CSE 250: Math Refresher

Lecture 3

August 30, 2024

Reminders

- AI Quiz due Sun, Sept 8 at 11:59 PM.
  - Your final submission must have a score of 1.0 to pass the class.
- PA 0 due Sun, Sept 8 at 11:59 PM.
  - All you need to do is make sure you have a working environment.
  - If you can't submit in autolab, let course staff know ASAP.
- WA1 released; due Sun, Sept 8 at 11:59 PM.

Math Refresher

1. Summations
2. Logarithms
3. Limits

Summations

$$\sum_{i=j}^{k} f(i) = f(j) + f(j+1) + \ldots + f(k)$$

Strategy

1. Find a rule with a pattern that matches the formula
2. Replace the formula with the right-hand side

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2. Replace the formula with the right-hand side

1. Split the summation.
2. Find the appropriate 'finisher'.
3. Apply the transformations to get to the right bounds.

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$$\sum_{i=j}^{k} c$$

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$$= (k - j + 1) \cdot c$$

Finishers

$$\sum_{i=j}^{k} c = (k - j + 1) \cdot c$$

$$\sum_{i=1}^{k} i = \frac{k(k+1)}{2} \cdot c$$

$$\sum_{i=0}^{k} 2^i = 2^{k+1} - 1$$

Finishers

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$$\sum_{i=0}^{k} 2^i = 2^{k+1} - 1$$

The trick is to get the summation into one of the above forms.

Useful Tricks

If c is a constant and $f(i)$ is a function of i:

$$\sum_{i=j}^{k} c \cdot f(i)$$

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$$\sum_{i=j}^{k} c \cdot f(i) = c \cdot f(j) + c \cdot f(j+1) + \ldots + c \cdot f(k)$$
$$= c \cdot (f(j) + f(j+1) + \ldots + f(k))$$

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$$= c \cdot (f(j) + f(j+1) + \ldots + f(k))$$

$$= c \cdot \sum_{i=j}^{k} f(i)$$

Useful Tricks

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If $f(i)$ and $g(i)$ are functions of i:

$$\sum_{i=j}^{k} f(i) + g(i) = (f(j) + g(j)) + \ldots + (f(k) + g(k))$$
$$= (f(j) + \ldots + f(k)) + (g(j) + \ldots + g(k))$$

Useful Tricks

If $f(i)$ and $g(i)$ are functions of i:

$$\begin{aligned}\sum_{i=j}^{k} f(i) + g(i) &= (f(j) + g(j)) + \ldots + (f(k) + g(k)) \\ &= (f(j) + \ldots + f(k)) + (g(j) + \ldots + g(k)) \\ &= \left(\sum_{i=j}^{k} f(i)\right) + \left(\sum_{i=j}^{k} g(i)\right)\end{aligned}$$

Useful Tricks

If $j < \ell \leq k$:

$$\sum_{i=j}^{k} f(i)$$

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Useful Tricks

If $j < \ell \leq k$:

$$\sum_{i=j}^{k} f(i) = f(j) + \ldots + f(k)$$

$$= f(j) + \ldots + f(\ell - 1) + f(\ell) \ldots + f(k)$$

$$= \left(\sum_{i=j}^{\ell-1} f(i)\right) + \left(\sum_{i=\ell}^{k} f(i)\right)$$

Useful Tricks

If $j < \ell \leq k$:

$$\left(\sum_{i=j}^{k} f(i)\right) = \left(\sum_{i=j}^{\ell-1} f(i)\right) + \left(\sum_{i=\ell}^{k} f(i)\right)$$

Useful Tricks

If $j < \ell \leq k$:

$$\left(\sum_{i=j}^{k} f(i)\right) = \left(\sum_{i=j}^{\ell-1} f(i)\right) + \left(\sum_{i=\ell}^{k} f(i)\right)$$

$$\left(\sum_{i=j}^{k} f(i)\right) - \left(\sum_{i=j}^{\ell-1} f(i)\right) = \left(\sum_{i=\ell}^{k} f(i)\right)$$

Series

$$\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$$

$$\sum_{i=0}^{k} 2^{k} = 2^{k+1} - 1$$

Summary

$$\sum_{i=j}^{k} c = (k - j + 1) \cdot c \tag{1}$$

$$\sum_{i=j}^{k} c \cdot f(i) = c \cdot \sum_{i=j}^{k} f(i) \tag{2}$$

$$\sum_{i=j}^{k} f(i) + g(i) = \left(\sum_{i=j}^{k} f(i)\right) + \left(\sum_{i=j}^{k} g(i)\right) \tag{3}$$

