CSE 250: Stacks and Queues

Lecture 15

Sept 30, 2024

Reminders

- Midterm 1 in class on Fri, Oct 04.
  - Covers: Asymptotics, Sequences/Lists, Arrays, Linked Lists, Recursion
  - Bounds: Tight Upper/Lower, Unqualified vs Amortized

Back to Sequence ADTs

- **Sequence**
  - `get(i)`, `set(i, v)`
- **List**
  - ... and `add(v)`, `add(i, v)`, `remove(i)`,

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- **Sequence**
  - `get(i)`, `set(i, v)`
- **List**
  - ... and `add(v)`, `add(i, v)`, `remove(i)`,
- **Stack**
  - `push(v)`, `pop()`, `peek()`
- **Queue**
  - `add(v)`, `remove()`, `peek()`

The Stack ADT

A stack of objects on top of one another.

- **Push**
  Put a new object on top of the stack.
- **Pop**
  Remove the object from the top of the stack.
- **Top**
  Peek at what's on top of the stack.

Stacks

Demo

Stacks in Practice

- Storing method-local variables ("call stack")
- Certain types of parsers ("context free")
- Backtracking search (more at the end)
- Reversing sequences

The Stack Interface

```
1  public interface Stack<E> extends List<E> {
2    public boolean empty();
3    public E peek();
4    public E pop();
5    public E push(E item);
6    /* ... */
7  }
```

LinkedListStack

```
1   class LinkedListStack<E> implements Stack<E> {
2     LinkedList<E> data = new LinkedList();
3
4     public boolean empty() { return data.size() == 0; }
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11        return data.remove(data.size()-1); }    O(N − i) = O(1)
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Java's `Stack` is implemented using a `Vector`.
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- $2\times$ capacity is wasteful when you have an upper limit.
- Each call to **pop** gives you another **push** credit.
- Keeping elements together in memory is worth the overhead.

The Queue ADT

Outside of the US, "queueing" is lining up.

- **Enqueue** (`add(item)` or `offer(item)`)
  Put a new object at the end of the queue.
- **Dequeue** (`remove()` or `poll()`)
  Remove the object from the front of the queue.
- **Peek** (`element()` or `peek()`)
  Peek at what's at the front of the queue.

Queues

Demo

Queues vs Stacks

- **Queue**
  First in, First out (FIFO)
- **Stack**
  Last in, First out (LIFO, FILO)

Queues in Practice

- Delivering network packets, emails, tootstagrexes.
- Scheduling the CPU
- Deferring long-running tasks

The Queue Interface

```
1  public interface Queue<E> {
2    public boolean add(E item);
3    public E remove();
4    public E peek();
5    /* ... */
6  }
```

LinkedListQueue

```
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ArrayListQueue

Thought Question:
How could you use an `ArrayList` to build a `Queue`?

ArrayListQueue (v1)

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4    public boolean add(E item) { data.add(item); }        O(1)
5    public E remove() { return data.remove(0); }          O(N)
6    public E peek() { return data.get(0); }               O(1)
7  }
```

ArrayListQueue (v2)

```
1  public interface ArrayListQueue<E> implements Queue<E> {
2    ArrayList<E> data = new ArrayList<>();
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4    public boolean add(E item) { data.add(item, 0); }
5    public E remove()
6                 { return data.remove(data.size()-1); }
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Circular Buffer

Demo

ArrayListQueue (final)

Active Array = [start, end)

- `Enqueue(item)`
  - Resize Array (if needed)
  - `data[end] = item`
  - `end = (end + 1) % capacity`
- `Dequeue(item)`
  - `return data[start]`
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This is called a **Circular** or **Ring Buffer**.

Balanced Parenthesis

What does it mean for parenthesis/braces to be balanced.

- Every opening symbol is matched by a closing symbol.
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- No nested overlaps (e.g., $\{(\}$) is not ok).

