CSE 250
Data Structures

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Lec 20: DFS and BFS

Announcements

- PA2 released
  - Testing phase due Sunday 10/22
  - Implementation due Sunday 11/5
  - AutoLab is now open

Depth-First Search

Primary Goals

- Visit every vertex in graph ***G* = (*V*,*E*)**
- Construct a spanning tree for every connected component
  - **Side Effect:** Compute connected components
  - **Side Effect:** Compute a path between all connected vertices
  - **Side Effect:** Determine if the graph is connected
  - **Side Effect:** Identify cycles
- Complete in time ***O*(|*V*| + |*E*|)**

Depth-First Search

DFS

Input: Graph $G = (V,E)$

Output: Label every edge as:

- Spanning Edge: Part of the spanning tree
- Back Edge: Part of a cycle

Depth-First Search

DFS

Input: Graph $G = (V,E)$

Output: Label every edge as:

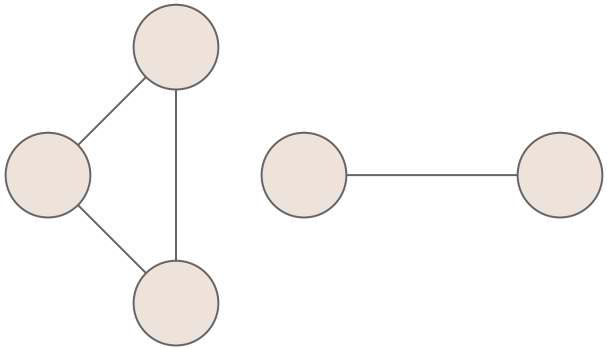
- Spanning Edge: Part of the spanning tree
- Back Edge: Part of a cycle

DFSOne

Input: Graph $G = (V,E)$, start vertex $v \in V$

Output: Label every edge in v's connected component

Depth-First Search



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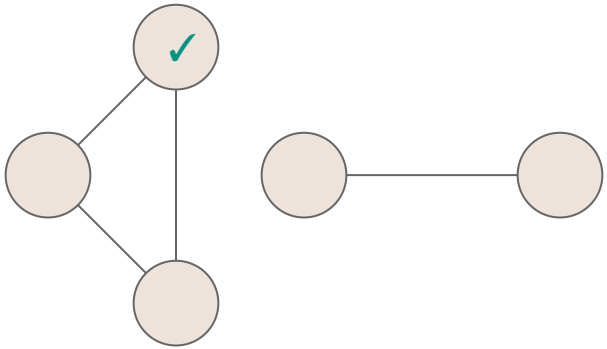

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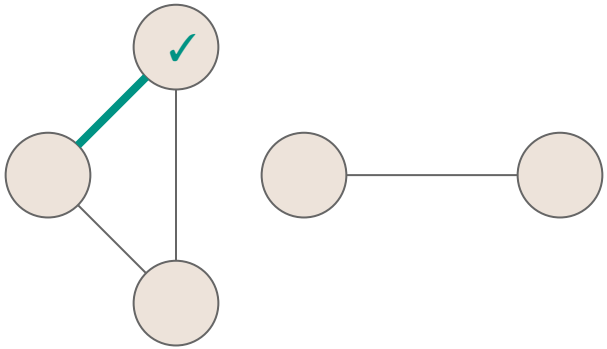

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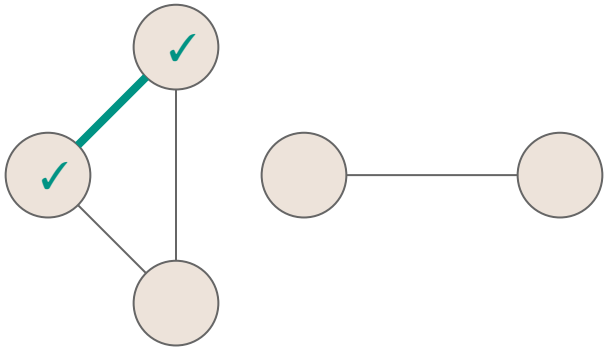

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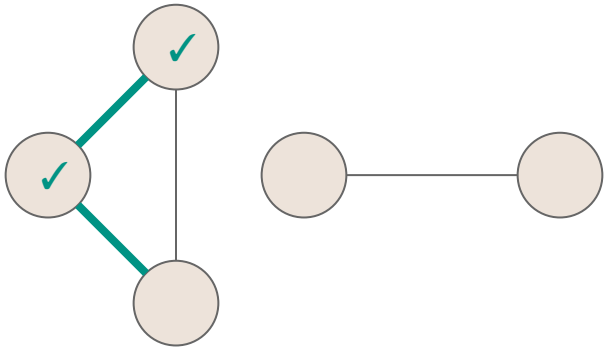

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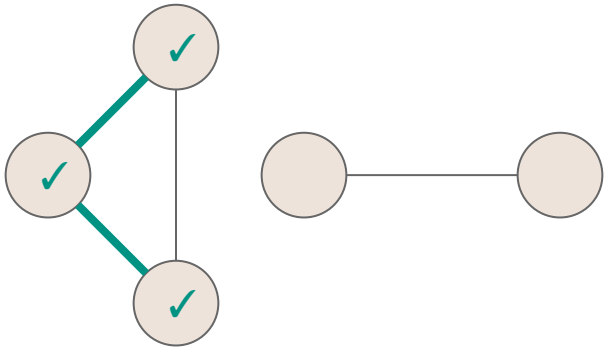

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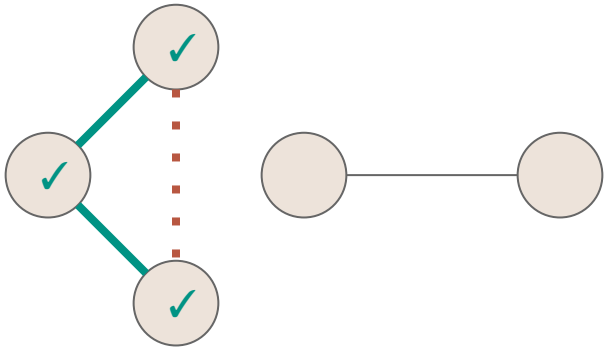

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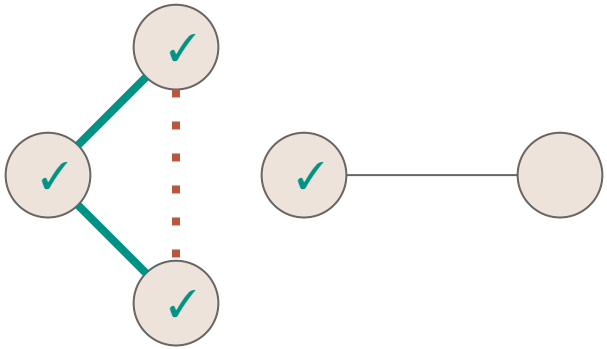

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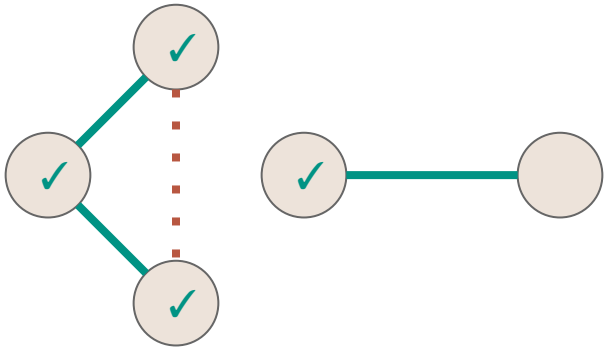

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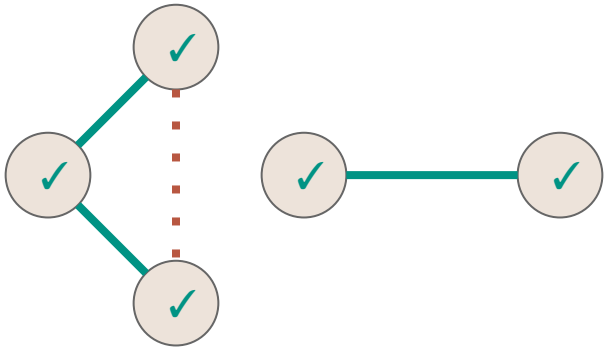

DFS

```
 1 public void DFS(Graph graph) {
 2   for (Vertex v : graph.vertices) {
 3     v.setLabel(UNEXPLORED);
 4   }
 5   for (Edge e : graph.edges) {
 6     e.setLabel(UNEXPLORED);
 7   }
 8   for (Vertex v : graph.vertices) {
 9     if (v.label == UNEXPLORED) {
10       DFSOne(graph, v);
11     }
12   }
13 }
```

DFS

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8    for (Vertex v : graph.vertices) {
9      if (v.label == UNEXPLORED) {
10       DFSOne(graph, v);
11     }
12   }
13 }
```

Initialize all vertices and edges to UNEXPLORED

DFS

```
1  public void DFS(Graph graph) {
2    for (Vertex v : graph.vertices) {
3      v.setLabel(UNEXPLORED);
4    }
5    for (Edge e : graph.edges) {
6      e.setLabel(UNEXPLORED);
7    }
8    for (Vertex v : graph.vertices) {
9      if (v.label == UNEXPLORED) {
10       DFSOne(graph, v);
11     }
12   }
13 }
```

Call DFSOne to label the connected component of every unexplored vertex

DFSOne

```
1  public void DFSOne(Graph graph, Vertex v) {
2    v.setLabel(VISITED);
3    for (Edge e : v.outEdges) {
4      if (e.label == UNEXPLORED) {
5        Vertex w = e.to;
6        if (w.label == UNEXPLORED) {
7          e.setLabel(SPANNING);
8          DFSOne(graph, w);
9        } else {
10         e.setLabel(BACK);
11       }
12     }
13 }}
```

DFSOne

```
1  public void DFSOne(Graph graph, Vertex v) {
2    v.setLabel(VISITED);
3    for (Edge e : v.outEdges) {
4      if (e.label == UNEXPLORED) {
5        Vertex w = e.to;
6        if (w.label == UNEXPLORED) {
7          e.setLabel(SPANNING);
8          DFSOne(graph, w);
9        } else {
10         e.setLabel(BACK);
11       }
12     }
13 }}
```

← Mark the vertex as VISITED (so we'll never try to visit it again)

DFSOne

```
1  public void DFSOne(Graph graph, Vertex v) {
2    v.setLabel(VISITED);
3    for (Edge e : v.outEdges) {
4      if (e.label == UNEXPLORED) {
5        Vertex w = e.to;
6        if (w.label == UNEXPLORED) {
7          e.setLabel(SPANNING);
8          DFSOne(graph, w);
9        } else {
10         e.setLabel(BACK);
11       }
12     }
13 }}
```

Check every outgoing edge (every possible way we could leave the current vertex)

DFSOne

```
1  public void DFSOne(Graph graph, Vertex v) {
2    v.setLabel(VISITED);
3    for (Edge e : v.outEdges) {
4      if (e.label == UNEXPLORED) {
5        Vertex w = e.to;
6        if (w.label == UNEXPLORED) {
7          e.setLabel(SPANNING);
8          DFSOne(graph, w);
9        } else {
10         e.setLabel(BACK);
11       }
12     }
13 }}
```

Follow the unexplored edges

DFSOne

```
1  public void DFSOne(Graph graph, Vertex v) {
2    v.setLabel(VISITED);
3    for (Edge e : v.outEdges) {
4      if (e.label == UNEXPLORED) {
5        Vertex w = e.to;
6        if (w.label == UNEXPLORED) {
7          e.setLabel(SPANNING);
8          DFSOne(graph, w);
9        } else {
10         e.setLabel(BACK);
11       }
12     }
13 }}
```

If it leads to an unexplored vertex, then it is a spanning edge. Recursively explore that vertex.

DFSOne

