CSE 250
Data Structures

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Lec 15: Stacks and Queues

Announcements

- WA1 grades have been released
- WA3 has been posted, due Oct 15th @ 11:59PM

Recap

QuickSort

- Divide and Conquer sorting algorithm like MergeSort
  - All of the work for Merge Sort happened during the combine step
  - QuickSort attempts to move the work to the divide step
- **Divide:** Move small elements to the left, and big elements to the right
- **Conquer:** Recursively call QuickSort on left and right halves
- **Combine:** …nothing

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QuickSort Review

Divide: Move *small* elements to the left and *big* elements to the right

How do we define what is *big* and what is *small*?

QuickSort Review

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Pick a pivot value

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[smaller than pivot], pivot, [larger than pivot]

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How do we pick a pivot?

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QuickSort Review

If our pivot was the median value, then our list would be split in half by the divide step, resulting in the same structure as MergeSort...

…but finding the median value is expensive…(it costs ***n*log(*n*)**).

So what if we pick one randomly instead?

Expected Value

If I roll a 6-sided die, the probability of a particular side being rolled is ⅙

If *X* is a random variable representing this die roll, then the expected value of *X* is:

$$E[X] = \frac{1}{6} \cdot 1 + \frac{1}{6} \cdot 2 + \frac{1}{6} \cdot 3 + \frac{1}{6} \cdot 4 + \frac{1}{6} \cdot 5 + \frac{1}{6} \cdot 6$$

$$E[X] = \sum_{i=1}^{6} \frac{1}{6} i = 3.5$$

Expected Value

If I roll a 20-sided die, the probability of a particular side being rolled is 1/20

If *X* is a random variable representing this die roll, then the expected value of *X* is:

$$E[X] = \frac{1}{20} \cdot 1 + \frac{1}{20} \cdot 2 + \ldots + \frac{1}{20} \cdot 20 = \sum_{i=1}^{20} \frac{1}{20} i$$

Expected Value

If I roll an *n*-sided die, the probability of a particular side being rolled is 1/*n*

If *X* is a random variable representing this die roll, then the expected value of *X* is:

$$E[X] = \frac{1}{n} \cdot 1 + \frac{1}{n} \cdot 2 + \ldots + \frac{1}{n} \cdot n = \sum_{i=1}^{n} \frac{1}{n} i$$

$$E[X] = \sum_{i} P_i \cdot X_i$$

QuickSort Review

Picking a pivot value randomly from the n elements of our sequence is the same as rolling an n-sided die.

There is a $1/n$ probability in any particular value being selected.

$X = k$ means that X is the kth largest value, and the expected value of X corresponds to the median value.

QuickSort Review

$$T(n) = \begin{cases} \Theta(1) & \textbf{if } n \leq 1 \\ T(0) + T(n-1) + \Theta(n) & \textbf{if } n > 1 \wedge X = 1 \\ T(1) + T(n-2) + \Theta(n) & \textbf{if } n > 1 \wedge X = 2 \\ T(2) + T(n-3) + \Theta(n) & \textbf{if } n > 1 \wedge X = 3 \\ .. & \\ T(n-2) + T(1) + \Theta(n) & \textbf{if } n > 1 \wedge X = n-1 \\ T(n-1) + T(0) + \Theta(n) & \textbf{if } n > 1 \wedge X = n \end{cases}$$

QuickSort Review

$$E[T(n)] = \begin{cases} \Theta(1) & \textbf{if } n \leq 1 \\ E[T(X-1) + T(n-X)] + \Theta(n) & \textbf{otherwise} \end{cases}$$

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Expected value of two independent events can be split up

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How are these two terms related?

QuickSort Review

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$$= \sum_{i=1}^{n} P_i \cdot T(X_i - 1)$$

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$$= \sum_{i=1}^{n} \frac{1}{n} \cdot T(n-i) \; = E[T(n-X)]$$

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They are equivalent!!