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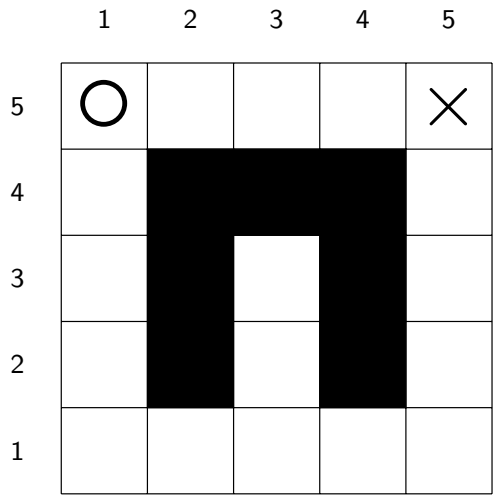
Balanced Parenthesis

Demo

Mazes

- ○ is the start, × is the objective.
  - There may be multiple paths.
  - Generally, we want the shortest
- **Approach 1**: Take the first available route in one direction.
  - **Right, Down, Left, or Up**
  - **Down, Right, Up, or Left**



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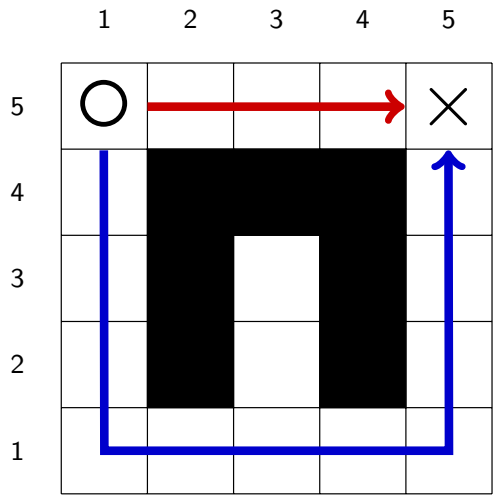

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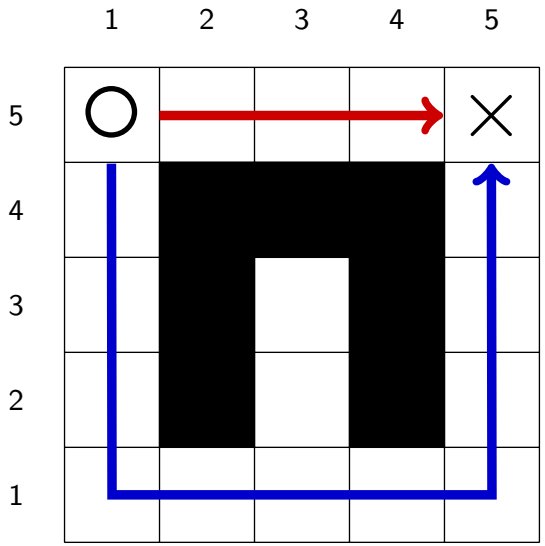

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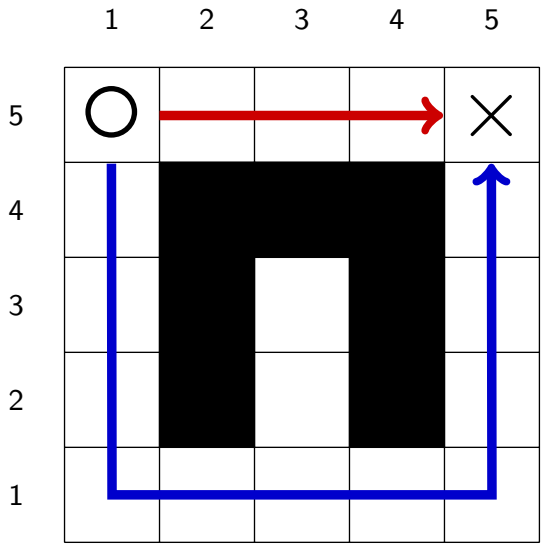