```
1  public void DFSOne(Graph graph, Vertex v) {
2    v.setLabel(VISITED);
3    for (Edge e : v.outEdges) {
4      if (e.label == UNEXPLORED) {
5        Vertex w = e.to;
6        if (w.label == UNEXPLORED) {
7          e.setLabel(SPANNING);
8          DFSOne(graph, w);
9        } else {
10         e.setLabel(BACK);
11       }
12     }
13 }}
```

Otherwise, we just found a cycle

Detailed Example

UNEXPLORED

VISITED

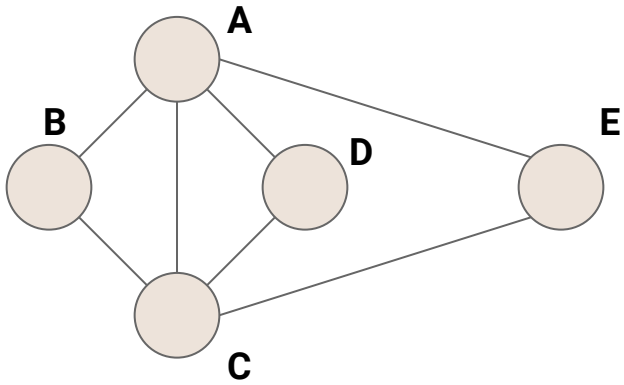
UNEXPLORED

SPANNING

BACK

Call Stack

(→ edges to list)



Detailed Example

UNEXPLORED

VISITED

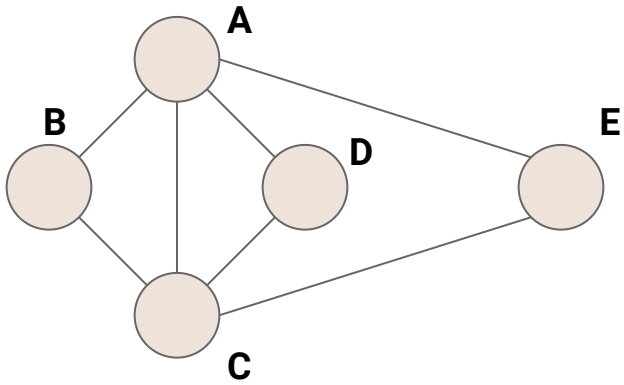
UNEXPLORED

SPANNING

BACK

Call Stack
DFS(G)

(→ edges to list)



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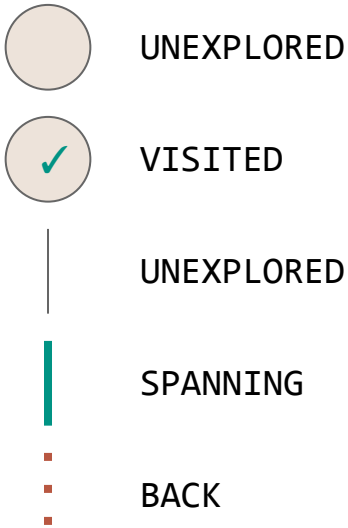
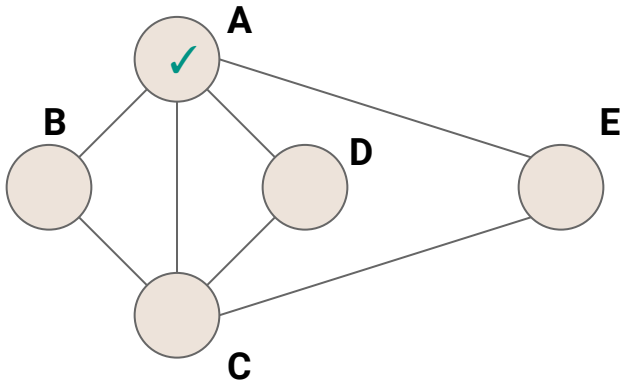


Call Stack

DFS(G)
DFSOne(G,A)

(→ edges to list)



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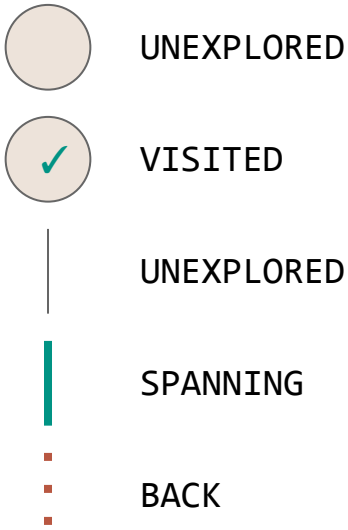
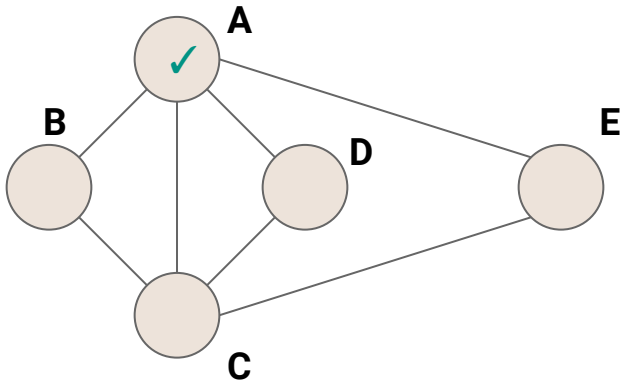


Call Stack (→ edges to list)

DFS(G)

DFSOne(G,A) (→ B, C, D, E)



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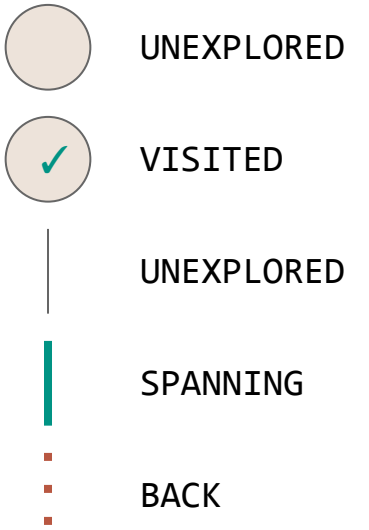


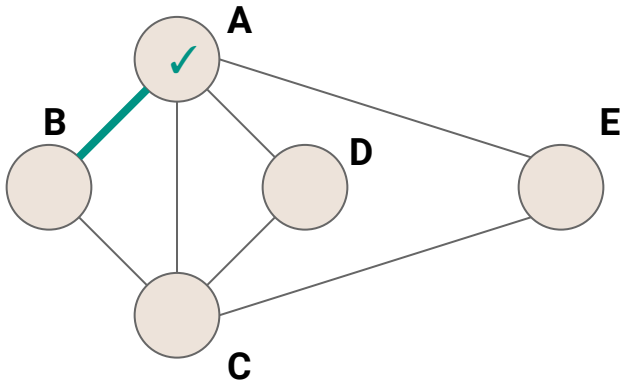
Call Stack

DFS(G)

DFSOne(G,A)

(→ edges to list)

(→ B, C, D, E)



Detailed Example

UNEXPLORED

VISITED

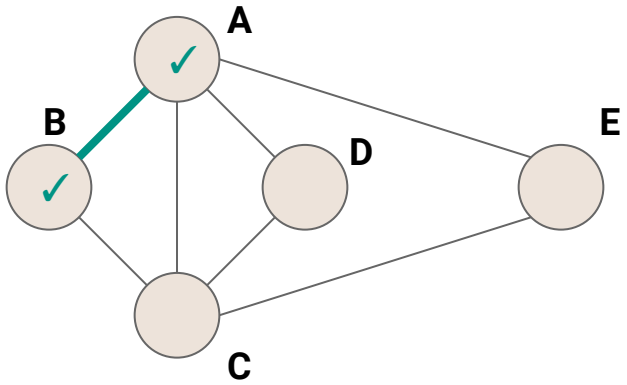
UNEXPLORED

SPANNING

BACK

<u>Call Stack</u> <u>(→ edges to list)</u>

```
DFS(G)
DFSOne(G,A)   ( → B, C, D, E)
DFSOne(G,B)   ( → A, C)
```



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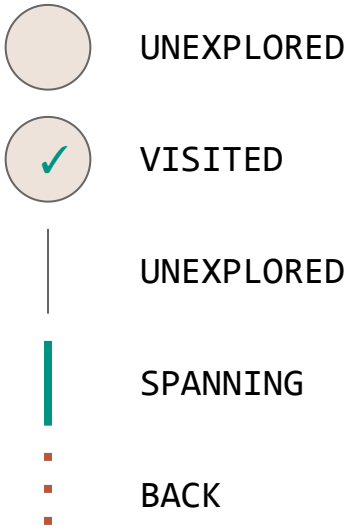


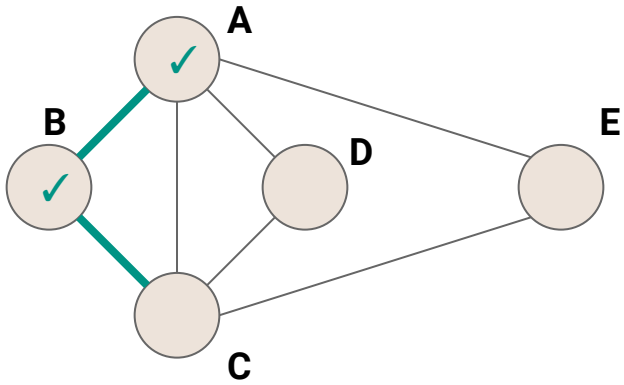
Call Stack