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$$E[T(n)] = \begin{cases} \Theta(1) & \textbf{if } n \leq 1 \\ 2E[T(X-1)] + \Theta(n) & \textbf{otherwise} \end{cases}$$

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Each T(X-1) is independent, so the expected values can be split out

QuickSort Review

$$E[T(n)] = \begin{cases} \Theta(1) & \textbf{if } n \leq 1 \\ \frac{2}{n}\left(\sum_{i=0}^{n-1} E[T(i)]\right) + \Theta(n) & \textbf{otherwise} \end{cases}$$

Back to Induction

Hypothesis: $E[T(n)] \in O(n \log(n))$

Base Case

Base Case: $E[T(2)] \leq c\ (2 \log(2))$

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$$\cancel{2} \cdot (T(0)/\cancel{2} + T(1)/\cancel{2}) + 2c_1 \leq 2c$$

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$$T(0) + T(1) + 2c_1 \leq 2c$$

$$2c_0 + 2c_1 \leq 2c$$

True for any $c \geq c_0 + c_1$

Inductive Case

Assume: $E[T(n')] \leq c\ (n' \log(n'))$ for **all** $n' < n$

Show: $E[T(n)] \leq c\ (n \log(n))$

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$$\frac{2}{n}\left(\sum_{i=0}^{n-1} E[T(i)]\right) + c_1 \leq cn\log(n)$$

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Show: $E[T(n)] \leq c\,(n \log(n))$

Our *i* here is always less than *n*, so we can use our assumption to substitute

$$\frac{2}{n}\left(\sum_{i=0}^{n-1} E[T(i)]\right) + c_1 \leq cn\log(n)$$

$$\frac{2}{n}\left(\sum_{i=0}^{n-1} ci\log(i)\right) + c_1 \leq cn\log(n)$$

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$$c\frac{2}{n}\left(\sum_{i=0}^{n-1} i\log(n)\right) + c_1 \leq cn\log(n)$$

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$$cn\log(n) - c\log(n) + c_1 \leq cn\log(n)$$

$$c_1 \leq c\log(n)$$

QuickSort

So…is QuickSort $O(n \log(n))$...?

No!

What guarantees do you get?

If *f*(*n*) is a Tight Bound

The algorithm always runs in $cf(n)$ steps

If *f*(*n*) is a Worst-Case Bound

The algorithm always runs in at most $cf(n)$

If *f*(*n*) is an Amortized Worst-Case Bound

n invocations of the algorithm **always** run in $cnf(n)$ steps

If *f*(*n*) is an Average Bound

…we don't have any guarantees

Current Road Map

Analysis Tools/Techniques	ADTs	Data Structures
Asymptotic Analysis, (Unqualified) Runtime Bounds		
	Sequence	Array, LinkedList
Amortized Runtime	List	ArrayList, LinkedList
Recursive analysis, divide and conquer, Average/Expected Runtime		
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Looking Ahead…

Analysis Tools/Techniques	ADTs	Data Structures
	Stack, Queue	ArrayList, LinkedList
Review recursive analysis	Graphs	EdgeList, AdjacencyList, AdjacencyMatrix
	Heaps, Trees	BST, AVL Tree, Red-Black Tree
Midterm #2		
Review expected runtime	HashTables	
Miscellaneous		

Stacks

Represents a stack of objects on top of one another

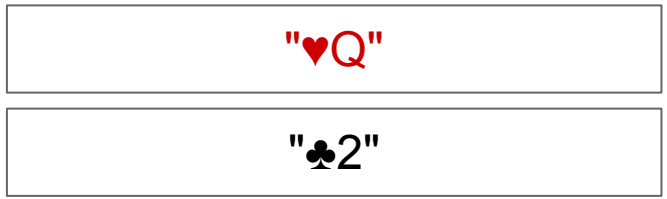
```
1  public class Stack<E> {
2
3    public void push(E value); // Add value to the "top" of the stack
4
5    public E pop();   // Remove and return the top of the stack
6
7    public E peek();  // Return the top of the stack
8
9  }
```