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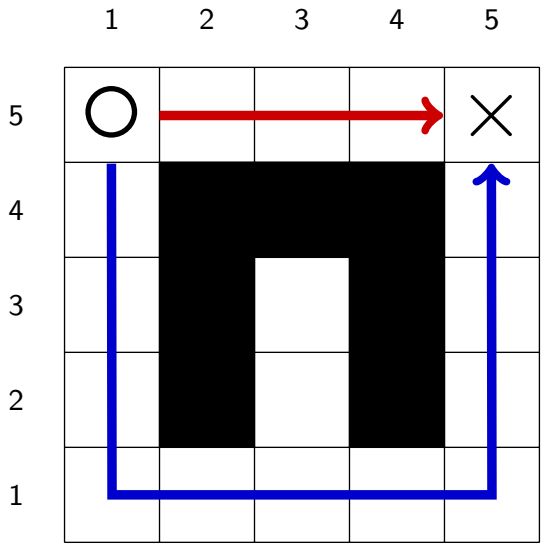
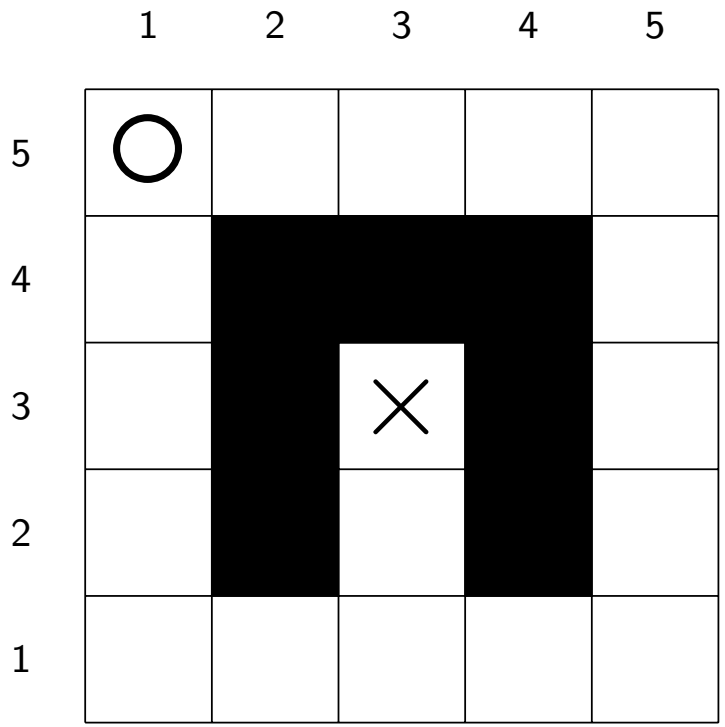


- How do we know which one is best?
- Are there any other problems?

Mazes



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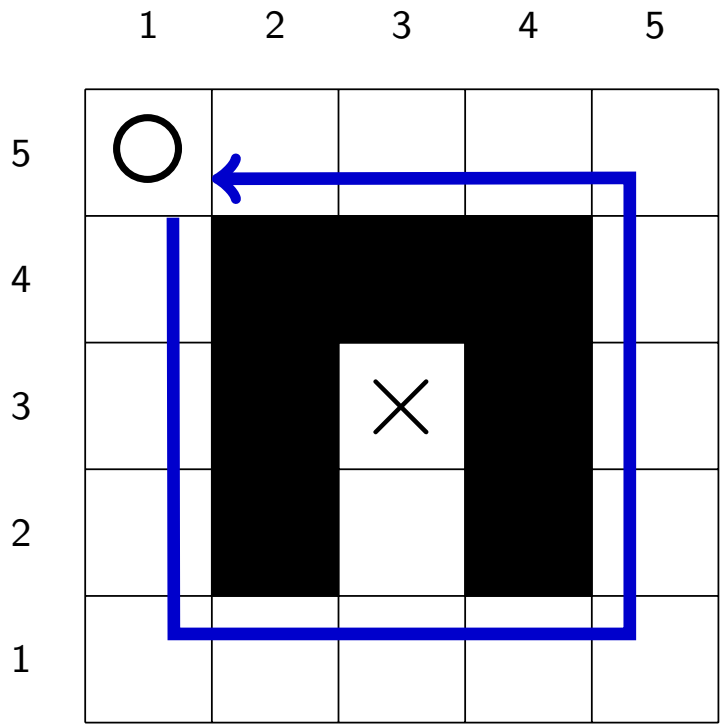

