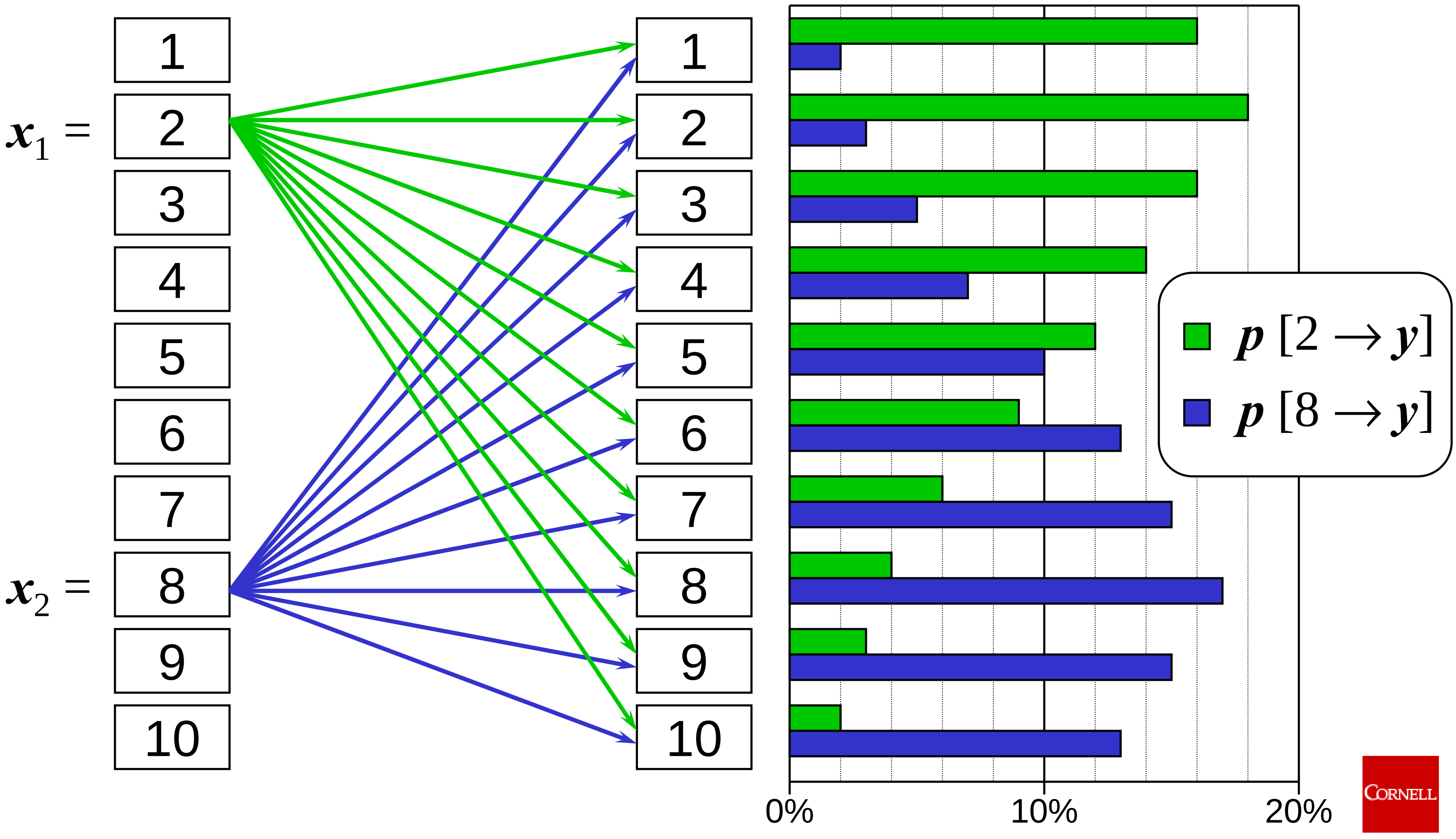

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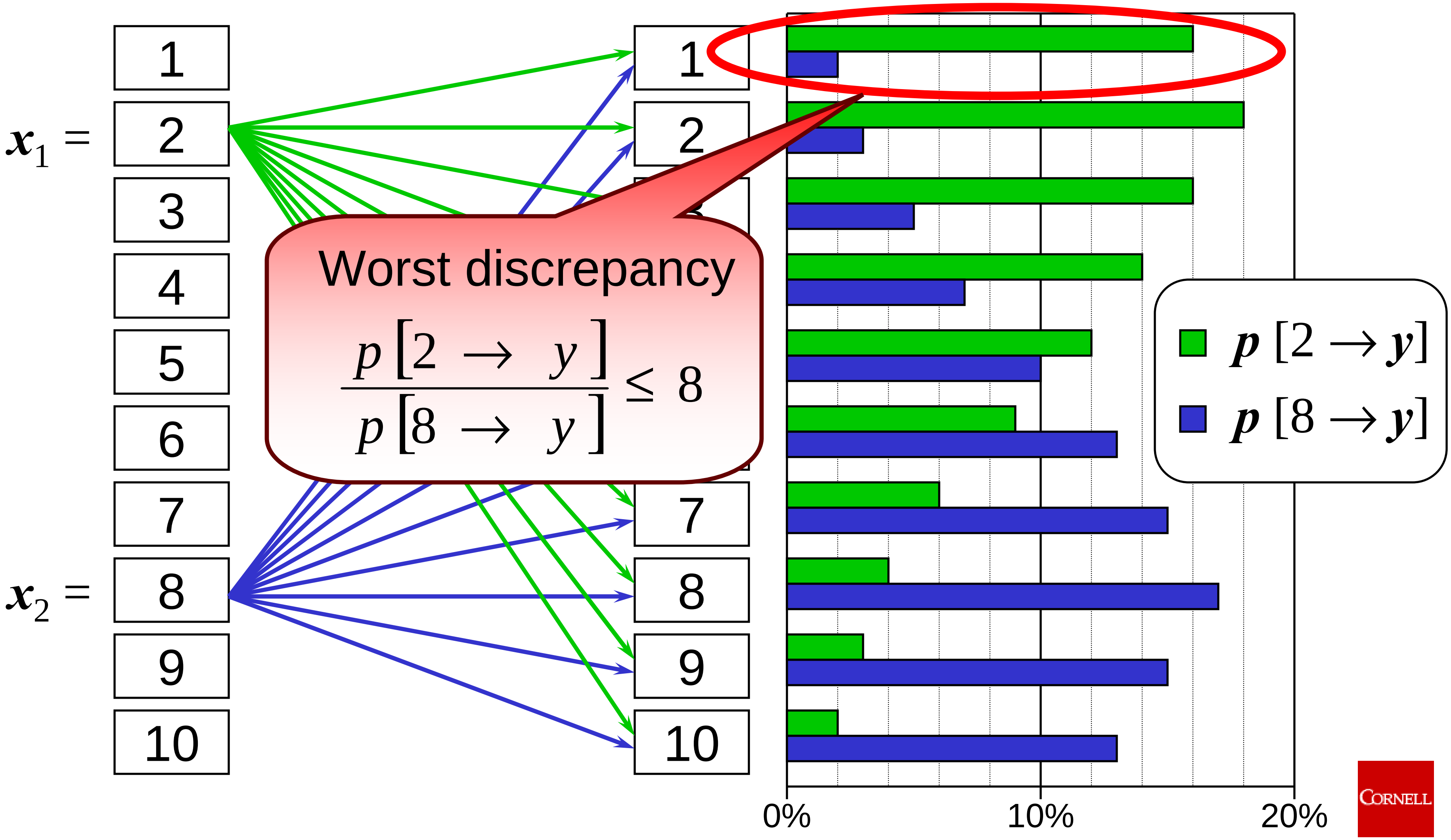

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Amplification Condition

Definition:

- Randomization operator $\boldsymbol{R}$ is called "at most γ-amplifying" if:

$$\max_{x_1, x_2} \max_{y} \frac{p[x_1 \to y]}{p[x_2 \to y]} \leq \gamma$$

  - Transition probabilities $\boldsymbol{p}\ [\boldsymbol{x} \to \boldsymbol{y}] = \text{Prob}\ [\boldsymbol{R}\ (\boldsymbol{x}) = \boldsymbol{y}]$ depend only on the operator $\boldsymbol{R}$ and not on data.
  - We assume that all $\boldsymbol{y}$ have a nonzero probability.
- The bigger γ is, the more may be revealed about $\boldsymbol{x}$.