```
DFS(G)
DFSOne(G,A)
DFSOne(G,B)
```

(→ edges to list)

(→ B, C, D, E)

(→ A, C)



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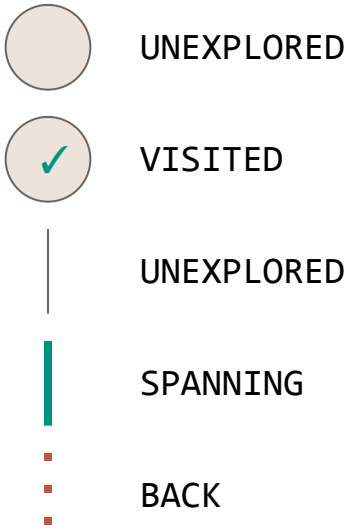
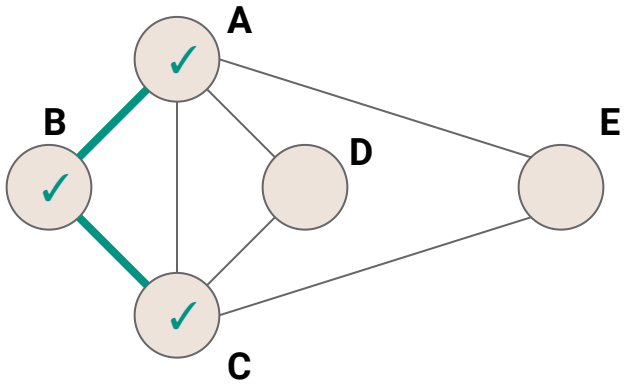


Call Stack	(→ edges to list)
DFS(G)	
DFSOne(G,A)	(→ B, C, D, E)
DFSOne(G,B)	(→ A, C)
DFSOne(G,C)	(→ B, A, D, E)



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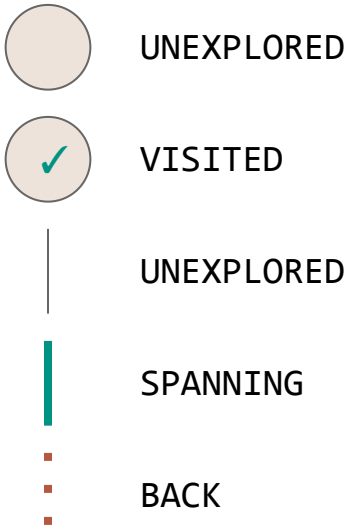

Call Stack	(→ edges to list)
DFS(G)	
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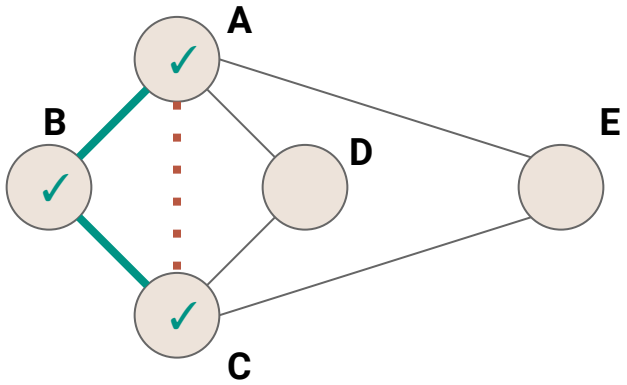
DFSOne(G,C)	(→ B, A, D, E)



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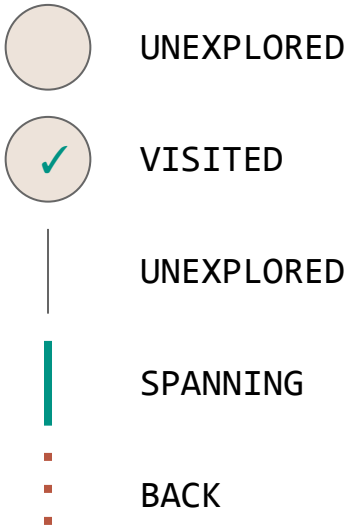
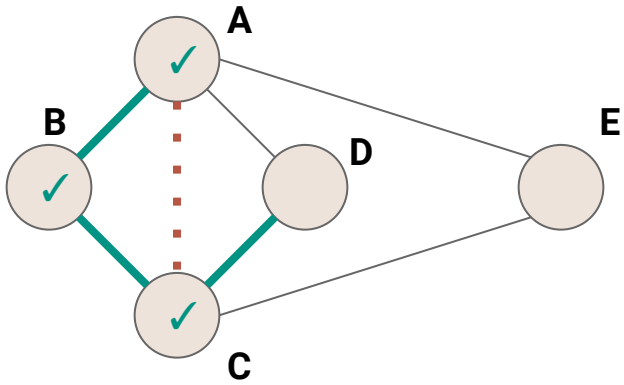


Call Stack	(→ edges to list)
DFS(G)	
DFSOne(G,A)	(→ B, C, D, E)
DFSOne(G,B)	(→ A, C)
DFSOne(G,C)	(→ B, A, D, E)



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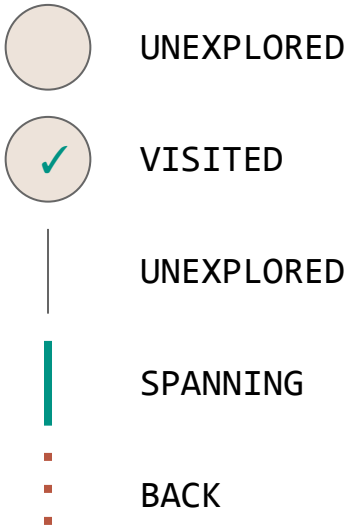
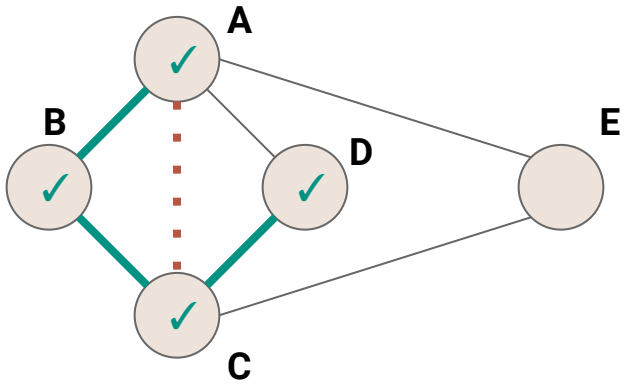


Call Stack	(→ edges to list)
DFS(G)	
DFSOne(G,A)	(→ B, C, D, E)
DFSOne(G,B)	(→ A, C)
DFSOne(G,C)	(→ B, A, D, E)
DFSOne(G,D)	(→ A, C)



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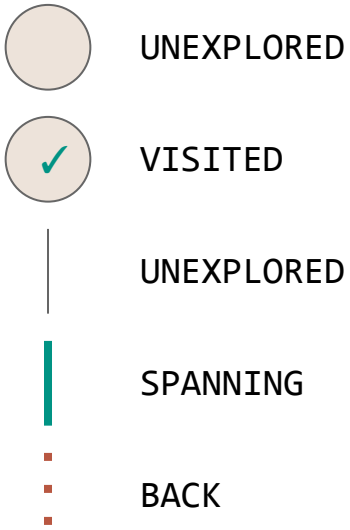

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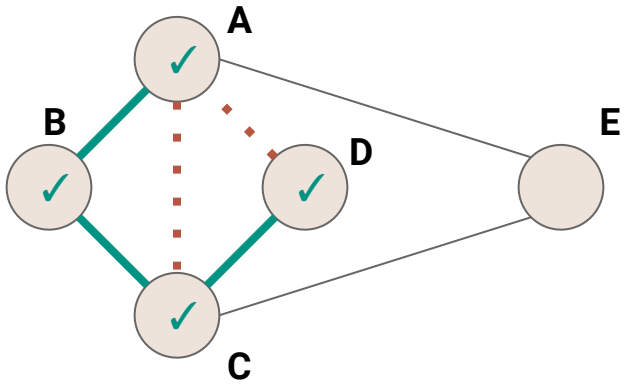

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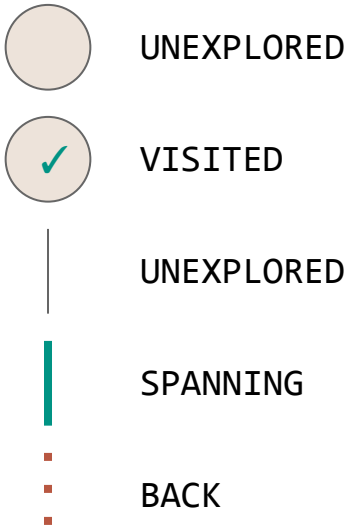

Call Stack	(→ edges to list)
DFS(G)	
DFSOne(G,A)	(→ B, C, D, E)
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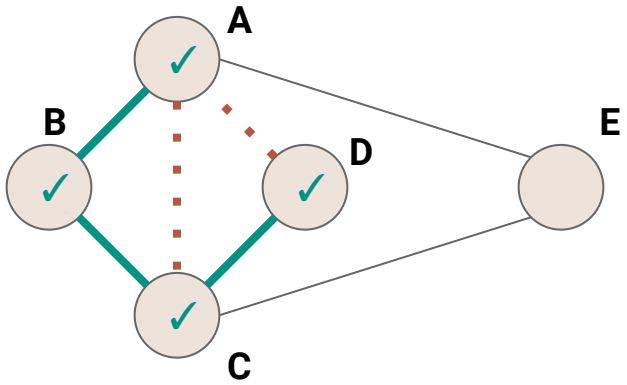

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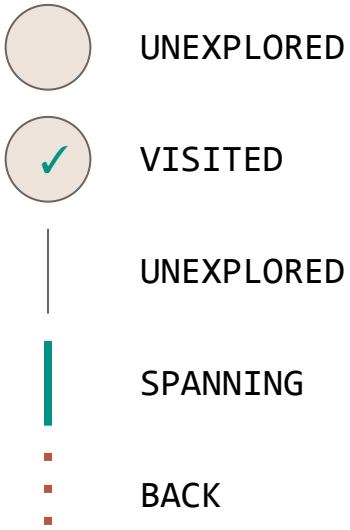

Call Stack	(→ edges to list)
DFS(G)	
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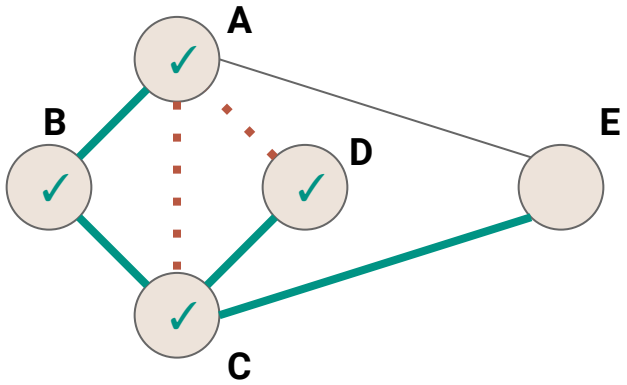

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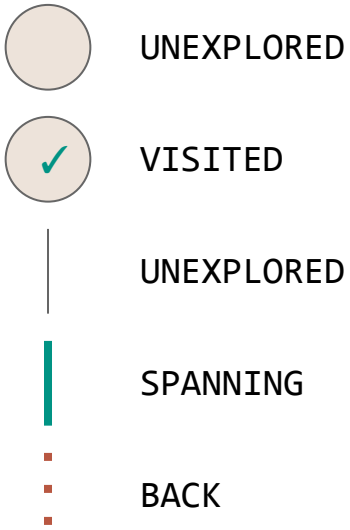

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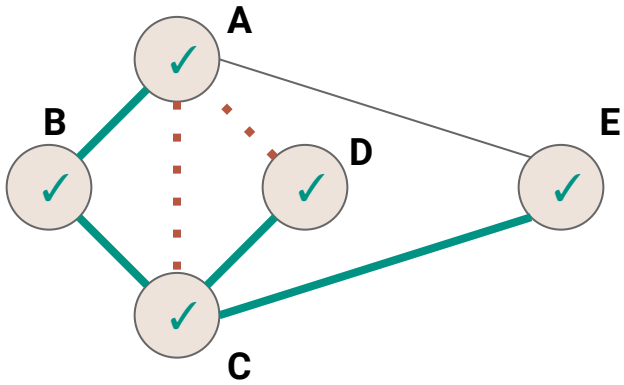

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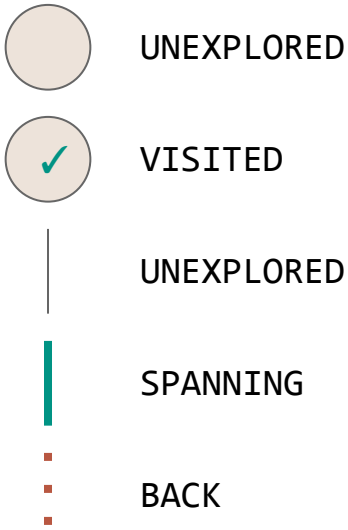

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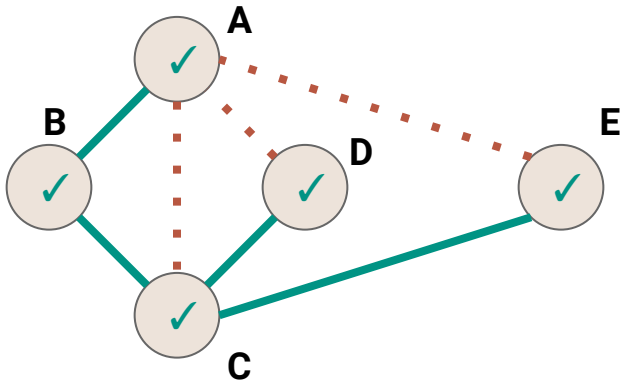
DFSOne(G,C)	(→ B, A, D, E)
DFSOne(G,E)	(→ A, C)



Detailed Example

UNEXPLORED

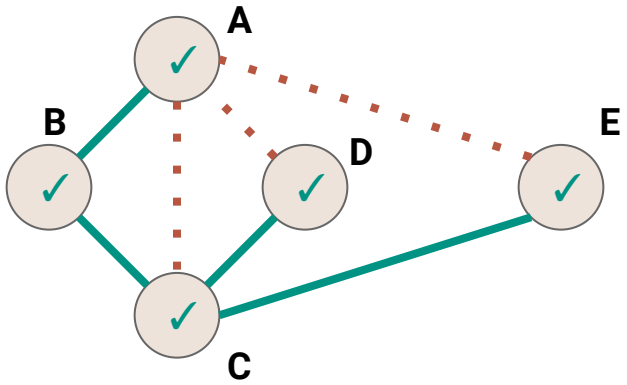
VISITED

UNEXPLORED

SPANNING

BACK

Call Stack	(→ edges to list)
DFS(G)	
DFSOne(G,A)	(→ B, C, D, E)
DFSOne(G,B)	(→ A, C)
DFSOne(G,C)	(→ B, A, D, E)



Detailed Example

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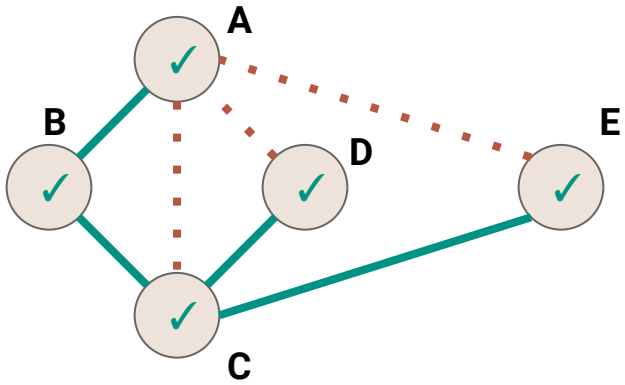
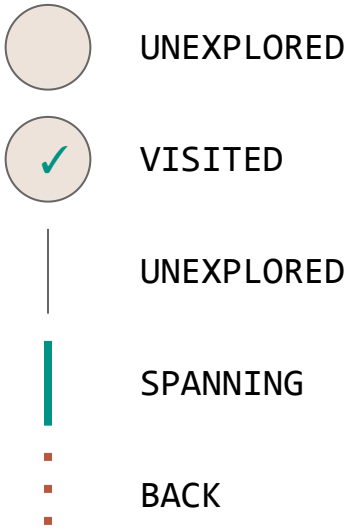


Detailed Example



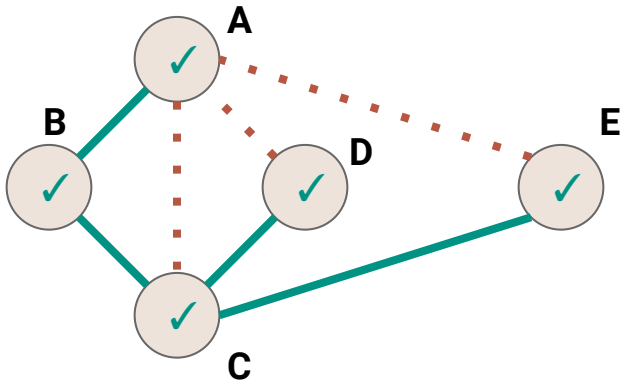
Call Stack

DFS(G)

DFSOne(G,A)

(→ edges to list)

(→ B, C, D, E)



Detailed Example

UNEXPLORED

VISITED

UNEXPLORED

SPANNING

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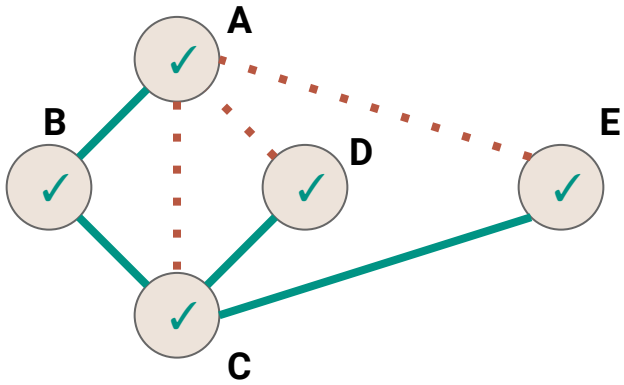
Call Stack
DFS(G)

(→ edges to list)



Detailed Example

UNEXPLORED

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UNEXPLORED

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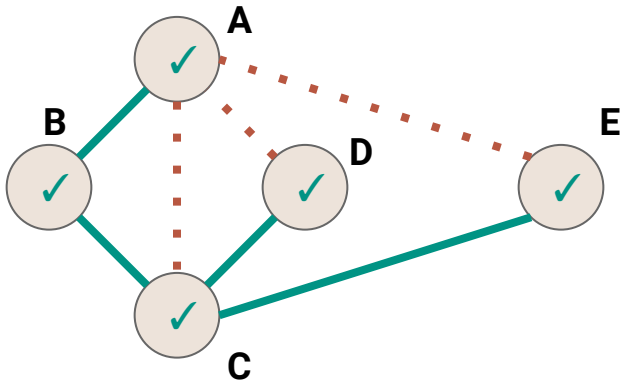
BACK

Call Stack

(→ edges to list)



DFS vs Mazes

The DFS algorithm is like our stack-based maze solver (kind of)

- Mark each grid square with **VISITED** as we explore it
- Mark each path with **SPANNING** or **BACK**
- Only visit each vertex once (this differs from our maze search)

DFS vs Mazes

The DFS algorithm is like our stack-based maze solver (kind of)

- Mark each grid square with **VISITED** as we explore it
- Mark each path with **SPANNING** or **BACK**
- Only visit each vertex once (this differs from our maze search)
  - DFS will not necessarily find the shortest paths

Depth-First Search Complexity

What's the complexity?

DFS