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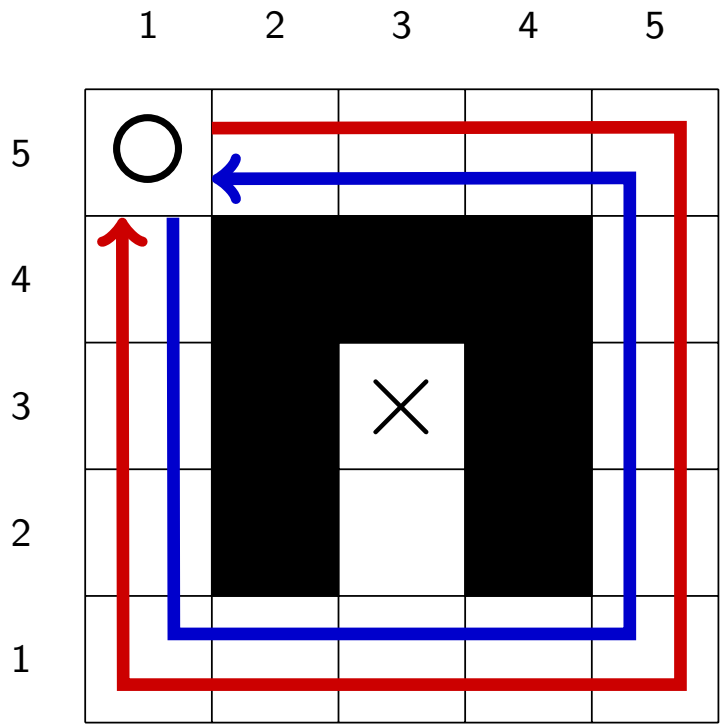

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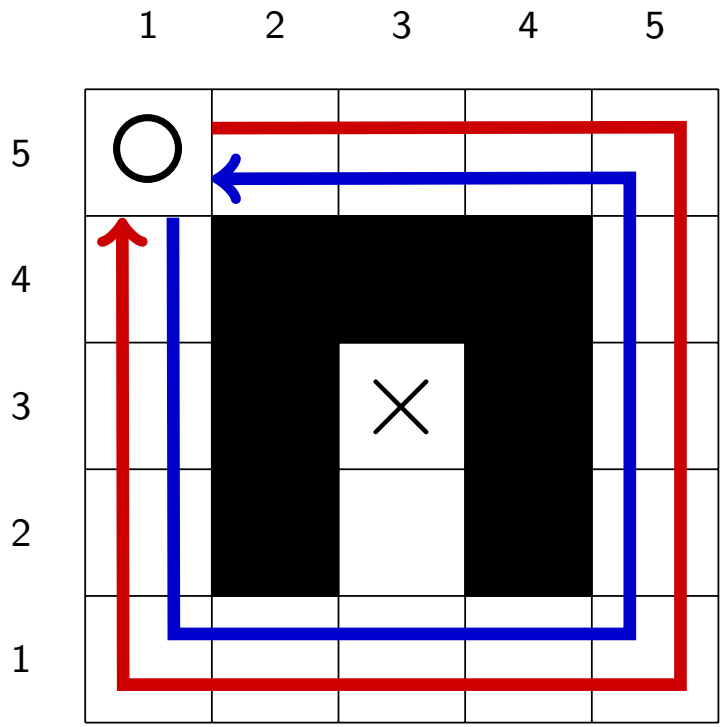


- Priority order doesn't guarantee exploring the entire maze

Formalizing Maze Solving

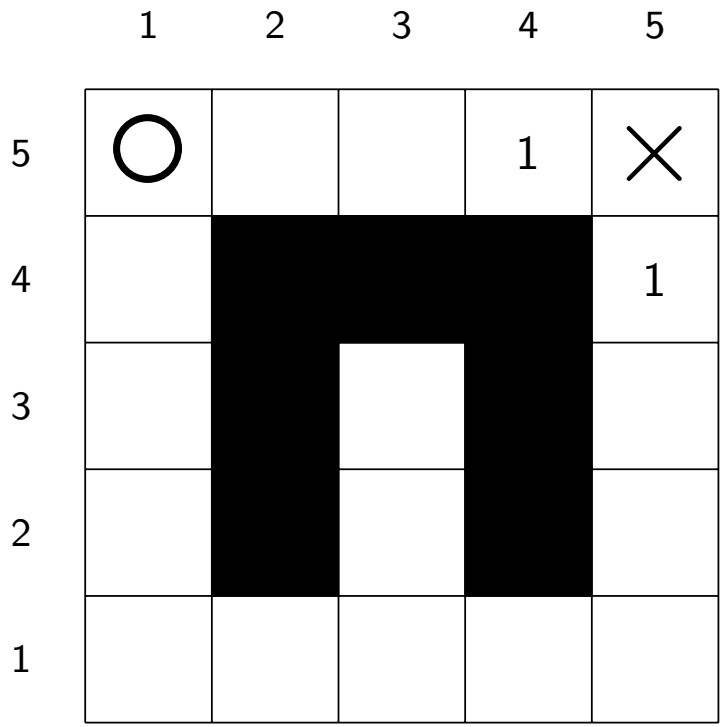
- **Inputs**
  - The map: An $n \times m$ grid of filled/empty squares.
  - The $\circ$ is at position `start`
  - The $\times$ is at position `dest`
- **Goal**
  - Compute `steps(start, dest)`, the minimum steps from start to end.