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The Bound on α-to-β Breaches

Statement:

- If randomization operator $\boldsymbol{R}$ is at most γ-amplifying, and if:

$$\gamma < \frac{\beta}{\alpha} \cdot \frac{1-\alpha}{1-\beta}$$

- Then, revealing $\boldsymbol{R}(\boldsymbol{X})$ to the server will never cause an α-to-β privacy breach.

See proof in [PODS 2003].

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The Bound on α-to-β Breaches

Examples:

- 5%-to-50% privacy breaches do not occur for $\gamma < 19$:

$$\frac{0.5}{0.05} \cdot \frac{1 - 0.05}{1 - 0.5} = 19$$

- 1%-to-98% privacy breaches do not occur for $\gamma < 4851$:

$$\frac{0.98}{0.01} \cdot \frac{1 - 0.01}{1 - 0.98} = 4851$$

- 50%-to-100% privacy breaches do not occur for any finite γ.

Amplification: Summary

- An α-to-β privacy breach w.r.t. property $\boldsymbol{P}(\boldsymbol{x})$ occurs when
  - Prob [$\boldsymbol{P}$ is true] $\leq \alpha$
  - Prob [$\boldsymbol{P}$ is true | $\boldsymbol{Y} = \boldsymbol{y}$] $\geq \beta$.
- Amplification methodology limits privacy breaches by just looking at transitional probabilities of randomization.
  - Does not use data distribution:

$$\max_{x_1, x_2} \max_{y} \frac{p[x_1 \to y]}{p[x_2 \to y]} \leq \gamma$$

Amplification In Practice

- Given transaction t of size m, construct $t' = R(t)$:

$t = \boxed{a, b, c, d, e, f, u, v, w}$

$t' =$

Definition of select-a-size

- Given transaction $\boldsymbol{t}$ of size $\boldsymbol{m}$, construct $\boldsymbol{t'} = \boldsymbol{R}\,(\boldsymbol{t})$:
  - Choose a number $\boldsymbol{j} \in \{0, 1, \ldots, \boldsymbol{m}\}$ with distribution $\{\boldsymbol{p}\,[\boldsymbol{j}]\}_{0..\boldsymbol{m}}$;

$$\boldsymbol{t} = \boxed{a, b, c, d, e, f, u, v, w}$$

$$\boldsymbol{t'} = \underbrace{\boxed{}}_{j\,=\,4}$$

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Definition of select-a-size

- Given transaction $\boldsymbol{t}$ of size $\boldsymbol{m}$, construct $\boldsymbol{t'} = \boldsymbol{R}(\boldsymbol{t})$:
  - Choose a number $j \in \{0, 1, \ldots, m\}$ with distribution $\{p[j]\}_{0..m}$;
  - Include exactly j items of t into t';

$$t = \boxed{a, \color{red}{b}, c, d, \color{red}{e}, f, \color{red}{u}, v, \color{red}{w}}$$

$$t' = \underbrace{\boxed{\color{red}{b}, \color{red}{e}, \color{red}{u}, \color{red}{w}}}_{j\,=\,4}$$

Definition of select-a-size

- Given transaction $\boldsymbol{t}$ of size $\boldsymbol{m}$, construct $\boldsymbol{t'} = \boldsymbol{R}(\boldsymbol{t})$:
  - Choose a number $\boldsymbol{j} \in \{0, 1, \ldots, \boldsymbol{m}\}$ with distribution $\{\boldsymbol{p}[\boldsymbol{j}]\}_{0..\boldsymbol{m}}$;
  - Include exactly $\boldsymbol{j}$ items of $\boldsymbol{t}$ into $\boldsymbol{t'}$;
  - Each other item (not from $\boldsymbol{t}$) goes into $\boldsymbol{t'}$ with probability ρ.

The choice of $\{\boldsymbol{p}[\boldsymbol{j}]\}_{0..\boldsymbol{m}}$ and ρ is based on the desired privacy level.