```
1  public void DFS(Graph graph) {
2    for (Vertex v : graph.vertices) {
3      v.setLabel(UNEXPLORED);
4    }
5    for (Edge e : graph.edges) {
6      e.setLabel(UNEXPLORED);
7    }
8    for (Vertex v : graph.vertices) {
9      if (v.label == UNEXPLORED) {
10       DFSOne(graph, v);
11     }
12   }
13 }
```

DFS

```
1  public void DFS(Graph graph) {
2    Θ(|V|)
3    for (Edge e : graph.edges) {
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DFS

```
1  public void DFS(Graph graph) {
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6        DFSOne(graph, v);
7      }
8    }
9  }
```

DFS

```
1  public void DFS(Graph graph) {
2    Θ(|V|)
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4    for (Vertex v : graph.vertices) {
5      if (v.label == UNEXPLORED) {
6        Θ(???)
7      }
8    }
9  }
```

DFSOne

```
1  public void DFSOne(Graph graph, Vertex v) {
2    v.setLabel(VISITED);
3    for (Edge e : v.outEdges) {
4      if (e.label == UNEXPLORED) {
5        Vertex w = e.to;
6        if (w.label == UNEXPLORED) {
7          e.setLabel(SPANNING);
8          DFSOne(graph, w);
9        } else {
10         e.setLabel(BACK);
11       }
12     }
13 }}
```

DFSOne

```
1  public void DFSOne(Graph graph, Vertex v) {
2    Θ(1)
3    for (Edge e : v.outEdges) {
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5        Θ(1)
6        if (w.label == UNEXPLORED) {
7          Θ(1)
8          Θ(???)
9        } else {
10         Θ(1)
11       }
12     }
13 }}
```

Depth-First Search Complexity

How many times do we call **DFSOne** *on each vertex?*

Depth-First Search Complexity

How many times do we call **`DFSOne`** *on each vertex?*

Observation: **`DFSOne`** is called on each vertex *at most* once

If **`v.label == VISITED`**, both **`DFS`**, and **`DFSOne`** skip it

Depth-First Search Complexity

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What's the runtime of **`DFSOne`** ***excluding the recursive calls?***

DFSOne

```
1  public void DFSOne(Graph graph, Vertex v) {
2    Θ(1)
3    for (Edge e : v.outEdges) {
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9        } else {
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13 }}
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DFSOne

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1  public void DFSOne(Graph graph, Vertex v) {
2    Θ(1)
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DFSOne

```
1  public void DFSOne(Graph graph, Vertex v) {
2    Θ(1)
3    for (Edge e : v.outEdges) {
4      Θ(1)
5    }
6  }
```

As long as we use an adjacency list this will be able to iterate through the adjacency edges in $\Theta(\deg(v))$ time

DFSOne

```
1  public void DFSOne(Graph graph, Vertex v) {
2    Θ(1)
3    Θ(deg(v))
4  }
```

Depth-First Search Complexity

How many times do we call **`DFSOne`** *on each vertex?*

Observation: **`DFSOne`** is called on each vertex *at most* once

If **`v.label == VISITED`**, both **`DFS`**, and **`DFSOne`** skip it

O(|***V***|) calls to **`DFSOne`**

What's the runtime of **`DFSOne`** ***excluding the recursive calls?***

Depth-First Search Complexity

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Observation: **`DFSOne`** is called on each vertex *at most* once

If **`v.label == VISITED`**, both **`DFS`**, and **`DFSOne`** skip it

O(|***V***|) calls to **`DFSOne`**

What's the runtime of **`DFSOne`** ***excluding the recursive calls?*** ***O***(deg(***v***))

Depth-First Search Complexity

What is the sum over all calls to **`DFSOne`**?

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$$\sum_{v \in V} O(deg(v))$$

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$$= O(2|E|)$$

Depth-First Search Complexity

What is the sum over all calls to **`DFSOne`**?

$$\sum_{v \in V} O(deg(v))$$

$$= O(\sum_{v \in V} deg(v))$$

$$= O(2|E|)$$

$$= O(|E|)$$

Depth-First Search Complexity

In summary…

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1. Mark the vertices **`UNVISITED`**

Depth-First Search Complexity

In summary…

1. Mark the vertices `UNVISITED` $O(|V|)$

Depth-First Search Complexity

In summary…

1. Mark the vertices **`UNVISITED`** $O(|V|)$
2. Mark the edges **`UNVISITED`**

Depth-First Search Complexity

In summary…

1. Mark the vertices `UNVISITED` $O(|V|)$
2. Mark the edges `UNVISITED` $O(|E|)$

Depth-First Search Complexity

In summary…

1. Mark the vertices `UNVISITED` $O(|V|)$
2. Mark the edges `UNVISITED` $O(|E|)$
3. `DFS` vertex loop

Depth-First Search Complexity

In summary…

1. Mark the vertices `UNVISITED` — $O(|V|)$
2. Mark the edges `UNVISITED` — $O(|E|)$
3. `DFS` vertex loop — $O(|V|)$ **iterations**

Depth-First Search Complexity

In summary…

1. Mark the vertices **`UNVISITED`**	$O(\|V\|)$
2. Mark the edges **`UNVISITED`**	$O(\|E\|)$
3. `DFS` vertex loop	**$O(\|V\|)$ iterations**
4. All calls to **`DFSOne`**	

Depth-First Search Complexity

In summary…

1. Mark the vertices **`UNVISITED`**	$O(\|V\|)$
2. Mark the edges **`UNVISITED`**	$O(\|E\|)$
3. `DFS` vertex loop	$O(\|V\|)$ **iterations**
4. All calls to **`DFSOne`**	$O(\|E\|)$ **total**

Depth-First Search Complexity

In summary…

1. Mark the vertices **`UNVISITED`**	$O(\|V\|)$
2. Mark the edges **`UNVISITED`**	$O(\|E\|)$
3. **`DFS`** vertex loop	$O(\|V\|)$ **iterations**
4. All calls to **`DFSOne`**	$O(\|E\|)$ **total**
	$O(\|V\| + \|E\|)$

DFS without Recursion

Our DFSOne implementation uses recursion for the search…

The recursive calls form a Stack…

Can we make a non-recursive implementation using a Stack explicitly?

```
1  public void DFSOneNoRecursion(Graph graph, Vertex v) {
2    Stack<Vertex> todo = new Stack<>();
3    v.setLabel(VISITED);
4    todo.push(v);
5    while (!todo.isEmpty()) {
6      Vertex curr = todo.pop();
7      for (Edge e : curr.outEdges) {
8        if (e.label == UNEXPLORED) {
9          Vertex w = e.to;
10         if (w.label == UNEXPLORED) {
11           w.setLabel(VISITED);
12           e.setLabel(SPANNING);
13           todo.push(w);
14         } else {
15           e.setLabel(BACK);
16         }
17 }}}}
```

```
1  public void DFSOneNoRecursion(Graph graph, Vertex v) {
2    Stack<Vertex> todo = new Stack<>();
3    v.setLabel(VISITED);
4    todo.push(v);
5    while (!todo.isEmpty()) {
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8        if (e.label == UNEXPLORED) {
9          Vertex w = e.to;
10         if (w.label == UNEXPLORED) {
11           curr.setLabel(VISITED);
12           e.setLabel(SPANNING);
13           todo.push(w);
14         } else {
15           e.setLabel(BACK);
16         }
17 }}}}
```

Use a stack to keep track of what vertices we want to visit (basically a running TODO list)

```
1  public void DFSOneNoRecursion(Graph graph, Vertex v) {
2    Stack<Vertex> todo = new Stack<>();
3    v.setLabel(VISITED);
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12           e.setLabel(SPANNING);
13           todo.push(w);
14         } else {
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16         }
17 }}}}
```

Pop a vertex from the Stack and check all of it's outgoing edges

