CSE 250 Data Structures

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Lec 03: Math Refresher

Announcements and Feedback

- Join Piazza! (Link on course website)
- Academic Integrity Quiz due 9/6 @ 11:59PM
- PA0 due 9/10 @ 11:59PM
- WA1 due 9/10 @ 11:59PM

Today's Topics

- Summations
- Logarithms
- Limits

Summations

$$\sum_{i=j}^{k} f(i) = f(j) + f(j+1) + \dots + f(k)$$

If **c** is a constant (with respect to **i**)

$$\sum_{i=j}^{k} c = (c + c + \dots + c)$$
(k - j + 1) times

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$$= (k - j + 1) \cdot c$$

If **c** is a constant and **f**(**i**) is a function of **i**:

$$\sum_{i=j}^{k} cf(i) = (cf(j) + cf(j+1) + \dots + cf(k))$$

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$$= c(f(j) + f(j+1) + \dots + f(k))$$
$$= c\sum_{i=j}^{k} f(i)$$

If f(i) and g(i) are functions of i:

$$\sum_{i=j}^{k} f(i) + g(i) = (f(j) + g(j)) + (f(j+1) + g(j+1)) + \dots + (f(k) + g(k))$$

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$$\sum_{i=j}^{k} f(i) + g(i) = (f(j) + g(j)) + (f(j+1) + g(j+1)) + \dots + (f(k) + g(k))$$

$$= (f(j) + f(j+1) + \dots + f(k)) + (g(j) + g(j+1) + \dots + g(k))$$

$$= \left(\sum_{i=j}^{k} f(i)\right) + \left(\sum_{i=j}^{k} g(i)\right)$$

If
$$\mathbf{j} < \mathbf{l} < \mathbf{k}$$
:
$$\sum_{i=j}^k f(i) = f(j) + \dots + f(k)$$

$$\sum_{i=j}^{k} f(i) = f(j) + \dots + f(k)$$
$$= f(j) + \dots + f(l-1) + f(l) + \dots + f(k)$$

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$$= f(j) + \dots + f(l-1) + f(l) + \dots + f(k)$$

$$= \left(\sum_{i=j}^{l-1} f(i)\right) + \left(\sum_{i=l}^{k} f(i)\right)$$

If *j* < *l* < *k*:

$$\left(\sum_{i=j}^{k} f(i)\right) = \left(\sum_{i=j}^{l-1} f(i)\right) + \left(\sum_{i=l}^{k} f(i)\right)$$

If *j* < *l* < *k*:

$$\left(\sum_{i=j}^{k} f(i)\right) = \left(\sum_{i=j}^{l-1} f(i)\right) + \left(\sum_{i=l}^{k} f(i)\right)$$

Subtract to other side

If *j* < *l* < *k*:

$$\left(\sum_{i=j}^{k} f(i)\right) = \left(\sum_{i=j}^{l-1} f(i)\right) + \left(\sum_{i=l}^{k} f(i)\right)$$

$$\left(\sum_{i=j}^{k} f(i)\right) - \left(\sum_{i=j}^{l-1} f(i)\right) = \left(\sum_{i=l}^{k} f(i)\right)$$

Series

Some common closed form solutions:

$$\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$$

$$\sum_{i=0}^{k} 2^{i} = 2^{k+1} - 1$$

Summary

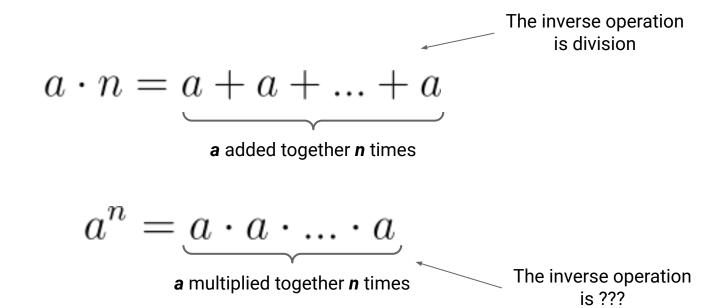
- The previous rules will always be provided on WAs and exams
- Usually the goal will be to reduce some complicated summation to a simpler form without a summation
 - Some of the rules get rid of summations
 - Some allow you to manipulate summations/bounds so that you can apply rules that get rid of summations
- Be cognizant of what variables are constant with respect to the summation variable and which one aren't

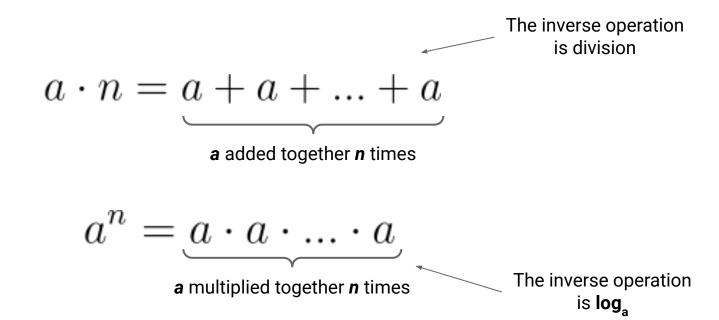
$$a \cdot n = \underbrace{a + a + \dots + a}_{\text{a added together } \textbf{n} \text{ times}}$$