$\boldsymbol{t}$ = ***a, b, c, d, e, f, u, v, w***

$\boldsymbol{t'}$ = ***b, e, u, w*** | ***œ, å, ß, §, ψ, €, ℵ, ъ, ђ, …***

$\boldsymbol{j} = 4$ ⟷ items inserted with prob. ρ

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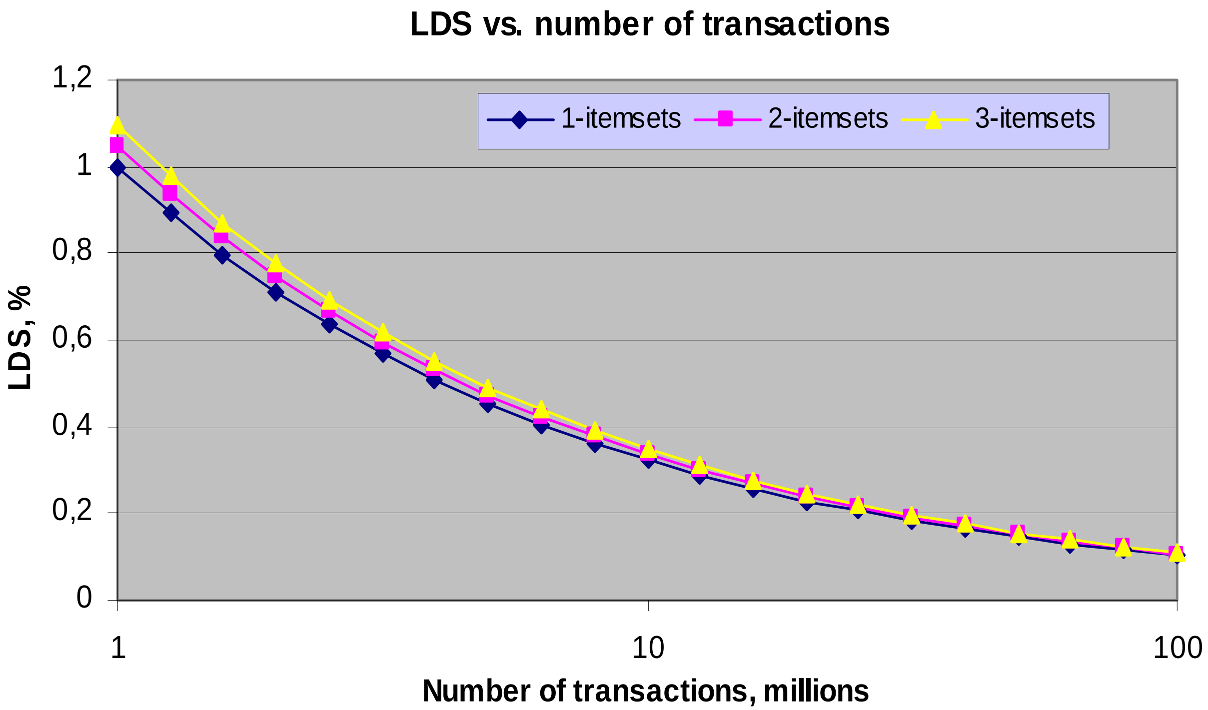
Lowest Discoverable Support

- LDS is s.t., when predicted, is 4σ away from zero.
- Roughly, LDS is proportional to $1 \big/ \sqrt{\#\ \text{trans.}}$