Formalizing Maze Solving

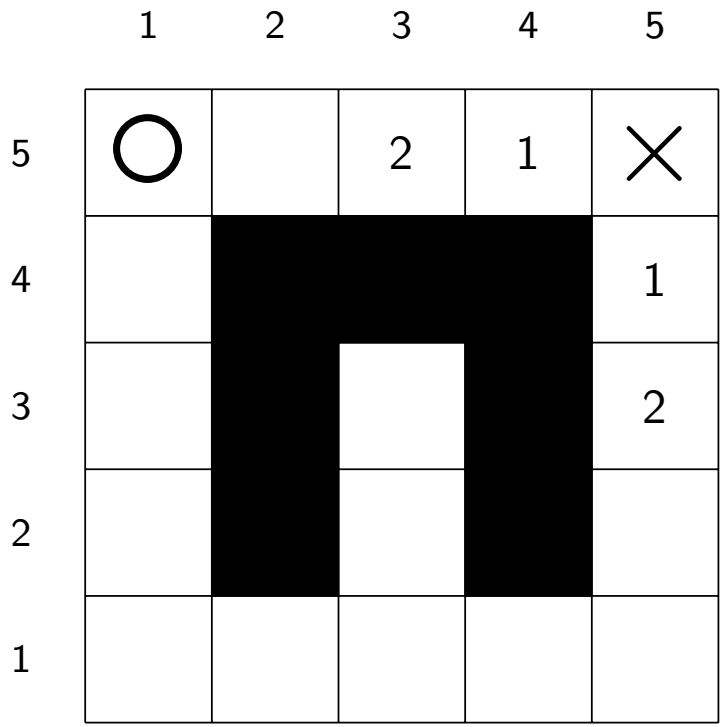
- **Inputs**
  - The map: An $n \times m$ grid of filled/empty squares.
  - The $\circ$ is at position `start`
  - The $\times$ is at position `dest`
- **Goal**
  - Compute `steps`(`start`, `dest`), the minimum steps from start to end.

How do we define `steps`?