Stacks

```
s.push("♣2")
```

Stacks

```
s.push("♣2")
s.push("❤️Q")
```



Stacks

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s.push("♣2")
s.push("❤️Q")
s.push("♠A")
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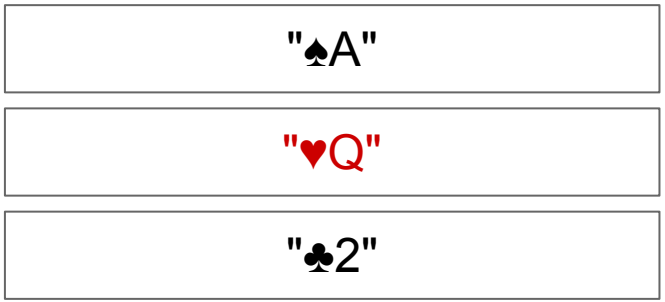


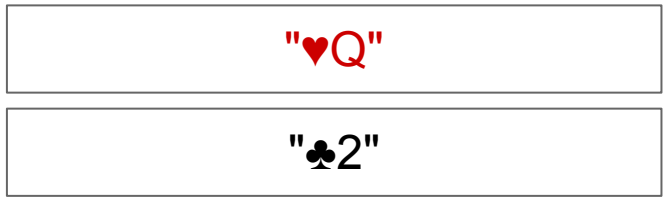
Stacks

```
s.push("♣2")

s.push("❤️Q")

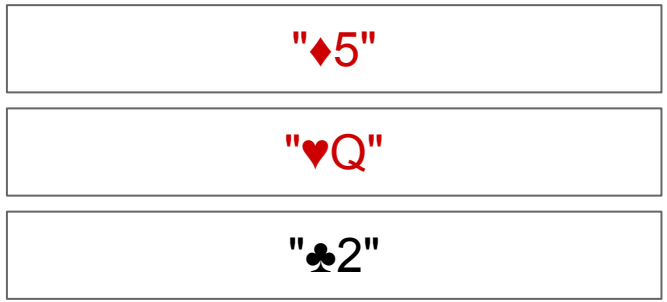
s.push("♠A")

card = s.pop() // Removes "♠A"
```



Stacks

```
s.push("♣2")
s.push("❤Q")
s.push("♠A")
card = s.pop() // Removes "♠A"
s.push("♦5")
```



Stacks in Practice

- Storing function variables in a "call stack"
- Certain types of parsers ("context free")
- Backtracking search
- Reversing Sequences

Stacks - LinkedList Implementation

```
1  public class ListStack<E> extends Stack<E> {
2    private LinkedList<E> data;
3
4    public void push(E value) {
5      /* ?? */
6    }
7    public E pop() {
8      /* ?? */
9    }
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Stacks - LinkedList Implementation

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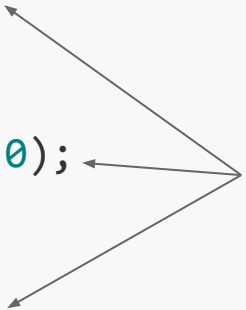

Θ(1) complexity in all cases

…and only requires a singly linked list

Stacks - ArrayList Implementation

```
1  public class ArrayStack<E> extends Stack<E> {
2    private ArrayList<E> data;
3
4    public void push(E value) {
5      /* ?? */
6    }
7    public E pop() {
8      /* ?? */
9    }
10   public E peek() {
11     /* ?? */
12   }
13 }
```

Stacks - ArrayList Implementation

```
1  public class ArrayStack<E> extends Stack<E> {
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Stacks - ArrayList Implementation

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1  public class ArrayStack<E> extends Stack<E> {
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3
4    public void push(E value) {
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6    }
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8      return data.remove(data.size() - 1);
9    }
10   public E peek() {
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***O*(*n*)**, Amortized **Θ(1)**

Θ(1)

Stacks in Java

Java's **`Stack`** implementation is based on **`Vector`**

(Like **`ArrayList`** but increases capacity by a constant, rather than doubling)

What does this mean for runtime of **`push`** in Java specifically?