```
1  public void DFSOneNoRecursion(Graph graph, Vertex v) {
2    Stack<Vertex> todo = new Stack<>();
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4    todo.push(v);
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8        if (e.label == UNEXPLORED) {
9          Vertex w = e.to;
10         if (w.label == UNEXPLORED) {
11           curr.setLabel(VISITED);
12           e.setLabel(SPANNING);
13           todo.push(w);
14         } else {
15           e.setLabel(BACK);
16         }
17 }}}}
```

When we find a new vertex, mark it as VISITED, and add it to our TODO list.

Remember, our TODO list is a stack (LIFO) so whatever we push last will be the next thing we pop (and explore)

Detailed Example

UNEXPLORED

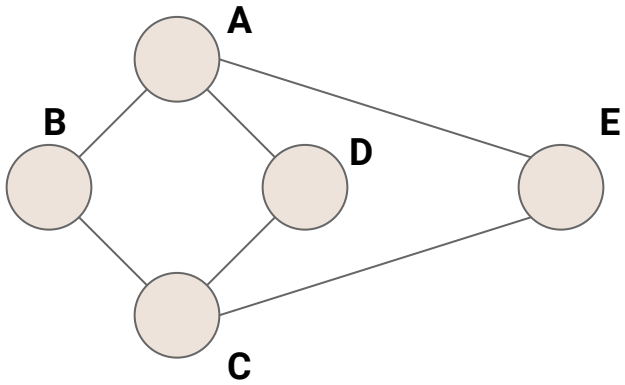
VISITED

UNEXPLORED

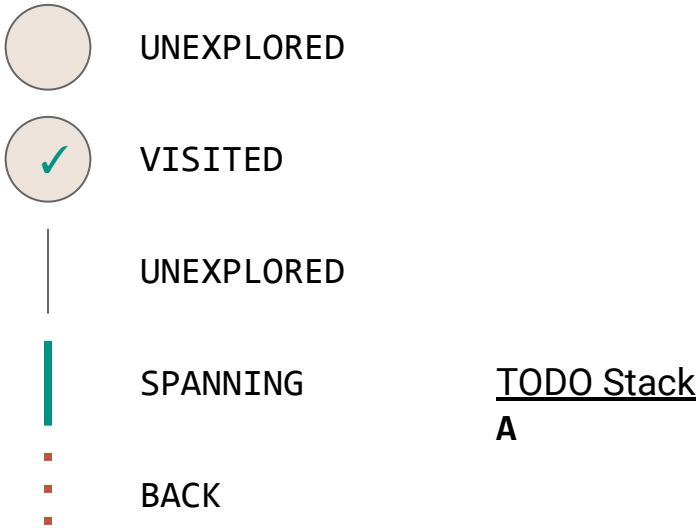
SPANNING

BACK

TODO Stack

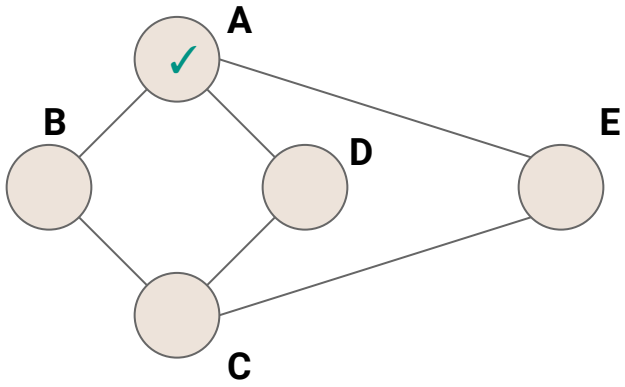


Detailed Example



TODO Stack

A



Detailed Example

UNEXPLORED

VISITED

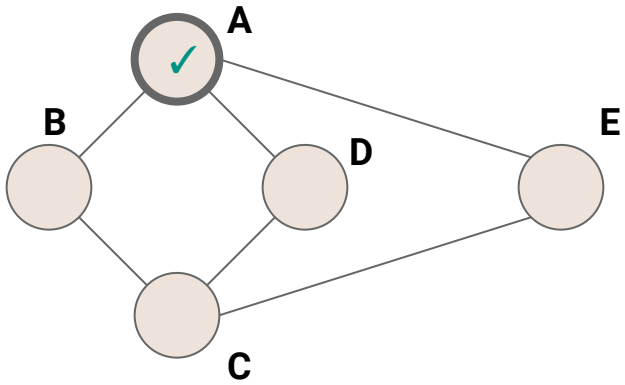
UNEXPLORED

SPANNING

BACK

Current Vertex: **A**

TODO Stack



Detailed Example

UNEXPLORED

VISITED

UNEXPLORED

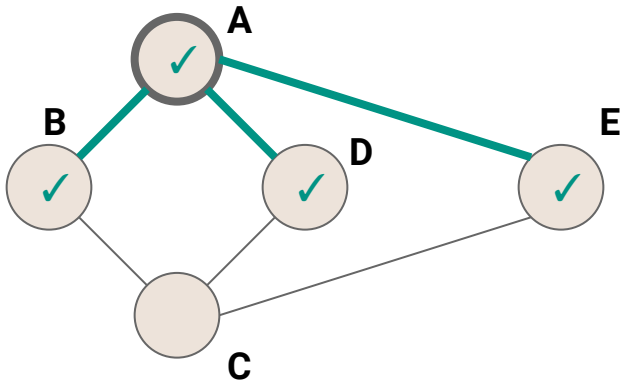
SPANNING

BACK

Current Vertex: **A**

TODO Stack

B
D
E



Detailed Example

UNEXPLORED

VISITED

UNEXPLORED

SPANNING

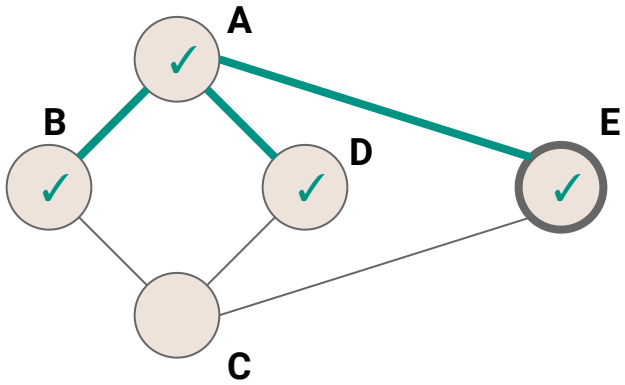
BACK

Current Vertex: **E**

TODO Stack

B

D



Detailed Example

UNEXPLORED

VISITED

UNEXPLORED

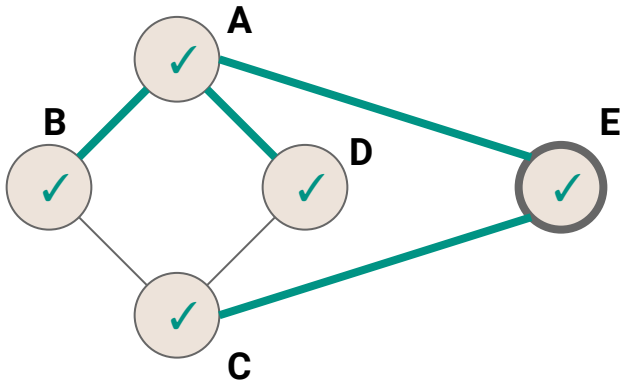
SPANNING

BACK

Current Vertex: **E**

<u>TODO Stack</u>

B
D
C



Detailed Example

UNEXPLORED

VISITED

UNEXPLORED

SPANNING

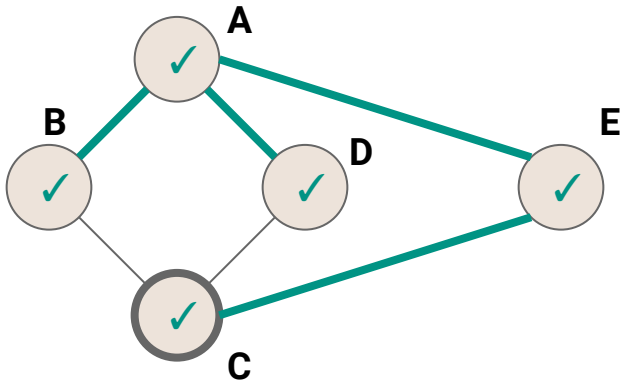
BACK

Current Vertex: **C**

TODO Stack

B

D



Detailed Example

UNEXPLORED

VISITED

UNEXPLORED

SPANNING

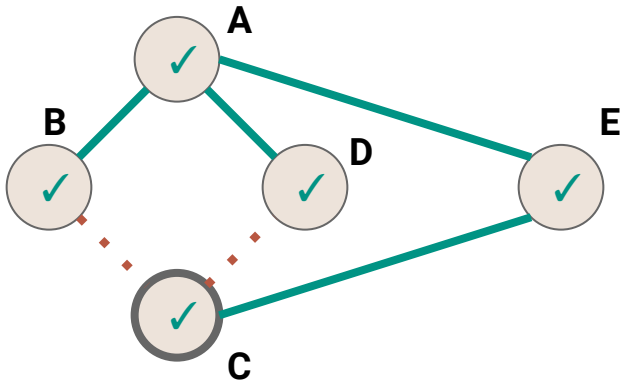
BACK

Current Vertex: **C**

TODO Stack

B

D



Detailed Example

UNEXPLORED

VISITED

UNEXPLORED

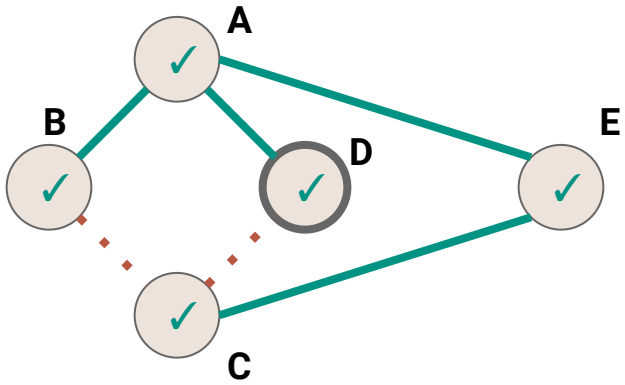
SPANNING

BACK

Current Vertex: **D**

TODO Stack

B



Detailed Example

UNEXPLORED

VISITED

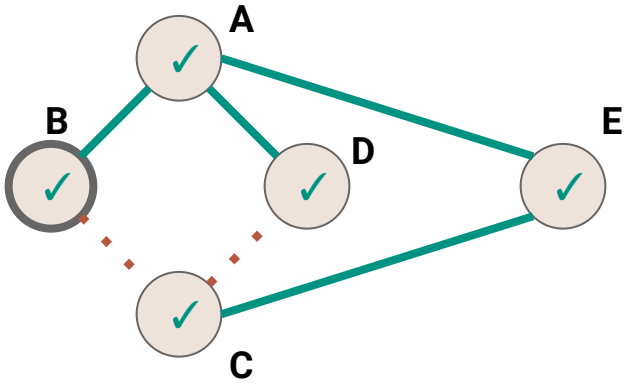
UNEXPLORED

SPANNING

BACK

Current Vertex: **B**

TODO Stack



```
1  public void DFSOneNoRecursion(Graph graph, Vertex v) {
2    Stack<Vertex> todo = new Stack<>();
3    v.setLabel(VISITED);
4    todo.push(v);
5    while (!todo.isEmpty()) {
6      Vertex curr = todo.pop();
7      for (Edge e : curr.outEdges) {
8        if (e.label == UNEXPLORED) {
9          Vertex w = e.to;
10         if (w.label == U
11           curr.setLabel(
12           e.setLabel(SPA
13           todo.push(w);
14         } else {
15           e.setLabel(BACK);
16         }
17 }}}}
```

Now back to our burning question…

What happens if we use a Queue to do our search instead of a Stack?

Breadth-First Search

Breadth-First Search

Primary Goals

- Visit every vertex in graph $G = (V,E)$
- Construct a spanning tree for every connected component
  - **Side Effect:** Compute connected components
  - **Side Effect:** Compute a path between all connected vertices
  - **Side Effect:** Determine if the graph is connected
  - **Side Effect:** Identify cycles
- Complete in time $O(|V| + |E|)$, with memory overhead $O(|V|)$

Breadth-First Search

Primary Goals

- Visit every vertex in graph $G = (V,E)$ **in increasing order of distance from the start**
- Construct a spanning tree for every connected component
  - **Side Effect:** Compute connected components
  - **Side Effect:** Compute a path between all connected vertices
  - **Side Effect:** Determine if the graph is connected
  - **Side Effect:** Identify cycles
  - **Side Effect: Identify shortest paths to the starting vertex**
- Complete in time $O(|V| + |E|)$, with memory overhead $O(|V|)$

BFS

```
1  public void BFS(Graph graph) {
2    for (Vertex v : graph.vertices) {
3      v.setLabel(UNEXPLORED);
4    }
5    for (Edge e : graph.edges) {
6      e.setLabel(UNEXPLORED);
7    }
8    for (Vertex v : graph.vertices) {
9      if (v.label == UNEXPLORED) {
10       BFSOne(graph, v);
11     }
12   }
13 }
```

Same as DFS driver function…just make sure that we explore EVERY vertex, even if the graph is disconnected

```
1  public void BFSOne(Graph graph, Vertex v) {
2    Queue<Vertex> todo = new Queue<>();
3    v.setLabel(VISITED);
4    todo.enqueue(v);
5    while (!todo.isEmpty()) {
6      Vertex curr = todo.dequeue();
7      for (Edge e : curr.outEdges) {
8        if (e.label == UNEXPLORED) {
9          Vertex w = e.to;
10         if (w.label == UNEXPLORED) {
11           w.setLabel(VISITED);
12           e.setLabel(SPANNING);
13           todo.enqueue(w);
14         } else {
15           e.setLabel(CROSS);
16         }
17 }}}}
```

```
1  public void BFSOne(Graph graph, Vertex v) {
2    Queue<Vertex> todo = new Queue<>();
3    v.setLabel(VISITED);
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6      Vertex curr = todo.dequeue();
7      for (Edge e : curr.outEdges) {
8        if (e.label == UNEXPLORED) {
9          Vertex w = e.to;
10         if (w.label == UNEXPLORED) {
11           w.setLabel(VISITED);
12           e.setLabel(SPANNING);
13           todo.enqueue(w);
14         } else {
15           e.setLabel(CROSS);
16         }
17 }}}}
```

Use a queue to keep track of what vertices we want to visit (basically a running TODO list)

```
1  public void BFSOne(Graph graph, Vertex v) {
2    Queue<Vertex> todo = new Queue<>();
3    v.setLabel(VISITED);
4    todo.enqueue(v);
5    while (!todo.isEmpty()) {
6      Vertex curr = todo.dequeue();
7      for (Edge e : curr.outEdges) {
8        if (e.label == UNEXPLORED) {
9          Vertex w = e.to;
10         if (w.label == UNEXPLORED) {
11           w.setLabel(VISITED);
12           e.setLabel(SPANNING);
13           todo.enqueue(w);
14         } else {
15           e.setLabel(CROSS);
16         }
17 }}}}
```

Dequeue a vertex from the Queue and check all of it's outgoing edges

```
1  public void BFSOne(Graph graph, Vertex v) {
2    Queue<Vertex> todo = new Queue<>();
3    v.setLabel(VISITED);
4    todo.enqueue(v);
5    while (!todo.isEmpty()) {
6      Vertex curr = todo.dequeue();
7      for (Edge e : curr.outEdges) {
8        if (e.label == UNEXPLORED) {
9          Vertex w = e.to;
10         if (w.label == UNEXPLORED) {
11           w.setLabel(VISITED);
12           e.setLabel(SPANNING);
13           todo.enqueue(w);
14         } else {
15           e.setLabel(CROSS);
16         }
17 }}}}
```

When we find a new vertex, mark it as VISITED, and add it to our TODO list.

Remember, our TODO list is a Queue (FIFO) so whatever we enqueud first will be the next thing we dequeue (and explore)