$|\, t \,| = 5$, 5%-50% privacy breaches are the worst allowed



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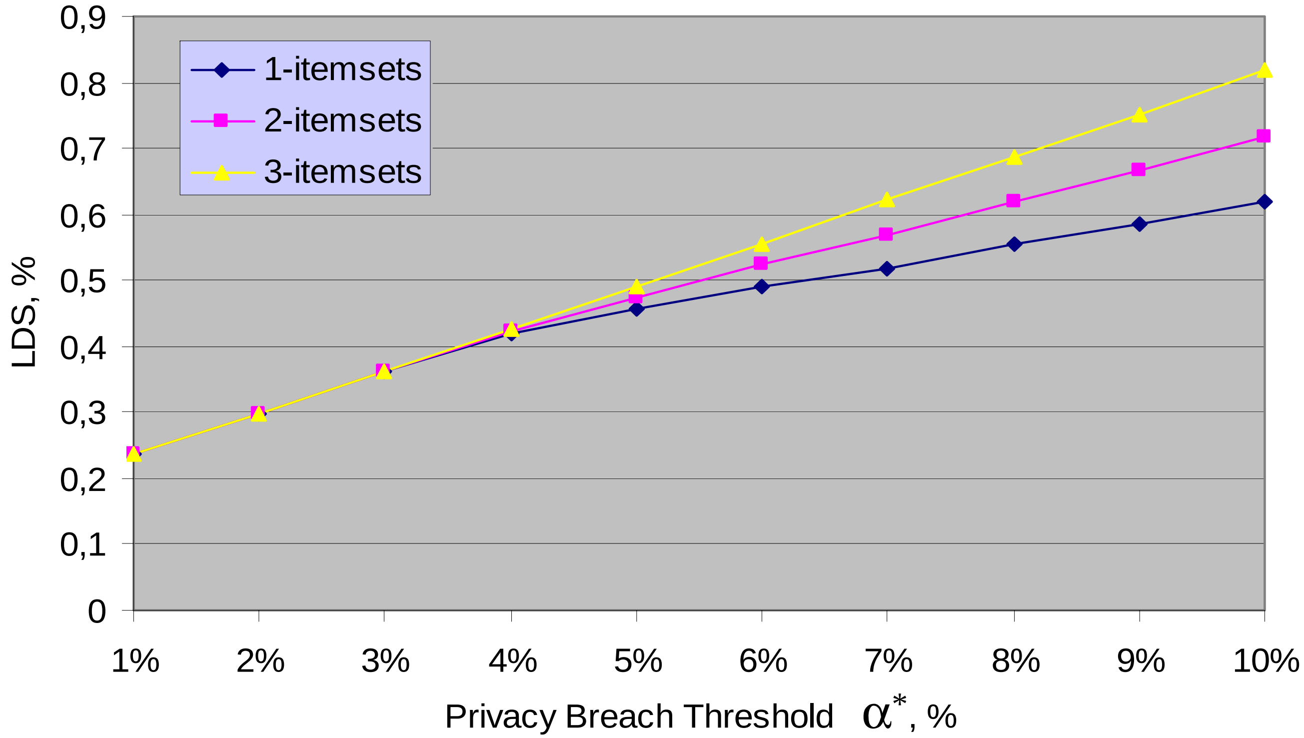
LDS vs. Breach Threshold α^*

$|\,\boldsymbol{t}\,| = 5$, 5 M trans., α-to-50% privacy breaches are limited to $\alpha \geq \alpha^*$



- Reminder: α-to-50% breach occurs when $\text{Prob}\,[\boldsymbol{P}\,(\boldsymbol{t})] \leq \alpha$ and $\text{Prob}\,[\boldsymbol{P}\,(\boldsymbol{t})\,|\,\boldsymbol{R}\,(\boldsymbol{t}) = \boldsymbol{t'}] \geq$ 50%.