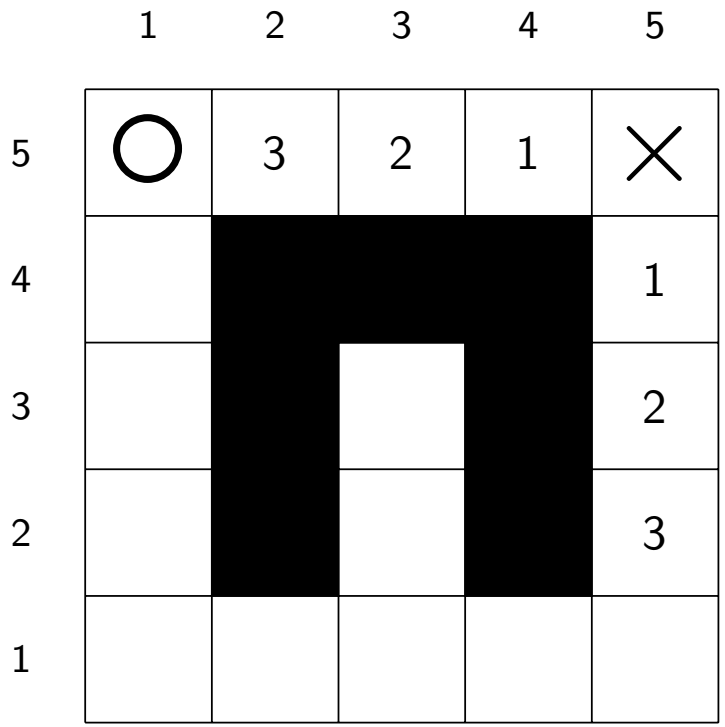
Formalizing Maze Solving



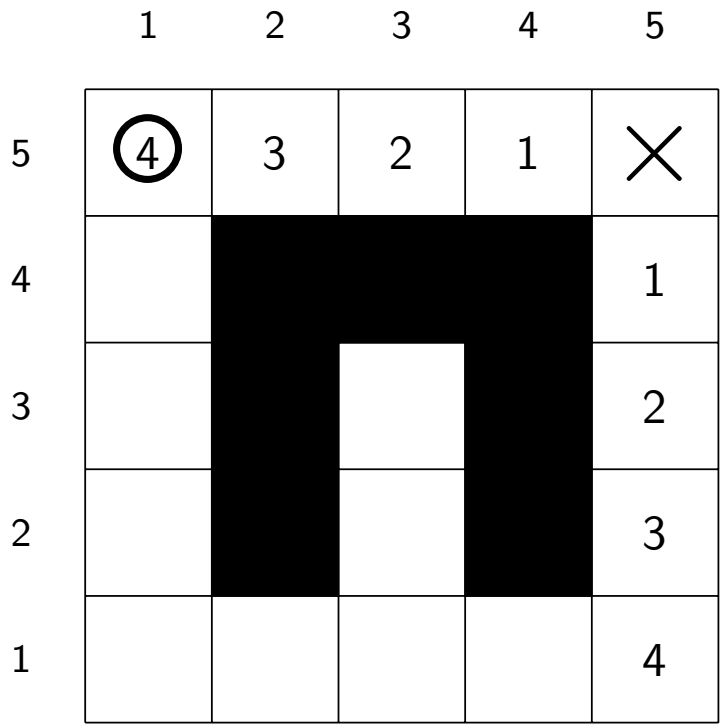
Formalizing Maze Solving



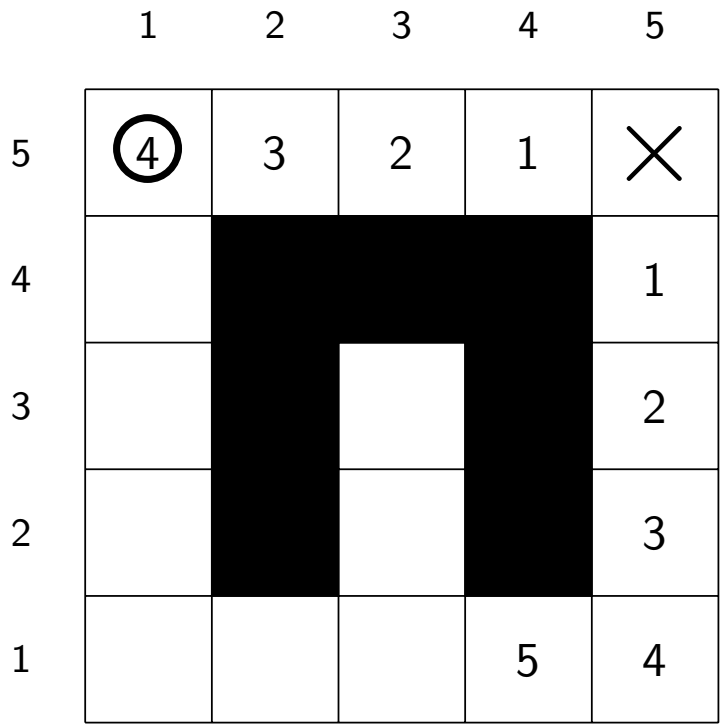
Formalizing Maze Solving



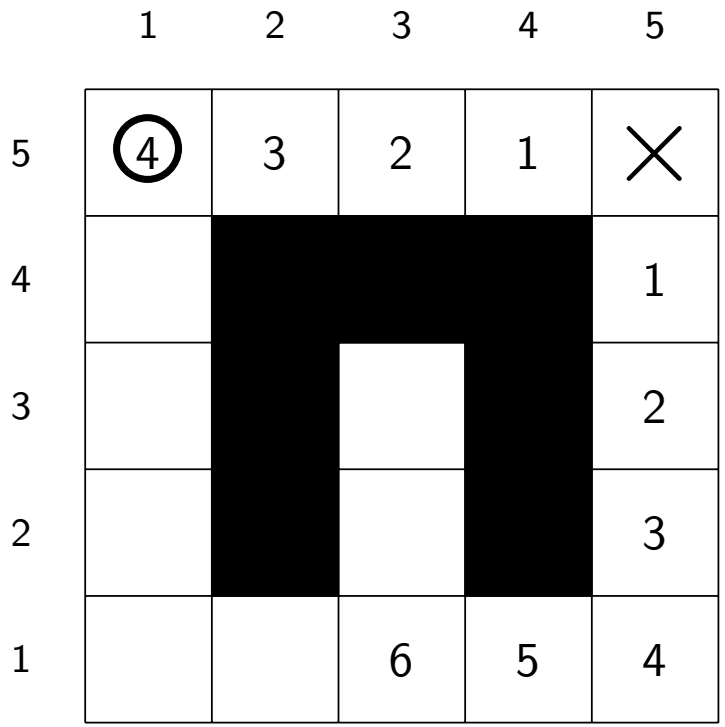
Formalizing Maze Solving



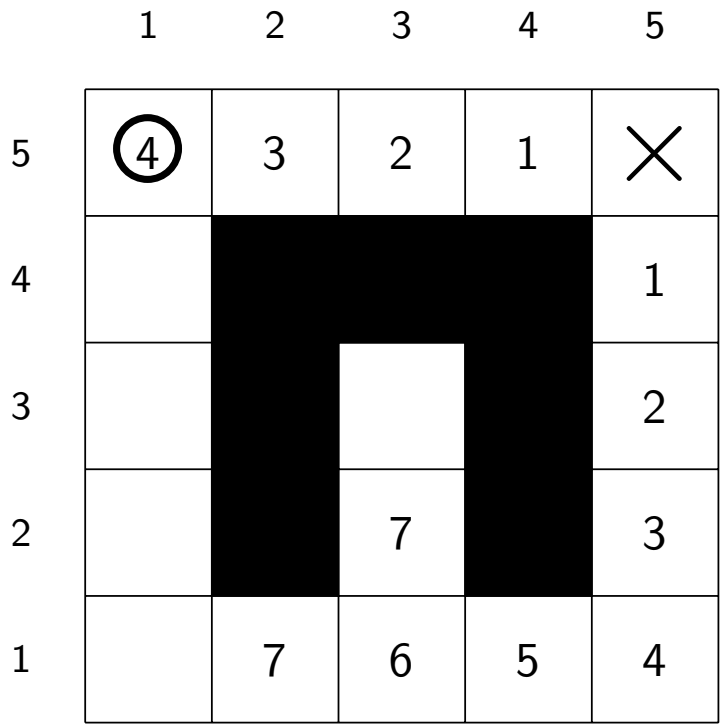
Formalizing Maze Solving