```
1  public void BFSOne(Graph graph, Vertex v) {
2    Queue<Vertex> todo = new Queue<>();
3    v.setLabel(VISITED);
4    todo.enqueue(v);
5    while (!todo.isEmpty()) {
6      Vertex curr = todo.dequeue();
7      for (Edge e : curr.outEdges) {
8        if (e.label == UNEXPLORED) {
9          Vertex w = e.to;
10         if (w.label == UNEXPLORED) {
11           w.setLabel(VISITED);
12           e.setLabel(SPANNING);
13           todo.enqueue(w);
14         } else {
15           e.setLabel(CROSS);
16         }
17 }}}}
```

When doing BFS we label edges that return to visited vertices as CROSS edges

Detailed Example

UNEXPLORED

VISITED

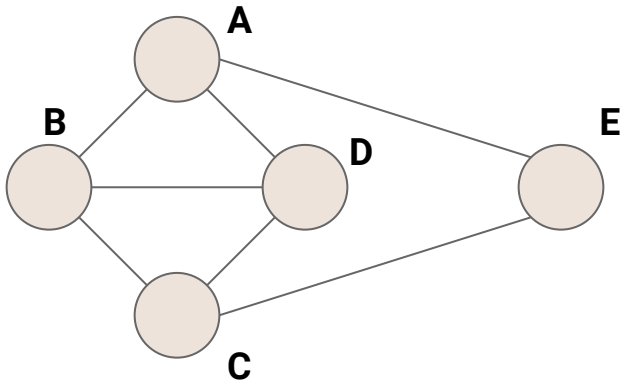
UNEXPLORED

SPANNING

CROSS

Call Stack

Work Queue



Detailed Example

UNEXPLORED

VISITED

UNEXPLORED

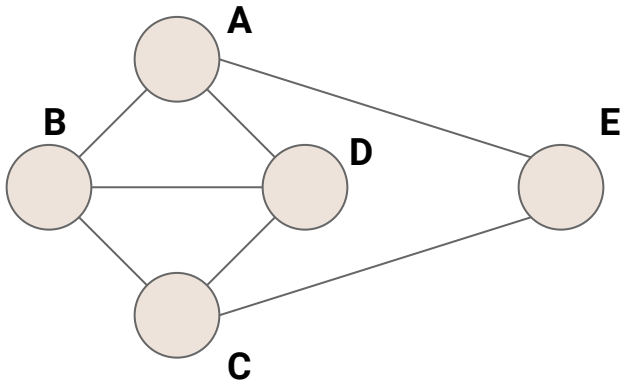
SPANNING

CROSS

Call Stack

BFS(G)

Work Queue



Detailed Example

UNEXPLORED

VISITED

UNEXPLORED

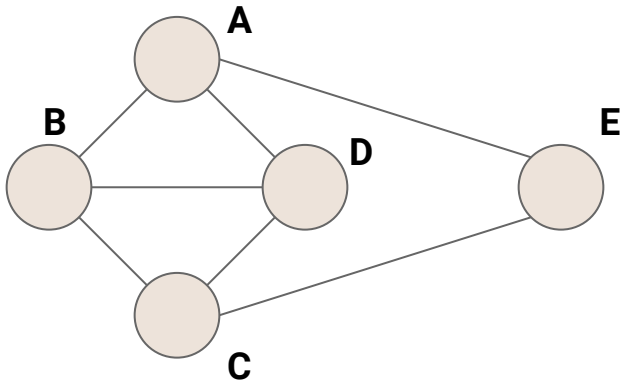
SPANNING

CROSS

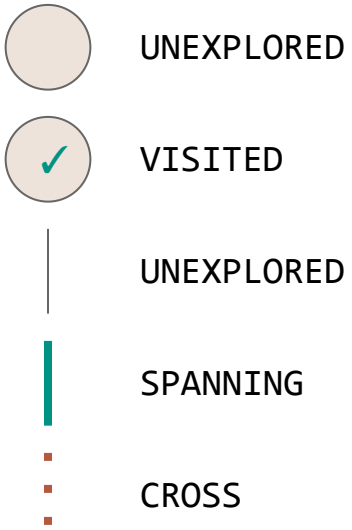
Call Stack

BFS(G)
BFSOne(G,A)

Work Queue

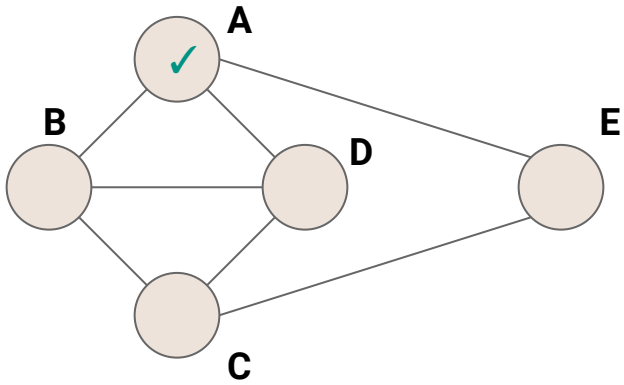


Detailed Example

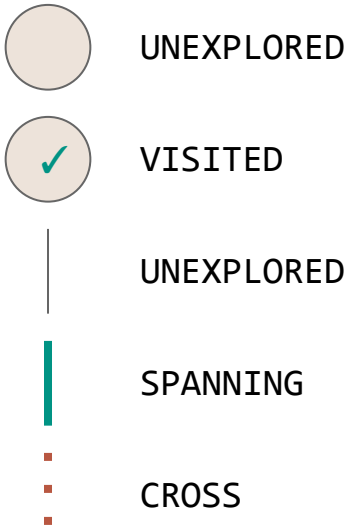


Call Stack
BFS(G)
BFSOne(G,A)

Work Queue
A

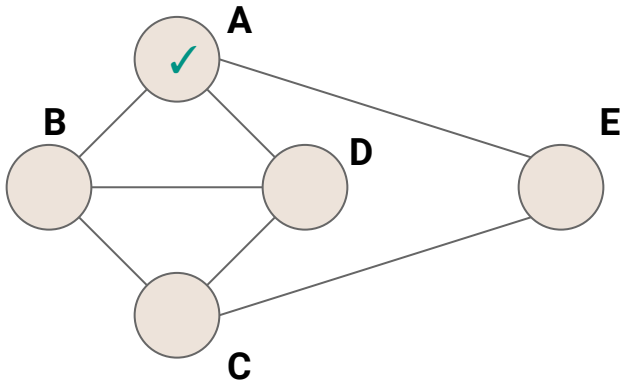


Detailed Example

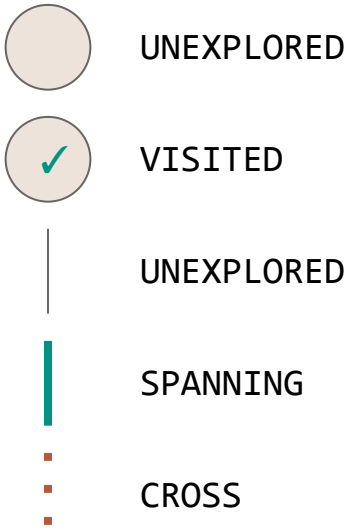


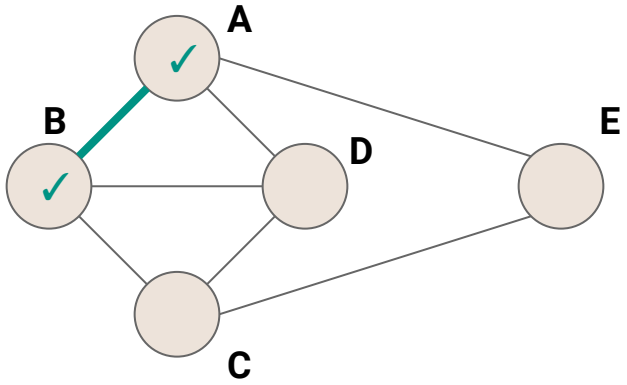
Call Stack
BFS(G)
BFSOne(G,A)

Work Queue
~~A~~



Detailed Example

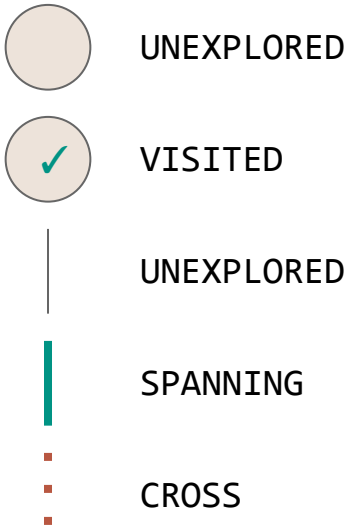


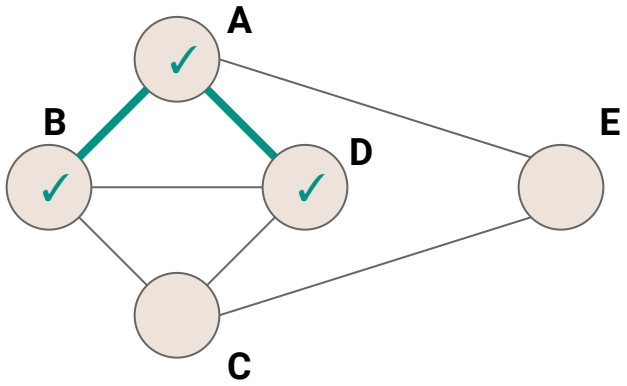


Call Stack
BFS(G)
BFSOne(G,A)

Work Queue
~~A~~
B

Detailed Example

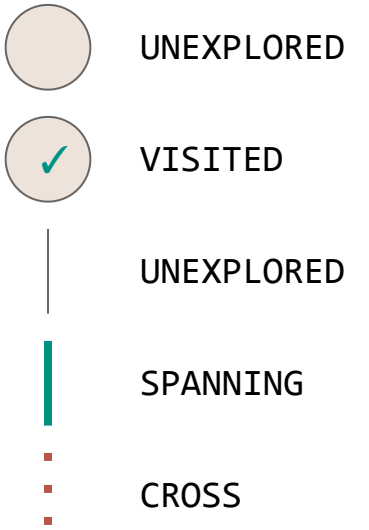


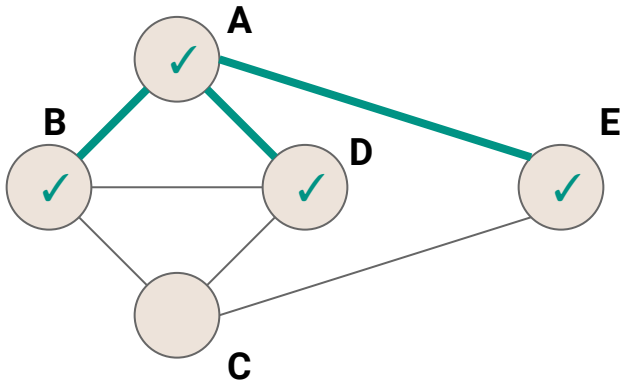


Call Stack
BFS(G)
BFSOne(G,A)

Work Queue
~~A~~
B
D

Detailed Example

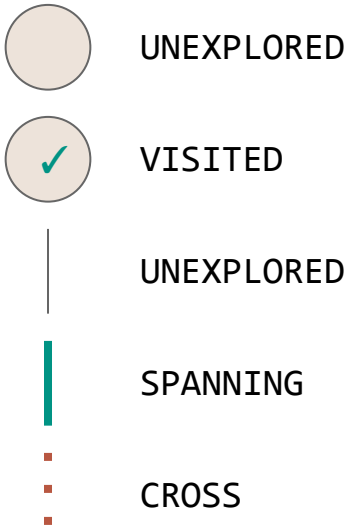


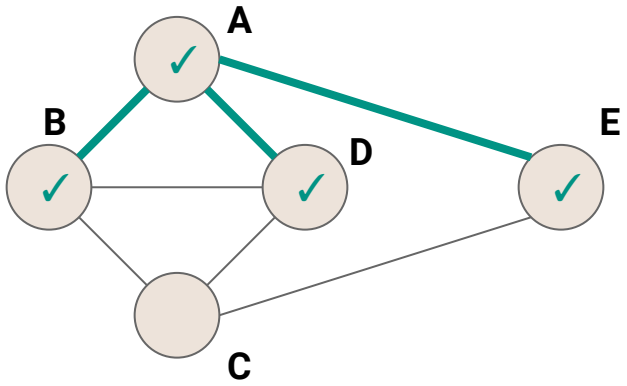


Call Stack
BFS(G)
BFSOne(G,A)

Work Queue
~~A~~
B
D
E

Detailed Example

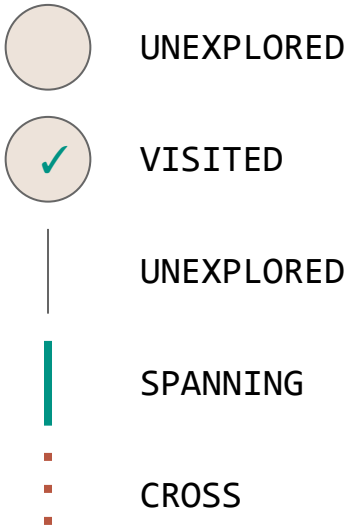


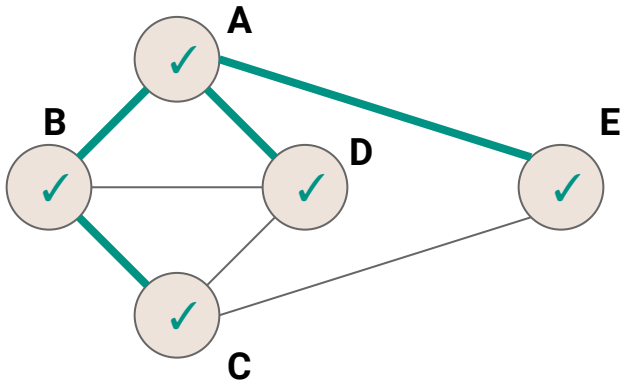


Call Stack
BFS(G)
BFSOne(G,A)

Work Queue
~~**A**~~
~~**B**~~
D
E

Detailed Example

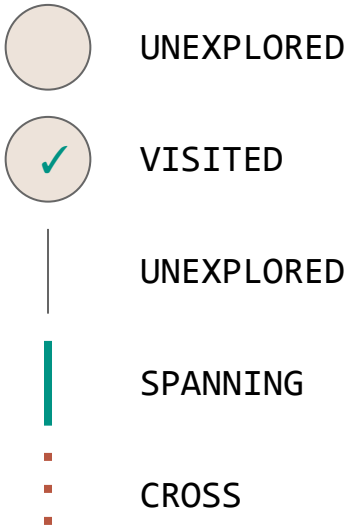


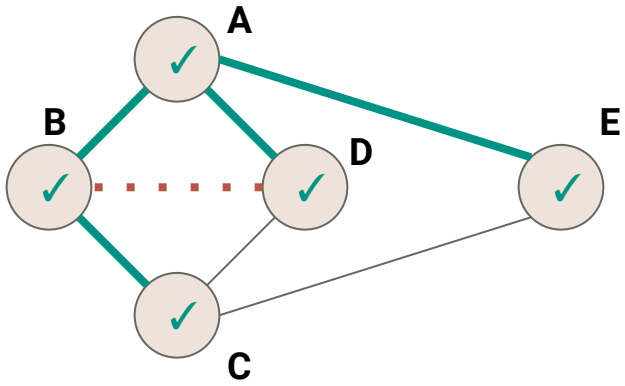


Call Stack
BFS(G)
BFSOne(G,A)

Work Queue
~~**A**~~
~~**B**~~
D
E
C

Detailed Example

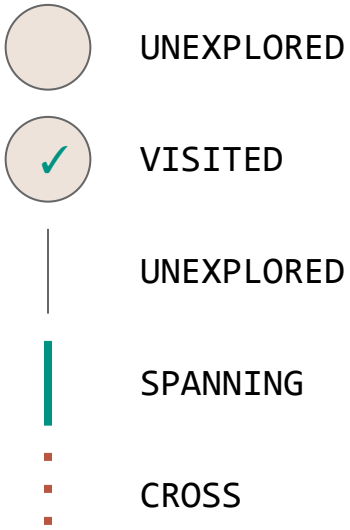


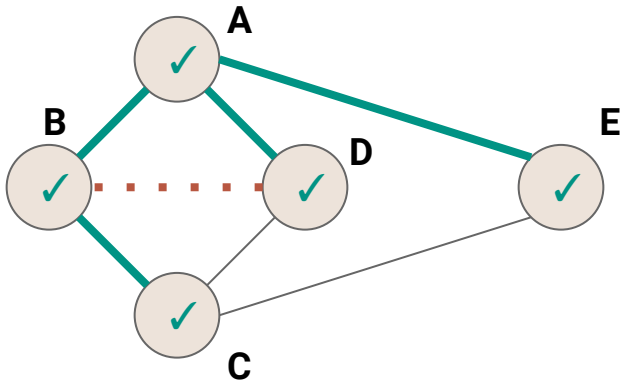


Call Stack
BFS(G)
BFSOne(G,A)

Work Queue
~~**A**~~
~~**B**~~
D
E
C

Detailed Example

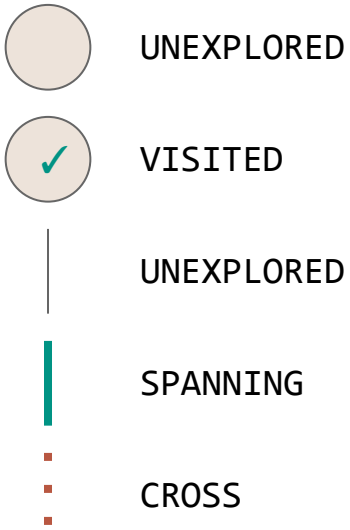




Call Stack
BFS(G)
BFSOne(G,A)

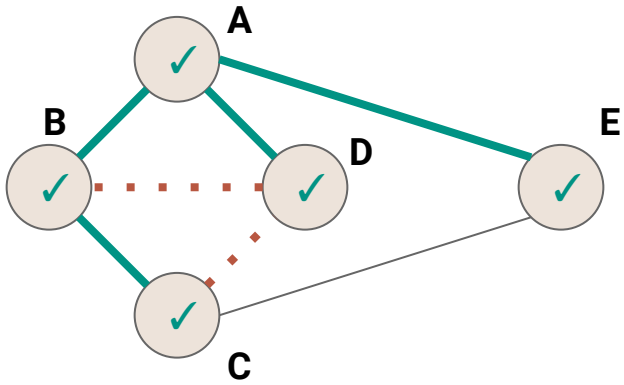
Work Queue
~~**A**~~
~~**B**~~
~~**D**~~
E
C

Detailed Example

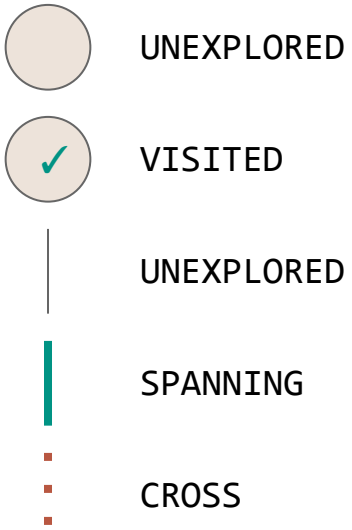


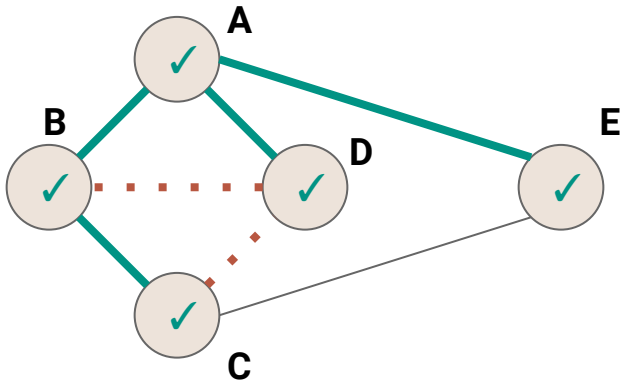
Call Stack
BFS(G)
BFSOne(G,A)

Work Queue
~~**A**~~
~~**B**~~
~~**D**~~
E
C



Detailed Example

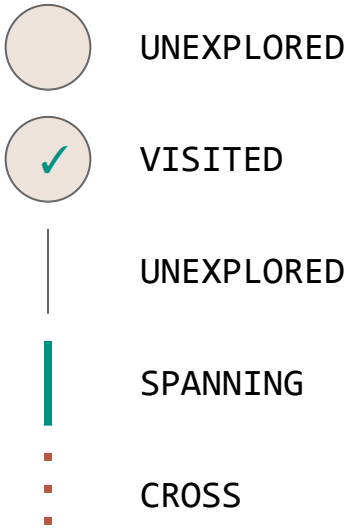


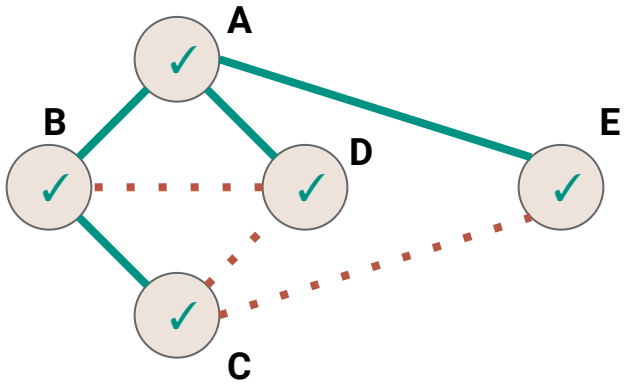


Call Stack
BFS(G)
BFSOne(G,A)

Work Queue
~~A~~
~~B~~
~~D~~
~~E~~
C

Detailed Example

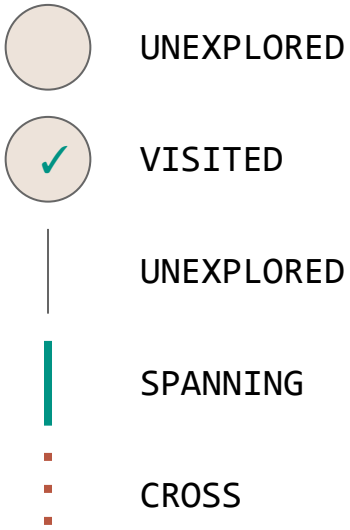


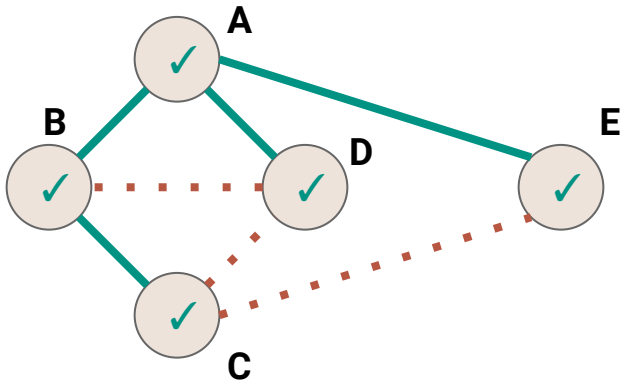


Call Stack
BFS(G)
BFSOne(G,A)

Work Queue
~~A~~
~~B~~
~~D~~
~~E~~
C

Detailed Example





Call Stack