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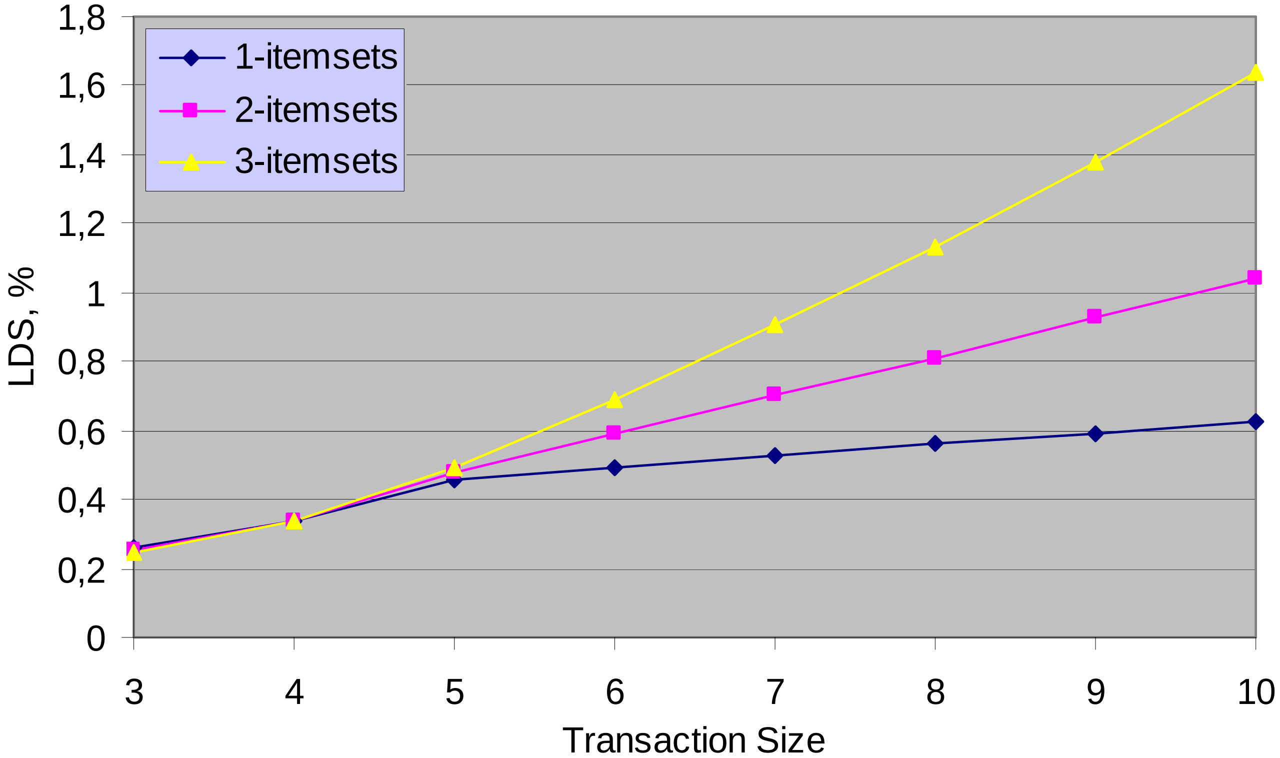
LDS vs. Transaction Size

5%-50% privacy breaches are the worst allowed, $|\boldsymbol{T}| = 5$ M



- Longer transactions are harder to use in support recovery

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Real datasets: soccer, mailorder

- Soccer is the clickstream log of WorldCup’98 web site, split into sessions of HTML requests.
  - 11 K items (HTMLs), 6.5 M transactions
  - Available at http://www.acm.org/sigcomm/ITA/
- Mailorder is a purchase dataset from a certain on-line store
  - Products are replaced with their categories
  - 96 items (categories), 2.9 M transactions

A small fraction of transactions are discarded as too long.

- longer than 10 (for soccer) or 7 (for mailorder)

Restricted Privacy Breaches

- Real data experiments used older approach [KDD 2002]
  - We constrained only $z \in t$ versus $A \subseteq t'$ privacy breaches
  - Restrictions in the form $\mathrm{Prob}\,[z \in t \mid A \subseteq t'] < \beta$
  - Older approach used some (minimal) information about data distribution to choose randomization parameters

Modified Apriori on Real Data

Breach level $\beta = 50\%$. Inserted 20-50% items to each transaction.

Soccer:

$s_{min} = 0.2\%$

$\sigma \approx 0.07\%$ for 3-itemsets

Itemset Size	True Itemsets	True Positives	False Drops	False Positives
1	266	254	12	31
2	217	195	22	45
3	48	43	5	26

Mailorder:

$s_{min} = 0.2\%$

$\sigma \approx 0.05\%$ for 3-itemsets

Itemset Size	True Itemsets	True Positives	False Drops	False Positives
1	65	65	0	0
2	228	212	16	28
3	22	18	4	5

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False Drops   False Positives