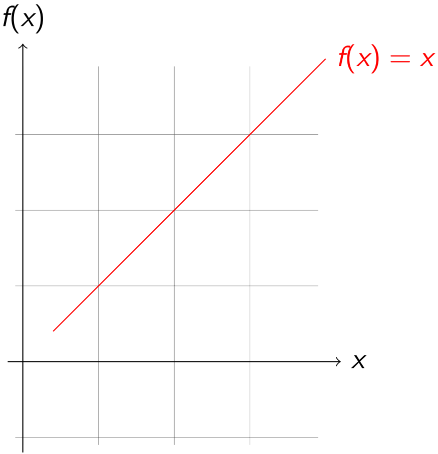
$$\sum_{i=j}^{k} f(i) = \left(\sum_{i=j}^{\ell-1} f(i)\right) + \left(\sum_{i=\ell}^{k} f(i)\right) \tag{4}$$

$$\left(\sum_{i=\ell}^{k} f(i)\right) = \left(\sum_{i=j}^{k} f(i)\right) - \left(\sum_{i=j}^{\ell-1} f(i)\right) \tag{5}$$

$$\sum_{i=1}^{k} i = \frac{k(k+1)}{2} \tag{6}$$

$$\sum_{i=0}^{k} 2^i = 2^{k+1} - 1 \tag{7}$$

Logarithms



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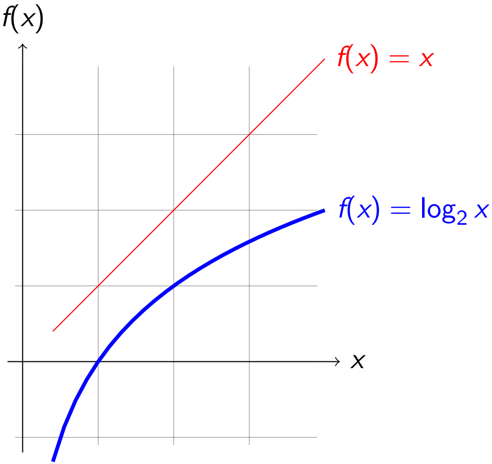

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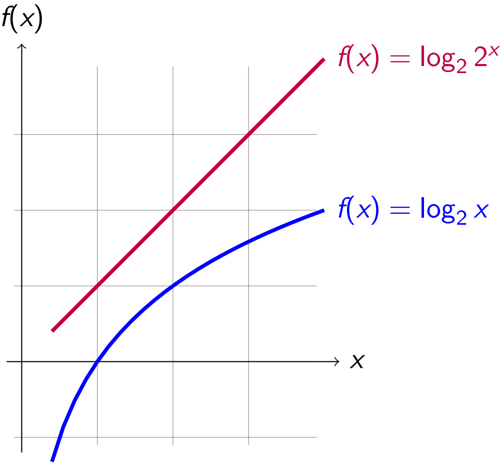

Stepping back

$$a \cdot n = \underbrace{a + a + \ldots + a}_{n \text{ times}}$$

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$$a^n = \underbrace{a \cdot a \cdot \ldots \cdot a}_{n \text{ times}}$$

Logarithms

$\log_a(b) =$ the number of times you multiply a together to get b

Examples

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Examples

$\log_2(32) =$

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$$\log_a(b) = \text{ the number of times you multiply } a \text{ together to get } b$$

Examples

$$\log_2(32) = 5 \qquad 32 = \underbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}_{\text{5 times}}$$

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Examples

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$\log_3(27) =$

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Examples

$$\log_2(32) = 5 \qquad 32 = \underbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}_{5 \text{ times}}$$

$$\log_3(27) = 3 \qquad 27 = \underbrace{3 \cdot 3 \cdot 3}_{3 \text{ times}}$$

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$$\log_2\left(\tfrac{1}{8}\right) =$$

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$\log_2(32) = 5$ $\qquad 32 = \underbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}_{5 \text{ times}}$

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$\log_2(\frac{1}{8}) = -3$ $\qquad \frac{1}{8} = 1 \underbrace{\div 2 \div 2 \div 2}_{-3 \text{ times}}$

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$\log_2(2^{10}) =$

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$\log_2(\frac{1}{8}) = -3$ $\qquad \frac{1}{8} = 1 \underbrace{\div 2 \div 2 \div 2}_{-3 \text{ times}}$

$\log_2(2^{10}) = 10$ $\qquad 2^{10} = \underbrace{2 \cdot \ldots \cdot 2}_{10 \text{ times}}$

Logarithm is the Inverse Exponent

$$b^{\log_b(n)} = n = \log_b(b^n)$$

The Product Rule

Let's say $n = a \cdot b$.
How are $\log_2(n)$, $\log_2(a)$, and $\log_2(b)$ related?