```
BFS(G)
BFSOne(G,A)
```

Work Queue

~~A~~
~~B~~
~~D~~
~~E~~
~~C~~

BFS Complexity

```
 1 public void BFS(Graph graph) {
 2   for (Vertex v : graph.vertices) {
 3     v.setLabel(UNEXPLORED);
 4   }
 5   for (Edge e : graph.edges) {
 6     e.setLabel(UNEXPLORED);
 7   }
 8   for (Vertex v : graph.vertices) {
 9     if (v.label == UNEXPLORED) {
10       BFSOne(graph, v);
11     }
12   }
13 }
```

BFS Complexity

```
1  public void BFS(Graph graph) {
2    Θ(|V|)
3    Θ(|E|)
4    for (Vertex v : graph.vertices) {
5      if (v.label == UNEXPLORED) {
6        BFSOne(graph, v);
7      }
8    }
9  }
```

BFS Complexity

```
1  public void BFS(Graph graph) {
2    Θ(|V|)
3    Θ(|E|)
4    for (Vertex v : graph.vertices) {
5      if (v.label == UNEXPLORED) {
6        Θ(???)
7      }
8    }
9  }
```

```
1  public void BFSOne(Graph graph, Vertex v) {
2    Queue<Vertex> todo = new Queue<>();
3    v.setLabel(VISITED);
4    todo.enqueue(v);
5    while (!todo.isEmpty()) {
6      Vertex curr = todo.dequeue();
7      for (Edge e : curr.outEdges) {
8        if (e.label == UNEXPLORED) {
9          Vertex w = e.to;
10         if (w.label == UNEXPLORED) {
11           curr.setLabel(VISITED);
12           e.setLabel(SPANNING);
13           todo.enqueue(w);
14         } else {
15           e.setLabel(CROSS);
16         }
17 }}}}
```

