CSE-250 Recitation

Sept 11-Sept 12: PA1, Inequalities, Logarithms, and Bounds

Questions?

- Java?
- PA1?
- Asymptotic Notation?
- Runtime Analysis?

PA1: Getting Started

- **PA1** will revolve around linked lists and how to implement them
- We will start **PA1** by writing tests
- Why Test Driven Development?
    - Deepens your understanding of the problem
    - Enables you to test your code without submitting to Autolab
    - Writing code before thinking about the problem will lead to disaster

PA1: Getting Started

- Remember, understanding the expected behavior of each method is more important than how to make your implementation when writing tests
- Some of the best tests are going to be written by asking “What situations could break my code”
- For our first exercise, we will try to come up with some good linked lists for testing
    - Side note: how can we make these lists without relying on methods like insert

Inequalities Cheat Sheet

$f(n) \geq g(n)$ is true if $f(n)/a \geq g(n)/a$ (for any $a > 0$)

$f(n) \geq g(n)$ is true if $f(n)*a \geq g(n)*a$ (for any $a > 0$)

$x + a \geq y + b$ is true if $x \geq y$ and $a \geq b$ (for any a, b)

$x \geq y$ is true if $x \geq a$ and $a \geq y$ (for any a)

Examples

Show that there is some **c** for which…

$12 \log(10 \times 2^n) \leq c\, n$

… for all **n** greater than some n_0

Examples

Show that there is some **c** for which…

$\mathbf{n^2 + n \log(n) \leq c\, 2^n}$

… for all **n** greater than some $\mathbf{n_0}$

Examples

Let **f(x)** be a function defined as follows:

- If **x** is odd, $\mathbf{f(x) = 10x}$
- If **x** is even, $\mathbf{f(x) = 100x^2}$

Show that there is some $\mathbf{c_{low}}$ and $\mathbf{c_{high}}$ for which…

$\mathbf{f(n) \geq c_{low}\, n}$ and $\mathbf{f(n) \leq c_{high}\, n^2}$

… for all **n** greater than some $\mathbf{n_0}$

Examples

Let **f(x)** be a function defined as follows:

- If **x** is odd, $\mathbf{f(x) = 10x}$
- If **x** is even, $\mathbf{f(x) = 100x^2}$

Is there a $\mathbf{c_{low}}$ and $\mathbf{c_{high}}$ for which…

$\mathbf{f(n) \geq c_{low}\, n^2}$ and $\mathbf{f(n) \leq c_{high}\, n}$

… for all **n** greater than some $\mathbf{n_0}$

Examples

For each of the functions to the right:

1. Compute the closed form of the summation
2. Provide a big-ϴ bound for the function.
3. Arrange them in order of complexity class.

$$f_1(n) = \sum_{i=1}^{5n} (n^2 2^i)$$

$$f_2(n) = \sum_{i=\frac{3}{4}n}^{n} i$$

$$f_3(n) = \sum_{i=1}^{n} \sum_{j=i}^{n} 2$$

$$f_4(n) = \sum_{i=1}^{\log(n)} (5 \cdot 2^i + n)$$

$$f_5(n) = \sum_{i=5}^{\log(n)+5} (3 \cdot i \cdot \log(n))$$

$$f_6(n) = \sum_{i=2}^{4} \log(i)$$

Logarithms Cheat Sheet

- Let $a, b, c, n > 0$
- **Exponent Rule:** $\log(n^a) = a\log(n)$
- **Product Rule:** $\log(an) = \log(a) + \log(n)$
- **Division Rule:** $\log\left(\frac{n}{a}\right) = \log(n) - \log(a)$
- **Change of Base from *b* to *c*:** $\log_b(n) = \frac{\log_c(n)}{\log_c(b)}$
- **Log/Exponent are Inverses:** $b^{\log_b(n)} = \log_b(b^n) = n$

Summation Cheat Sheet

1. $\sum_{i=j}^{k} c = (k - j + 1)c$

2. $\sum_{i=j}^{k}(cf(i)) = c\sum_{i=j}^{k} f(i)$

3. $\sum_{i=j}^{k}(f(i) + g(i)) = \left(\sum_{i=j}^{k} f(i)\right) + \left(\sum_{i=j}^{k} g(i)\right)$

4. $\sum_{i=j}^{k}(f(i)) = \left(\sum_{i=\ell}^{k}(f(i))\right) - \left(\sum_{i=\ell}^{j-1}(f(i))\right)$ (for any $\ell < j$)

5. $\sum_{i=j}^{k} f(i) = f(j) + f(j+1) + \ldots + f(k-1) + f(k)$

6. $\sum_{i=j}^{k} f(i) = f(j) + \ldots + f(\ell - 1) + \left(\sum_{i=\ell}^{k} f(i)\right)$ (for any $j < \ell \leq k$)

7. $\sum_{i=j}^{k} f(i) = \left(\sum_{i=j}^{\ell} f(i)\right) + f(\ell + 1) + \ldots + f(k)$ (for any $j \leq \ell < k$)

8. $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$

9. $\sum_{i=0}^{k} 2^i = 2^{k+1} - 1$