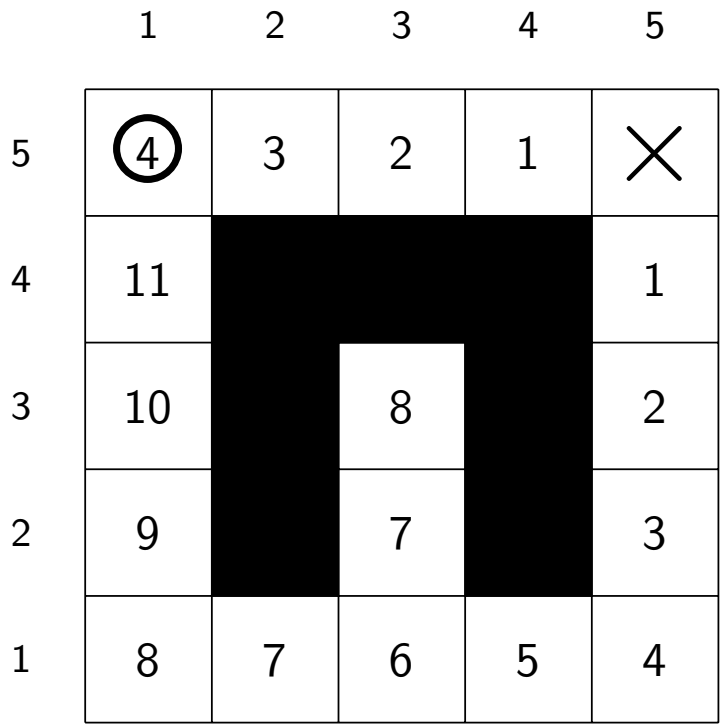
Formalizing Maze Solving



Formalizing Maze Solving



Formalizing Maze Solving



Formalizing Maze Solving

	1	2	3	4	5
5	④	3	2	1	✕
4	11	∞	∞	∞	1
3	10	∞	8	∞	2
2	9	∞	7	∞	3
1	8	7	6	5	4

Formalizing Maze Solving

$$\texttt{steps}(\texttt{pos}, \texttt{dest}) = \begin{cases} 0 & \textbf{if } \texttt{pos} = \texttt{dest} \\ \end{cases}$$