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A common assumption for Stacks (and Queues) is they will have a limited size. The contiguous nature of array memory usage has some benefits…

Queues

Outside of the US, "queueing" is lining up, ie at Starbucks

```
1  public class Queue<E> {
2
3    public void add(E value); // Add value to the "back" of the queue
4
5    public E remove();   // Remove and return the front of the queue
6
7    public E peek();  // Return the front of the stack
8
9  }
```

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5    public E remove();    // Remo                  e
6
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8
9  }
```

In context of queues we will often refer to add/remove as enqueue/dequeue

Queues

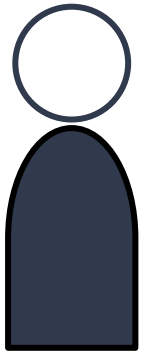
Front

Back

Queues

```
enqueue("Michael")
```

Front

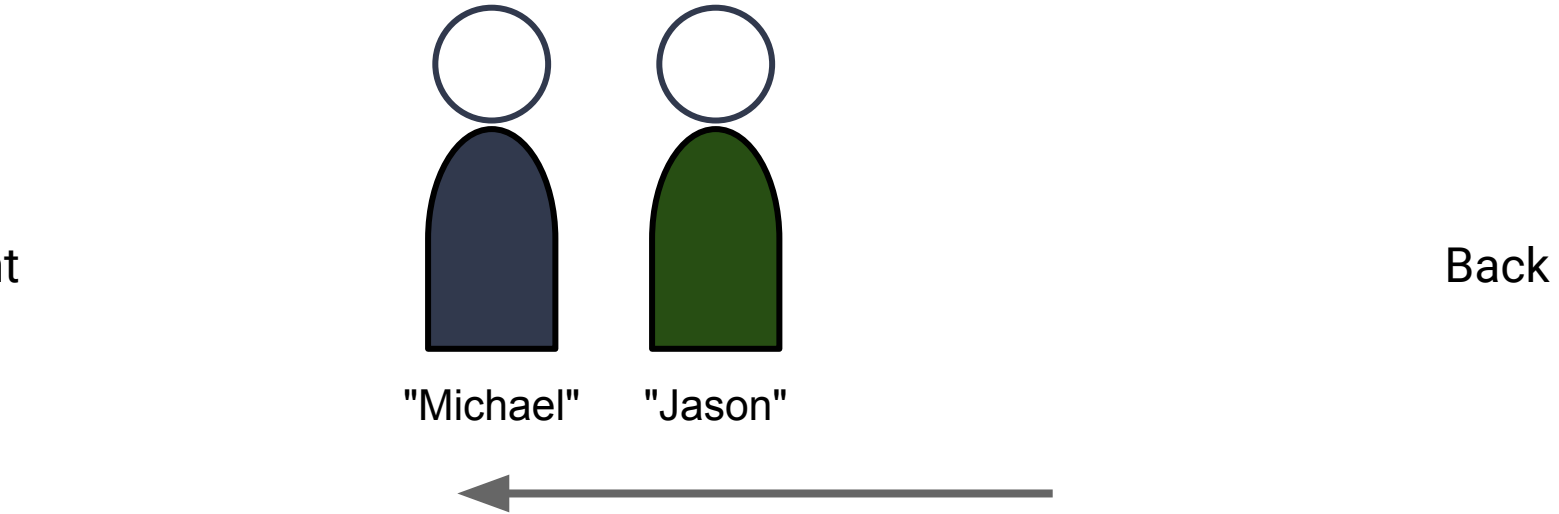


"Michael"

Back

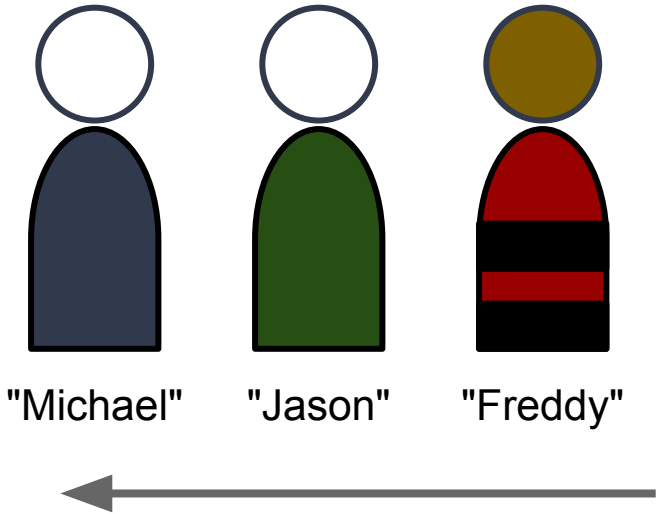
Queues