```
1  public void BFSOne(Graph graph, Vertex v) {
2    Θ(1)
3    while (!todo.isEmpty()) {
4      Vertex curr = todo.dequeue();
5      for (Edge e : curr.outEdges) {
6        if (e.label == UNEXPLORED) {
7          Vertex w = e.to;
8          if (w.label == UNEXPLORED) {
9            curr.setLabel(VISITED);
10           e.setLabel(SPANNING);
11           todo.enqueue(w);
12         } else {
13           e.setLabel(CROSS);
14         }
15 }}}}
```

```
1  public void BFSOne(Graph graph, Vertex v) {
2    Θ(1)
3    while (!todo.isEmpty()) {
4      Θ(1)
5      for (Edge e : curr.outEdges) {
6        if (e.label == UNEXPLORED) {
7          Vertex w = e.to;
8          if (w.label == UNEXPLORED) {
9            curr.setLabel(VISITED);
10           e.setLabel(SPANNING);
11           todo.enqueue(w);
12         } else {
13           e.setLabel(CROSS);
14         }
15 }}}}
```

```
1  public void BFSOne(Graph graph, Vertex v) {
2    Θ(1)
3    while (!todo.isEmpty()) {
4      Θ(1)
5      for (Edge e : curr.outEdges) {
6        if (e.label == UNEXPLORED) {
7          Θ(1)
8          if (w.label == UNEXPLORED) {
9            Θ(1)
10           todo.enqueue(w);
11         } else {
12           Θ(1)
13         }
14 }}}}
15
```

```
1  public void BFSOne(Graph graph, Vertex v) {
2    Θ(1)
3    while (!todo.isEmpty()) {
4      Θ(1)
5      for (Edge e : curr.outEdges) {
6        if (e.label == UNEXPLORED) {
7          Θ(1)
8          if (w.label == UNEXPLORED) {
9            Θ(1)
10           Θ(1)
11         } else {
12           Θ(1)
13         }
14 }}}}
15
```

```
1  public void BFSOne(Graph graph, Vertex v) {
2    Θ(1)
3    while (!todo.isEmpty()) {
4      Θ(1)
5      for (Edge e : curr.outEdges) {
6        Θ(1)
7      }
8    }
9  }
```

```
1  public void BFSOne(Graph graph, Vertex v) {
2    Θ(1)
3    while (!todo.isEmpty()) {
4      Θ(1)
5      Θ(deg(v))
6    }
7  }
8
9
```

```
1  public void BFSOne(Graph graph, Vertex v) {
2    Θ(1)
3    while (!todo.isEmpty()) {
4      Θ(1)
5      Θ(deg(v))
6    }
7  }
8
9
```

How many iterations will this while loop run?

```
1  public void BFSOne(Graph graph, Vertex v) {
2    Θ(1)
3    while (!todo.isEmpty()) {
4      Θ(1)
5      Θ(deg(v))
6    }
7  }
8
9
```

How many iterations will this while loop run?
Each vertex will be enqueued exactly ONCE

```
1  public void BFSOne(Graph graph, Vertex v) {
2    Θ(1)
3    while (!todo.isEmpty()) {
4      Θ(1)
5      Θ(deg(v))
6    }
7  }
8
9
```

How many iterations will this while loop run?
Each vertex will be enqueued exactly ONCE
The cost to process each vertex is deg(v)

Breadth-First Search Complexity

What is the sum over all iterations in **BFSOne**?

Breadth-First Search Complexity

What is the sum over all iterations in **BFSOne**?

$$\sum_{v \in V} O(deg(v))$$

Breadth-First Search Complexity

What is the sum over all iterations in **`BFSOne`**?

$$\sum_{v \in V} O(deg(v))$$

$$= O(\sum_{v \in V} deg(v))$$

Breadth-First Search Complexity

What is the sum over all iterations in **`BFSOne`**?

$$\sum_{v \in V} O(deg(v))$$

$$= O(\sum_{v \in V} deg(v))$$

$$= O(2|E|)$$

Breadth-First Search Complexity

What is the sum over all iterations in **`BFSOne`**?

$$\sum_{v \in V} O(deg(v))$$

$$= O(\sum_{v \in V} deg(v))$$

$$= O(2|E|)$$

$$= O(|E|)$$

Breadth-First Search Complexity

In summary…

1. Mark the vertices **`UNVISITED`**

Breadth-First Search Complexity

In summary…

1. Mark the vertices **`UNVISITED`** $O(|V|)$

Breadth-First Search Complexity

In summary…

1. Mark the vertices **`UNVISITED`** $O(|V|)$
2. Mark the edges **`UNVISITED`**

Breadth-First Search Complexity

In summary…

1. Mark the vertices `UNVISITED` $O(|V|)$
2. Mark the edges `UNVISITED` $O(|E|)$

Breadth-First Search Complexity

In summary…

1. Mark the vertices `UNVISITED` $O(|V|)$
2. Mark the edges `UNVISITED` $O(|E|)$
3. Add each vertex to the work queue

Breadth-First Search Complexity

In summary…

1. Mark the vertices `UNVISITED` $O(|V|)$
2. Mark the edges `UNVISITED` $O(|E|)$
3. Add each vertex to the work queue $O(|V|)$

Breadth-First Search Complexity

In summary…

1. Mark the vertices `UNVISITED` $O(|V|)$
2. Mark the edges `UNVISITED` $O(|E|)$
3. Add each vertex to the work queue $O(|V|)$
4. Process each vertex

Breadth-First Search Complexity

In summary…

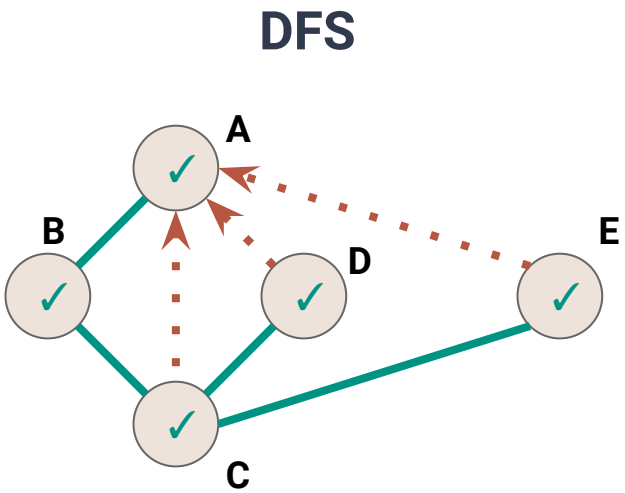
1. Mark the vertices `UNVISITED` $O(|V|)$
2. Mark the edges `UNVISITED` $O(|E|)$
3. Add each vertex to the work queue $O(|V|)$
4. Process each vertex $O(|E|)$

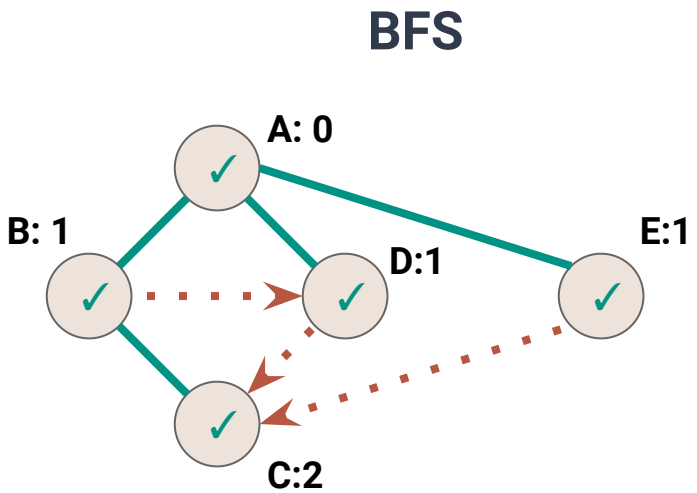
Breadth-First Search Complexity

In summary…

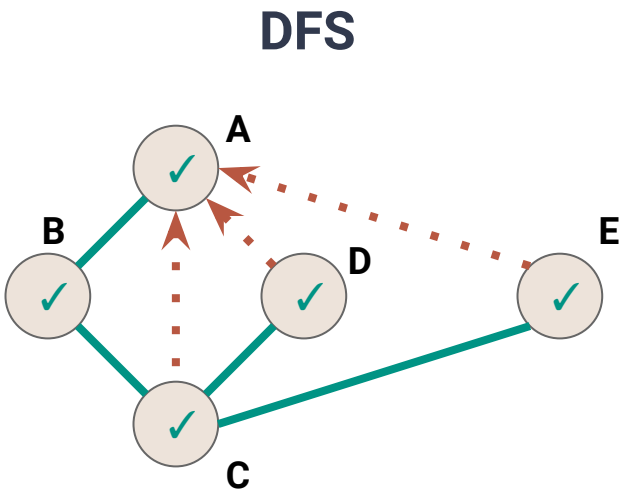
1.	Mark the vertices `UNVISITED`	$O(\|V\|)$
2.	Mark the edges `UNVISITED`	$O(\|E\|)$
3.	Add each vertex to the work queue	$O(\|V\|)$
4.	Process each vertex	$O(\|E\|)$
		$O(\|V\| + \|E\|)$

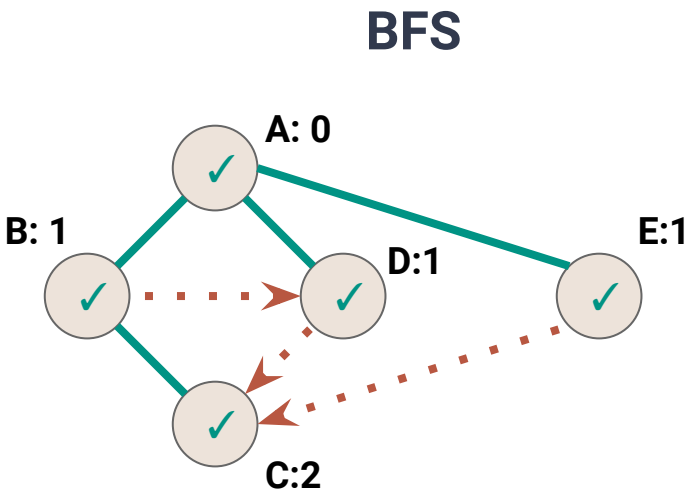
DFS vs BFS





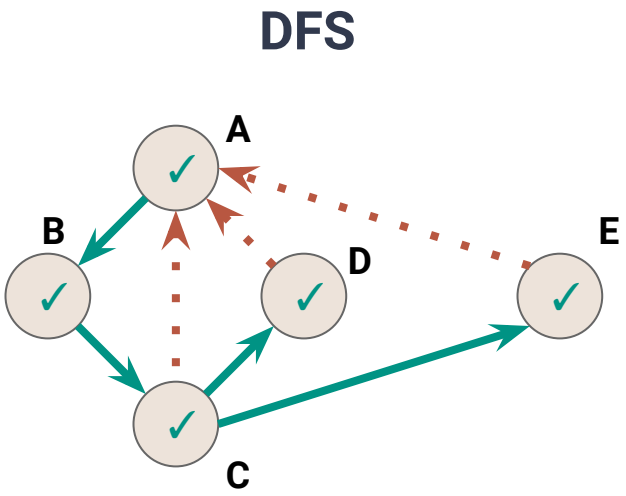
DFS vs BFS

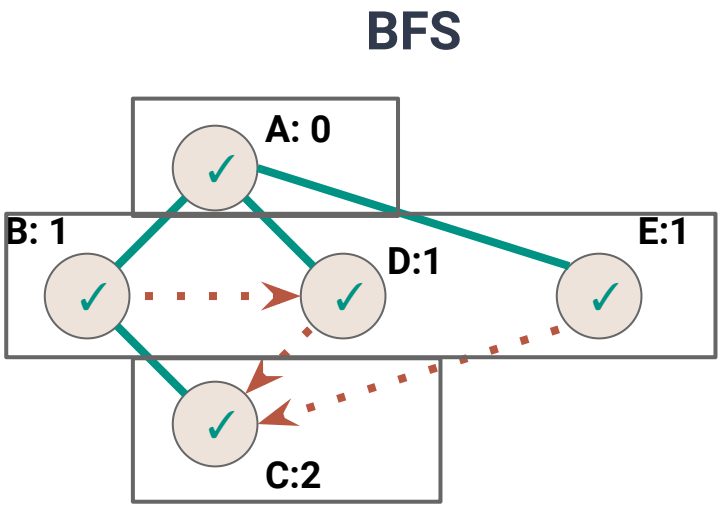




BACK Edge(*v*,*w*): ***w*** is an ancestor of ***v*** in the discovery tree

DFS vs BFS





BACK Edge(*v*,*w*): ***w*** is an ancestor of ***v*** in the discovery tree

CROSS Edge(*v*,*w*): ***w*** is at the same or next level as ***v***

DFS Traversal vs BFS Traversal

Application	DFS	BFS
Spanning Trees	✓	✓
Connected Components	✓	✓
Paths/Connectivity	✓	✓
Cycles	✓	✓
Shortest Paths		✓
Articulation Points	✓	