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$$n = \underbrace{2 \cdot \ldots \cdot 2}_{\log_2(n) \text{ times}} = \underbrace{2 \cdot \ldots \cdot 2}_{\log_2(a) \text{ times}} \cdot \underbrace{2 \cdot \ldots \cdot 2}_{\log_2(b) \text{ times}}$$

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$$\log_2(n) = \log_2(ab) = \log_2(a) + \log_2(b)$$

The Exponent Rule

$$\log_2(a^n)$$

The Exponent Rule

$$\log_2(a^n) = \log_2(\underbrace{a \cdot \ldots \cdot a}_{n \text{ times}})$$

The Exponent Rule

$$\begin{aligned}\log_2(a^n) &= \log_2(\underbrace{a \cdot \ldots \cdot a}_{n \text{ times}}) \\ &= \log_2(a) + \log_2(\underbrace{a \cdot \ldots \cdot a}_{n-1 \text{ times}}) \\ &= \log_2(a) + \log_2(a) + \log_2(\underbrace{a \cdot \ldots \cdot a}_{n-2 \text{ times}})\end{aligned}$$

The Exponent Rule

$$
\begin{aligned}
\log_2(a^n) &= \log_2(\underbrace{a \cdot \ldots \cdot a}_{n \text{ times}}) \\
&= \log_2(a) + \log_2(\underbrace{a \cdot \ldots \cdot a}_{n-1 \text{ times}}) \\
&= \log_2(a) + \log_2(a) + \log_2(\underbrace{a \cdot \ldots \cdot a}_{n-2 \text{ times}}) \\
&= \underbrace{\log_2(a) + \ldots + \log_2(a)}_{n \text{ times}}
\end{aligned}
$$

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&= \log_2(a) + \log_2(\underbrace{a \cdot \ldots \cdot a}_{n-1 \text{ times}}) \\
&= \log_2(a) + \log_2(a) + \log_2(\underbrace{a \cdot \ldots \cdot a}_{n-2 \text{ times}}) \\
&= \underbrace{\log_2(a) + \ldots + \log_2(a)}_{n \text{ times}} \\
&= n \cdot \log_2(a)
\end{aligned}
$$

The Division Rule

$$\log_2\left(\frac{a}{b}\right)$$

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$$\log_2\left(\frac{a}{b}\right) = \log_2\left(a \cdot \frac{1}{b}\right)$$

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$$\log_2\left(\frac{a}{b}\right) = \log_2\left(a \cdot \frac{1}{b}\right)$$
$$= \log_2(a) + \log_2\left(\frac{1}{b}\right)$$

The Division Rule

$$\log_2\left(\frac{a}{b}\right) = \log_2\left(a \cdot \frac{1}{b}\right)$$
$$= \log_2(a) + \log_2\left(\frac{1}{b}\right)$$
$$= \log_2(a) + \log_2(b^{-1})$$

The Division Rule

$$\begin{aligned}
\log_2\left(\frac{a}{b}\right) &= \log_2\left(a \cdot \frac{1}{b}\right) \\
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\end{aligned}$$

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&= \log_2(a) + \log_2(b^{-1}) \\
&= \log_2(a) + (-1) \cdot \log_2(b) \\
&= \log_2(a) - \log_2(b)
\end{aligned}
$$

Change of Base

$$b^m = n$$

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Change of Base

$$
\begin{aligned}
b^m &= n \\
\log_c(b^m) &= \log_c(n) \\
m \log_c(b) &= \log_c(n) \\
m &= \frac{\log_c(n)}{\log_c(b)}
\end{aligned}
$$

Change of Base

$$b^m = n$$
$$\log_c(b^m) = \log_c(n)$$
$$m \log_c(b) = \log_c(n)$$
$$m = \frac{\log_c(n)}{\log_c(b)}$$
$$\log_b(n) = \frac{\log_c(n)}{\log_c(b)}$$

Summary

Exponent Rule	$\log(n^a) = a\log(n)$
Product Rule	$\log(ab) = \log(a) + \log(b)$
Division Rule	$\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$
Change of Base	$\log_b(n) = \frac{\log_c(n)}{\log_c(b)}$
Inverse	$b^{\log_b(n)} = \log_b(b^n) = n$

Summary

Exponent Rule	$\log(n^a) = a\log(n)$
Product Rule	$\log(ab) = \log(a) + \log(b)$
Division Rule	$\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$
Change of Base	$\log_b(n) = \frac{\log_c(n)}{\log_c(b)}$
Inverse	$b^{\log_b(n)} = \log_b(b^n) = n$

For this class, assume base-2 logarithms unless stated otherwise.

Limits

$$\lim_{i \to c} f(i)$$

The value that $f(i)$ converges to as i approaches c
(Even if $f(c)$ is not defined)

Limit Examples

$$\lim_{i \to \infty} \frac{1}{i} = 0$$

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$$\lim_{i \to \infty} i = \infty$$

Limit Examples

$$\lim_{i \to \infty} \frac{1}{i} = 0$$

$$\lim_{i \to \infty} 6 = 6$$

$$\lim_{i \to \infty} 6 + \frac{1}{i} = 6$$

$$\lim_{i \to \infty} i = \infty$$

$$\lim_{i \to \infty} i - i = 0$$