CSE 250: DFS+BFS; Dijkstra's Algorithm

Lecture 21

Oct 20, 2023

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Reminders

- PA2 released yesterday: Implement **Map Routing**
  1. Create an adjacency list (discussed today)
  2. Find a path from A to B with the fewest intersections
  3. Find a path from A to B with the shortest distance
- PA2 test cases due Sun, Oct 22 at 11:59 PM
- PA2 implementation due Sun, Nov 5 at 11:59 PM

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Depth First Search

Start with some node A.

1. If we've explored all edges from A, return
2. Pick an unexplored edge
3. If the edge connects to an explored vertex, it's a back edge
4. Otherwise recur with A = the other vertex.

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Depth First Search

Mark Vertices `UNVISITED`	$O(N)$
Mark Edges `UNVISITED`	$O(M)$
`DFS` Vertex Loop	$O(N)$
All calls to `DFSOne`	$O(M)$
	$O(N+M)$

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Breadth First Search

Set up a todo list with your start node. Until the todo list is empty:

1. Take the next vertex on the todo list.
2. For each edge leaving the vertex node:
    1. If the edge is explored, leave it alone.
    2. If the edge connects to an explored vertex, mark it as a back edge.
    3. If the vertex is unexplored, mark it explored and add it to the todo list.

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To summarize...

Mark Vertices `UNVISITED`	$O(N)$
Mark Edges `UNVISITED`	$O(M)$
Add each vertex to the queue	$O(N)$
Process all vertices	$O(M)$
	$O(N + M)$

(But requires $O(N)$ memory for the queue)

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DFS vs BFS

DFS	**BFS**
Long Paths	Shortest Paths
Low Memory	Mid/High Memory

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Other questions

What if we want the path?

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Todo

```
1  public class Todo
2  {
3    Vertex vertex;
4    List<Vertex> path;
5  }
```

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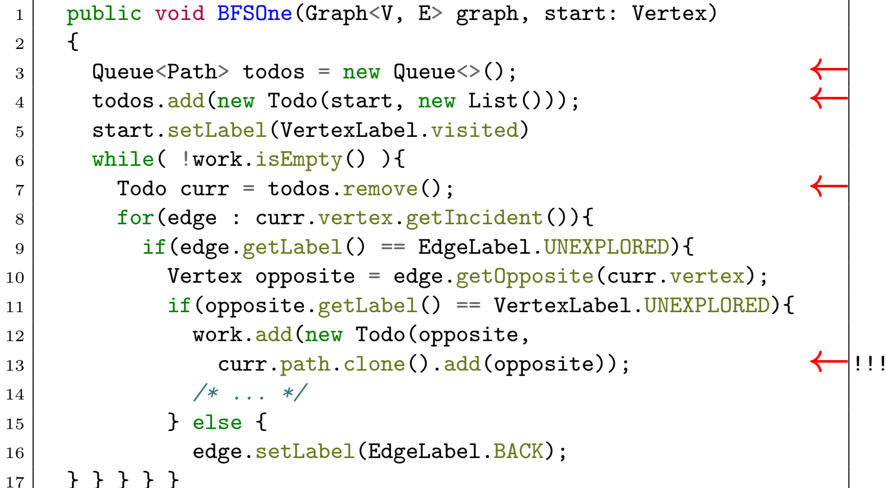
BFSOne - Pathfinding



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BFSOne - Pathfinding

Problem: "Copying" the path can be $O(N)$

Idea: Store the 'previous hop' for each vertex.