Soccer

Pred. supp%, when true supp ≥ **0.2%**

Size	< 0.1	0.1-0.15	0.15-0.2	≥0.2
1	0	2	10	254
2	0	5	17	195
3	0	1	4	43

True supp%, when pred. supp ≥ **0.2%**

Size	< 0.1	0.1-0.15	0.15-0.2	≥0.2
1	0	7	24	254
2	7	10	28	195
3	5	13	8	43

Mailorder

Pred. supp%, when true supp ≥ **0.2%**

Size	< 0.1	0.1-0.15	0.15-0.2	≥0.2
1	0	0	0	65
2	0	1	15	212
3	0	1	3	18

True supp%, when pred. supp ≥ **0.2%**

Size	< 0.1	0.1-0.15	0.15-0.2	≥0.2
1	0	0	0	65
2	0	0	28	212
3	1	2	2	18

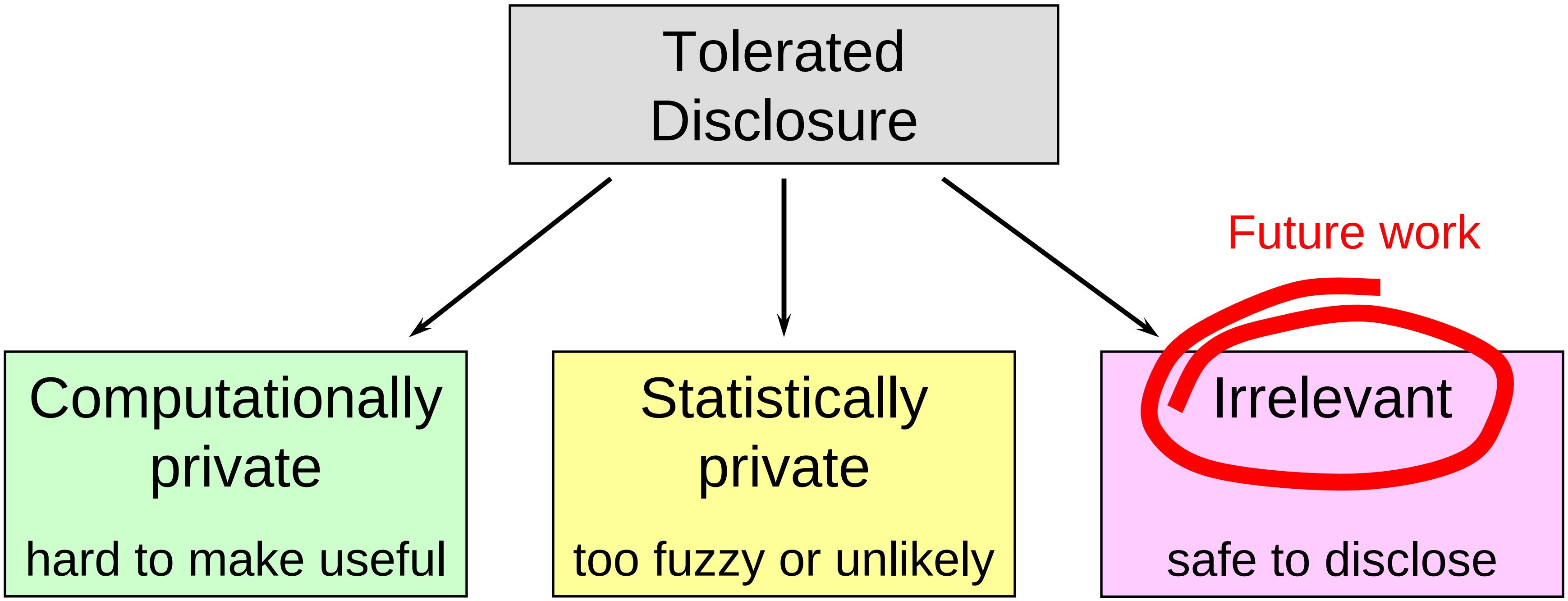
Actual Privacy Breaches

- Verified actual privacy breach levels
- The breach probabilities $\mathrm{Prob}\,[z \in t \mid A \subseteq t']$ are counted in the datasets for frequent and near-frequent itemsets.
- With the right choice of randomization parameters, even worst-case breach levels fluctuated around 50%
  - At most 53.2% for soccer,
  - At most 55.4% for mailorder.

Ongoing Research

- Using randomization and traditional secure multiparty computation together
  - Privacy preserving two-party join size computation with sketches
- What if we cannot guarantee amplification condition?
  - Probabilistic privacy breaches and amplification "on average"
- Information theory and statistical privacy
  - A slightly modified information measure that provably bounds privacy breaches

Future Work



- Can statistical privacy be extended so that we can prove "orthogonality" between disclosure and sensitive questions?

Conclusion

- We defined privacy using statistics, not computational hardness
- Randomization can guarantee statistical privacy
  - Demonstrated for association mining
- A simple property of randomization operators provably bounds privacy breaches

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Thank You!

Questions?