University at Buffalo — CSE-250 — Fall 2023

Written Assignment 3: Induction

Due: Sunday Oct 15, 2023 before 11:59 PM

Expect this assignment to take 4-6 hours. The total point value of all 1 problems in this assignment is 100. This assignment is worth 5% of your overall grade.

Your written solution may be either handwritten and scanned or typeset. Either way, you must produce a PDF that is legible and displays reasonably on a typical PDF reader. This PDF should be submitted via autolab. You should view your submission after you upload it to make sure that it is not corrupted or malformed. **Submissions that are rotated**, **upside-down**, or that do not load will not receive credit. Illegible submissions may also lose credit depending on what can be read.

Problem 1: Induction (100 points)

Recall the recursive runtime function for Merge Sort

 $T_{merge}(N) = \begin{cases} \theta(1) & \text{if } N \leq 1\\ \theta(N) + \theta(1) + 2 \cdot T_{merge}\left(\frac{N}{2}\right) & \text{otherwise} \end{cases}$

Use induction to prove that $T_{merge}(N) \in \Omega(N \cdot \log(N))$. You may limit your proof to only show that powers of two meet the inductive hypothesis (i.e., that $T(N) \in \Omega(N \cdot \log(N))$) only for $N \in 2^i$). Your answer should clearly include, and label:

- A proof of the base case [30 pt]
- Your inductive assumption case [20 pt]
- A proof of the inductive step [50 pt]