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BFSOne - Pathfinding

```
1  public void BFSOne(Graph<V, E> graph, start: Vertex)
2  {
3    Queue<Vertex> todos = new Queue<>();
4    todos.add(start)
5    start.setLabel(VertexLabel.visited)
6    while( !work.isEmpty() ){
7      Vertex current = todos.remove();
8      for(edge : current.getIncident()){
9        if(edge.getLabel() == EdgeLabel.UNEXPLORED){
10         Vertex opposite = edge.getOpposite(current);
11         if(opposite.getLabel() == VertexLabel.UNEXPLORED){
12           /* ... */
13           prevHops.put(opposite, current);
14         } else {
15           edge.setLabel(EdgeLabel.BACK);
16 } } } } }
```

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BFSOne - Pathfinding

To compute the path from A to B:

1. B is the last element of the path.
2. `prevHops`$[B]$ is the second-to last element of the path.
3. `prevHops`[`prevHops`$[B]$] is the third-to last element of the path.
4. ...
5. A is the first element of the path.

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How about

What happens if we use a `Stack` instead of a `Queue`?

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Not Quite BFS

```
1  public void NotQuiteBFSOne(Graph<V, E> graph, start: Vertex)
2  {
3    Stack<Vertex> todos = new Stack<>();
4    todos.add(start)
5    start.setLabel(VertexLabel.visited)
6    while( !work.isEmpty() ){
7      Vertex curr = todos.remove();
8      for(edge : curr.getIncident()){
9        if(edge.getLabel() == EdgeLabel.UNEXPLORED){
10         Vertex opposite = edge.getOpposite(curr);
11         if(opposite.getLabel() == VertexLabel.UNEXPLORED){
12           work.add(opposite);
13           opposite.setLabel(VertexLabel.VISITED);
14           edge.setLabel(EdgeLabel.SPANNING);
15         } else {
16           edge.setLabel(EdgeLabel.BACK);
17 } } } } }
```

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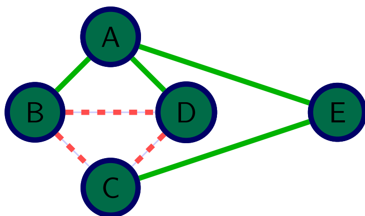
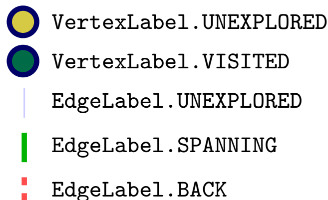
Not quite BFS





Call Stack

`NotQuiteBFS(G)`
`NotQuiteBFSOne(G, A)`

Work Stack

A
B
D
E
C

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DFS

DFS is BFS with a Stack

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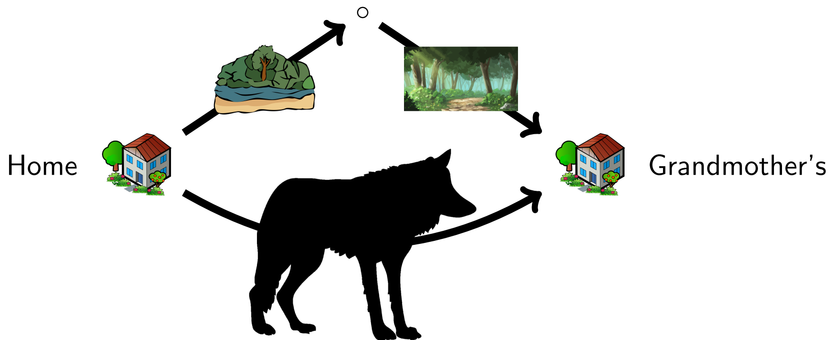
Shortest Path



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Shortest Path

Problem: BFS always finds the path with the *Fewest Edges*

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Weighted Graphs

A **weighted graph** is:

- A graph $G = (V, E)$
- A weight function $\omega(e)$ that assigns a real (called a **edge weight**) to each edge $e \in E$.

Examples of Weighted Graphs

- Latency of a network connection
- Distance between two cities
- Time between two subway/bus stops
- Flow capacity between two points in a series of tubes.

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Shortest Path

Given

- A weighted graph $G = (V, E, \omega)$
- A start vertex `start` $\in V$
- A end vertex `end` $\in V$

Goal

- Produce a simple path P from `start` to `end`
- ... that minimizes the sum of edge weights in P.