```
enqueue("Michael")
enqueue("Jason")
```



Queues

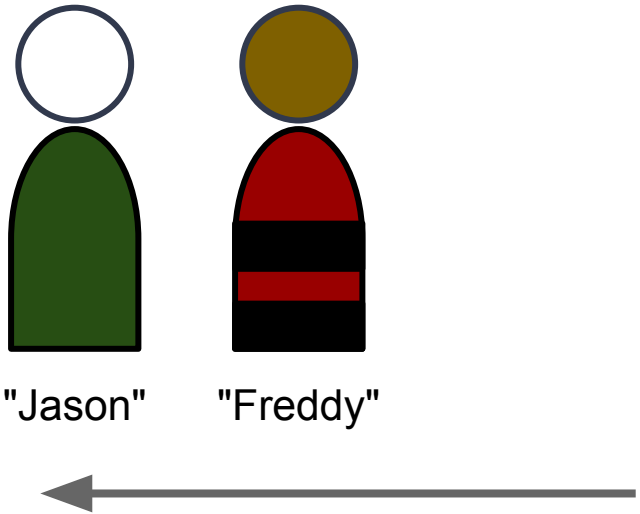
```
enqueue("Michael")
enqueue("Jason")
enqueue("Freddy")
```



Queues

```
enqueue("Michael")
enqueue("Jason")
enqueue("Freddy")
dequeue()
```

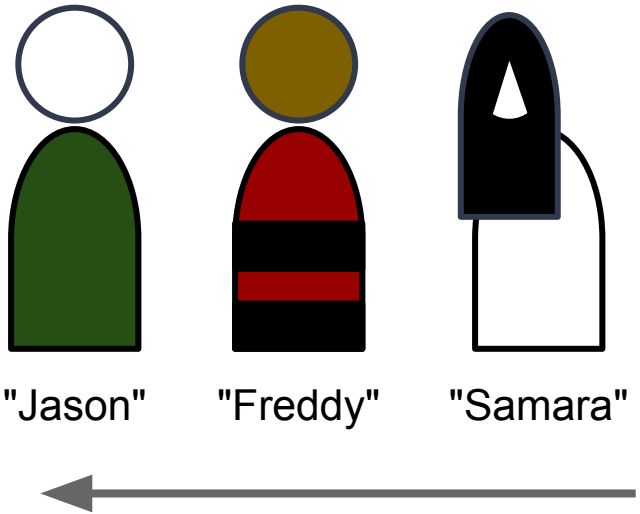
Front



Back

Queues

```
enqueue("Michael")
enqueue("Jason")
enqueue("Freddy")
dequeue()
enqueue("Samara")
```



Queues vs Stacks

Queue First in, First Out (FIFO)

Stacks Last in, First Out (LIFO / FILO)

Queues in Practice

- Delivering network packets, emails, twitter/tiktok/instagram
- Scheduling CPU cycles
- Deferring long-running tasks

Queues - LinkedList Implementation

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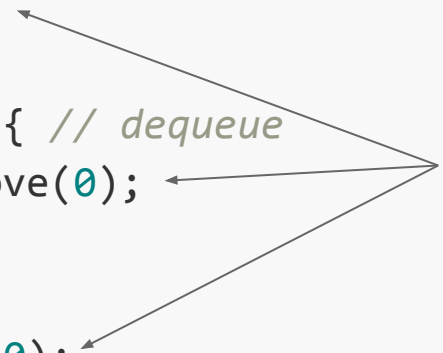

Θ(1) complexity in all cases as long as we have a reference to the last node in the list

Queues

Thought Experiment: How can we use an array to build a queue?