University at Buffalo — CSE-250 — Fall 2023

Written Assignment 1: Summations, Exponents, and Limits

Due: Sunday Sept 10, 2023 before 11:59 PM

Expect this assignment to take 4-6 hours. The total point value of all 2 problems in this assignment is 100. This assignment is worth 5% of your overall grade.

Your written solution may be either handwritten and scanned or typeset. Either way, you must produce a PDF that is legible and displays reasonably on a typical PDF reader. This PDF should be submitted via autolab. You should view your submission after you upload it to make sure that it is not corrupted or malformed. **Submissions that are rotated, upside-down, or that do not load will not receive credit**. Illegible submissions may also lose credit depending on what can be read.

Problem 1 (70 points)

Simplify each of the following equations $f_i(n)$. Your final result should be a sum-free equation in terms of n. You may use any of the rules discussed in-class (see the summary below). Show your work as a sequence of steps. For each step, indicate the specific rule that relates the equation to the previous one. All logarithms are base-2.

$$f_1(n) = \sum_{i=1}^{n} \frac{i}{3} \qquad (10\text{ pt})$$

$$f_2(n) = \sum_{i=0}^{n} (2 + 2^i) \qquad (10\text{ pt})$$

$$f_3(n) = \sum_{i=n}^{2n} 5i \qquad (10\text{ pt})$$

$$f_4(n) = \sum_{i=3}^{n} \log(2^i) \qquad (10\text{ pt})$$

$$f_5(n) = \sum_{i=2}^{n} (2^i + 7) \qquad (5\text{ pt})$$

$$f_6(n) = \sum_{i=1}^{n} \sum_{j=1}^{i} 3 \qquad (5\text{ pt})$$

$$f_7(n) = \sum_{i=1}^{2^n} \sum_{j=0}^{\log(i)-1} 2^j \qquad (5\text{ pt})$$

$$f_8(n) = \sum_{i=8}^{10} \frac{1}{i} \qquad (5\text{ pt})$$

$$f_9(n) = \sum_{i=0}^{n} \sum_{j=2}^{n} 2^i \qquad (5\text{ pt})$$

$$f_{10}(n) = \sum_{i=0}^{n} \log(2^{2^i} \cdot 4^i) \qquad (5\text{ pt})$$

Problem 2 (30 points)

For each of the equations $g_i(n)$ below, compute the limit $\lim_{n\to\infty} g_i(n)$. You do not need to show your work, but are encouraged to do so.

$$g_1(n) = n + 10 \quad \text{(5 pt)}$$

$$g_2(n) = \frac{1}{n} \quad \text{(5 pt)}$$

$$g_3(n) = \frac{n + 10}{5n} \quad \text{(5 pt)}$$

$$g_4(n) = \frac{3n^2}{n} \quad \text{(5 pt)}$$

$$g_5(n) = \frac{\log_2(n)}{\log_3(n)} \quad \text{(5 pt)}$$

$$g_6(n) = \frac{\log_2(n)}{n^2} \quad \text{(5 pt)}$$

Summations

The following works for any functions f, g (even constants). c is any constant relative to $i, j, k, \ell \in \mathbb{Z}$. Any sum $\sum_{i=j}^{k} f(i)$ is always 0 if $k < j$.

$$\sum_{i=j}^{k} c = (k - j + 1)c \qquad \textbf{Rule S1}$$

$$\sum_{i=j}^{k} (cf(i)) = c \sum_{i=j}^{k} f(i) \qquad \textbf{Rule S2}$$

$$\sum_{i=j}^{k} (f(i) + g(i)) = \left(\sum_{i=j}^{k} f(i)\right) + \left(\sum_{i=j}^{k} g(i)\right) \qquad \textbf{Rule S3}$$

$$\sum_{i=j}^{k} (f(i)) = \left(\sum_{i=\ell}^{k} (f(i))\right) - \left(\sum_{i=\ell}^{j-1} (f(i))\right) \qquad \text{(for any } \ell < j\text{)} \qquad \textbf{Rule S4}$$

$$\sum_{i=j}^{k} f(i) = f(j) + f(j+1) + \ldots + f(k-1) + f(k) \qquad \textbf{Rule S5}$$

$$\sum_{i=j}^{k} f(i) = f(j) + \ldots + f(\ell - 1) + \left(\sum_{i=\ell}^{k} f(i)\right) \qquad \text{(for any } j < \ell \leq k\text{)} \qquad \textbf{Rule S6}$$

$$\sum_{i=j}^{k} f(i) = \left(\sum_{i=j}^{\ell} f(i)\right) + f(\ell + 1) + \ldots + f(k) \qquad \text{(for any } j \leq \ell < k\text{)} \qquad \textbf{Rule S7}$$

$$\sum_{i=1}^{k} i = \frac{k(k+1)}{2} \qquad \textbf{Rule S8}$$

$$\sum_{i=0}^{k} 2^i = 2^{k+1} - 1 \qquad \textbf{Rule S9}$$

Logarithms

$$\log(n^a) = a \log(n) \qquad \textbf{Rule L1}$$

$$\log(an) = \log(a) + \log(n) \qquad \textbf{Rule L2}$$

$$\log\left(\frac{n}{a}\right) = \log(n) - \log(a) \qquad \textbf{Rule L3}$$

$$\log_b(n) = \frac{\log_c(n)}{\log_c(b)} \qquad \textbf{Rule L4}$$

$$\log(2^n) = 2^{\log(n)} = n \qquad \textbf{Rule L5}$$

Limits

$$\lim_{n\to\infty} f(n) + g(n) = \lim_{n\to\infty} (f(n)) + \lim_{n\to\infty} (g(n)) \qquad \text{(for any functions } f \text{ and } g\text{)} \qquad \textbf{Rule I1}$$

$$\lim_{n\to\infty} c = c \qquad \text{(for any } \textit{constant } c\text{)} \qquad \textbf{Rule I2}$$

$$\lim_{n\to\infty} f(n) = \infty \qquad \text{(for any } \textit{increasing function } f\text{)} \qquad \textbf{Rule I3}$$

$$\lim_{n\to\infty} \frac{1}{f(n)} = 0 \qquad \text{(for any } \textit{increasing function } f\text{)} \qquad \textbf{Rule I4}$$