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BFS with Level

Set up a todo list with [start node, level 0]. Until the todo list is empty:

1. Take the next [vertex, level] on the todo list.
   - **Consequence:** Dequeue reads vertices in ascending order of level.
   - **Therefore:** The first time we reach a vertex, it is via the fewest number of edges.
2. For each edge leaving the vertex node:
   1. If the edge is explored, leave it alone.
   2. If the edge connects to an explored vertex, mark it as a back edge.
   3. If the other vertex is unexplored, mark it explored and add [other, level+1] to the todo list.
      - BFS always adds 1 to the level when exploring a new node; One edge adds 1 to the level.

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Shortest Path

In this example, the 'distance' is based on the number of edges. Each edge has a 'weight' of one. ($\omega(e) = 1$ for all $e \in E$)

What if we let ω vary??

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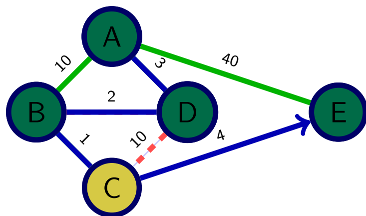
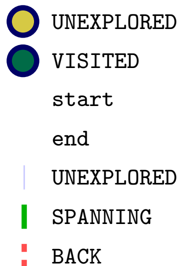
BFS with Varying ω





How do we find this path?

Call Stack

BFS(G)
BFSOne(G, A)

Work Queue

A
B
D
E

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Shortest Path

Question: How do we find the shortest path, when not every edge is created equal?

At any given point, what vertex should we explore next?

Idea: Explore the smallest edge available.

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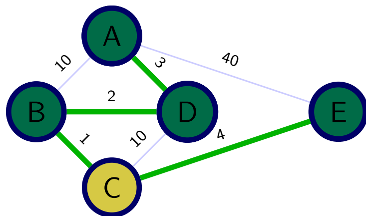
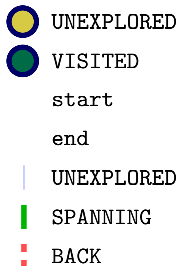
Attempt 1: Explore the Shortest Edge





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Attempt 1: Explore the Shortest Edge

Question: Will exploring the shortest available edge always work?

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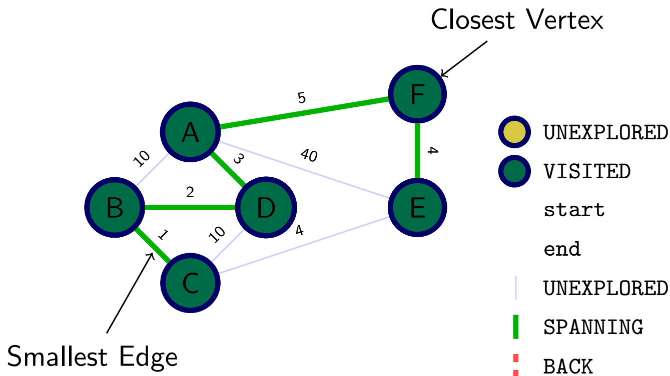
Attempt 1: Explore the Shortest Edge



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Dijkstra's Algorithm

Set up a todo list with [start node, level 0]. Until the todo list is empty:

1. Take the next [vertex, level] on the todo list in ascending order of level... but how?.
2. For each edge leaving the vertex node:
    1. If the edge is explored, leave it alone.
    2. If the edge connects to an explored vertex, mark it as a back edge.
    3. If the other vertex is unexplored, mark it explored and add [other, level+1ω(edge)] to the todo list.

This is **Dijkstra's Algorithm**.

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New ADT: Priority Queue

`PriorityQueue<E>` (`E` must be `Comparable``Comparable`)

- `public void add(E e)`: Add e to the queue.
- `public E peek()`: Return the *leastleast* element added.
- `public E remove()`: Remove and return the *leastleast* element added.

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