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$$a^n = \underbrace{a \cdot a \cdot \dots \cdot a}_{}$$

a multiplied together **n** times





 $\log_a(b)$ = the number of times you multiply a together to get b

$$\log_2(32) = 5$$

$$\log_3(27) = 3$$

$$\log_2(\frac{1}{8}) = -3$$

$$\log_2(2^{10}) = 10$$

Logarithm is the inverse exponent

$$b^{\log_b(n)} = n = \log_b(b^n)$$

Let's say $n = a \cdot b$

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$$a = 2 \cdot \dots \cdot 2$$

$$b=2\cdot\ldots\cdot2$$

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$$a = \underbrace{2 \cdot \ldots \cdot 2}_{\text{log_2(a) times}}$$

$$b = \underbrace{2 \cdot \dots \cdot 2}_{\log_2(b) \text{ times}}$$

Let's say $n = a \cdot b$

$$a=2\cdot\ldots\cdot 2 \qquad \qquad b=2\cdot\ldots\cdot 2$$

$$n=\underbrace{2\cdot\ldots\cdot 2}_{\log_2(\textbf{n})\text{ times}}=\underbrace{2\cdot\ldots\cdot 2}_{\log_2(\textbf{a})\text{ times}}\cdot\underbrace{\log_2(\textbf{b})\text{ times}}$$

Let's say $n = a \cdot b$

$$a = 2 \cdot \dots \cdot 2 \qquad \qquad b = 2 \cdot \dots \cdot 2$$

$$n = \underbrace{2 \cdot \ldots \cdot 2}_{\log_2(\mathbf{n}) \text{ times}} = \underbrace{2 \cdot \ldots \cdot 2}_{\log_2(\mathbf{a}) \text{ times}} \cdot \underbrace{2 \cdot \ldots \cdot 2}_{\log_2(\mathbf{b}) \text{ times}}$$

$$\log_2(n) = \log_2(ab) = \log_2(a) + \log_2(b)$$

$$\log_2(a^n) = \log_2(a \cdot \dots \cdot a)$$

$$\log_2(a^n) = \log_2(\overbrace{a \cdot \ldots \cdot a})$$

$$\log_2(a^n) = \log_2(\overbrace{a \cdot \ldots \cdot a}) \qquad \text{n-1 times}$$

$$= \log_2(a) + \log_2(\overbrace{a \cdot \ldots \cdot a}) \qquad \text{n-2 times}$$

$$= \log_2(a) + \log_2(a) + \log_2(\overbrace{a \cdot \ldots \cdot a})$$

$$= \log_2(a) + \ldots + \log_2(a)$$

$$= \log_2(a) + \ldots + \log_2(a)$$

$$\text{n times}$$

Exponent Rule

$$\log_2(a^n) = \log_2(a \cdot \dots \cdot a)$$

$$= \log_2(a) + \log_2(a \cdot \dots \cdot a)$$

$$= \log_2(a) + \log_2(a) + \log_2(a \cdot \dots \cdot a)$$

$$= \log_2(a) + \dots + \log_2(a)$$

$$= n \cdot \log_2(a)$$

$$\log_2(\frac{a}{b}) = \log_2(a \cdot \frac{1}{b})$$

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$$= \log_2(a) + \log_2(b^{-1})$$

$$\log_2(\frac{a}{b}) = \log_2(a \cdot \frac{1}{b})$$

$$= \log_2(a) + \log_2(\frac{1}{b})$$

$$= \log_2(a) + \log_2(b^{-1})$$

$$= \log_2(a) - \log_2(b)$$

$$b^m = n$$

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$$\log_c(b^m) = \log_c(n)$$

$$b^{m} = n$$
$$\log_{c}(b^{m}) = \log_{c}(n)$$
$$m \cdot \log_{c}(b) = \log_{c}(n)$$

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$$m = \frac{\log_{c}(n)}{\log_{c}(b)}$$

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$$m = \frac{\log_{c}(n)}{\log_{c}(b)}$$
$$\log_{b}(n) = \frac{\log_{c}(n)}{\log_{c}(b)}$$

Summary

Exponent Rule
$$\log(n^a) = a \log(n)$$

Product Rule $\log(ab) = \log(a) + \log(b)$
Division Rule $\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$
Change of Base $\log_b(n) = \frac{\log_c(n)}{\log_c(b)}$
Inverse $\log_b(n^a) = \log_b(b^a) = n$

^{*} for this class, always assume base 2 unless otherwise stated *

Limits

$$\lim_{i \to c} f(i)$$

The value that f(i) converges to as i approaches c (even if f(c) is not defined)

$$\lim_{i\to\infty}\frac{1}{i}=0$$

$$\lim_{i\to\infty} 6=6$$

$$\lim_{i\to\infty}6+\frac{1}{i}=6$$

$$\lim_{i\to\infty}i=\infty$$

$$\lim_{i\to\infty}i-i=0$$