Formalizing Maze Solving

$$\texttt{steps}(\texttt{pos}, \texttt{dest}) = \begin{cases} 0 & \textbf{if } \texttt{pos} = \texttt{dest} \\ \infty & \textbf{if } \texttt{pos} \textbf{ is filled} \end{cases}$$

Formalizing Maze Solving

$$\texttt{steps}(\texttt{pos}, \texttt{dest}) = \begin{cases} 0 & \textbf{if } \texttt{pos} = \texttt{dest} \\ \infty & \textbf{if } \texttt{pos} \textbf{ is filled} \\ 1 + \texttt{min\_nearby}(\texttt{pos}, \texttt{dest}) & \textbf{otherwise} \end{cases}$$

Formalizing Maze Solving

$$\texttt{steps}(\texttt{pos}, \texttt{dest}) = \begin{cases} 0 & \textbf{if } \texttt{pos} = \texttt{dest} \\ \infty & \textbf{if } \texttt{pos} \textbf{ is filled} \\ 1 + \texttt{min\_nearby}(\texttt{pos}, \texttt{dest}) & \textbf{otherwise} \end{cases}$$

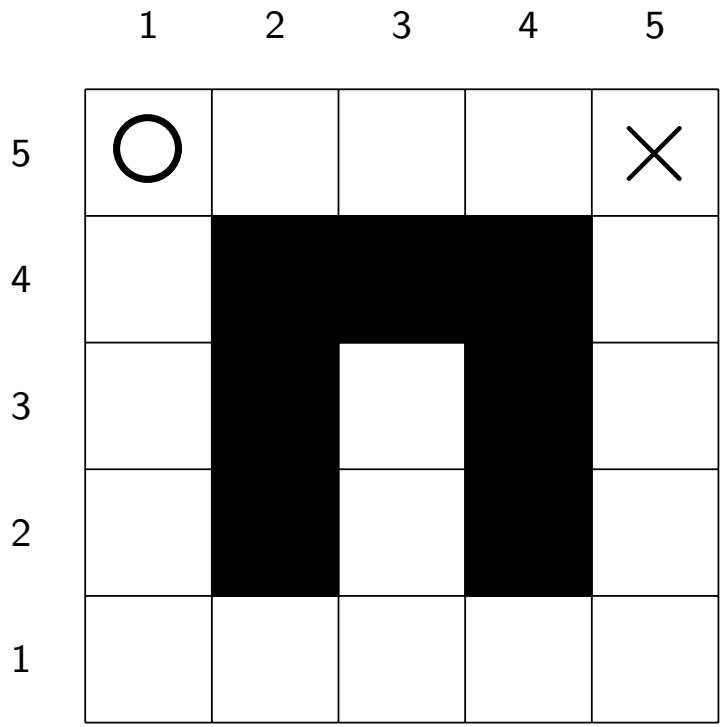
Where...

$$\texttt{min\_nearby}(\texttt{pos}, \texttt{dest}) = \min \begin{cases} \texttt{steps}(\texttt{moveRight}(\texttt{pos}), \texttt{dest}) \\ \texttt{steps}(\texttt{moveDown}(\texttt{pos}), \texttt{dest}) \\ \texttt{steps}(\texttt{moveLeft}(\texttt{pos}), \texttt{dest}) \\ \texttt{steps}(\texttt{moveUp}(\texttt{pos}), \texttt{dest}) \end{cases}$$

Formalizing Maze Solving

```
1   public int steps(Point pos, Point dest)
2   {
3     if(pos == dest){ return 0; }
4     else if(is_filled(pos){ return ∞; }
5     else {
6       return 1 + Math.min(
7         steps(pos.moveRight, dest),
8         steps(pos.moveDown, dest),
9         steps(pos.moveLeft, dest),
10        steps(pos.moveUp, dest)
11      );
12    }
13  }
```

Formalizing Maze Solving



Formalizing Maze Solving

Problem: Infinite Loop

Formalizing Maze Solving

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Insight: A path with a loop in it can't be shorter than one without the loop.

Formalizing Maze Solving

Problem: Infinite Loop

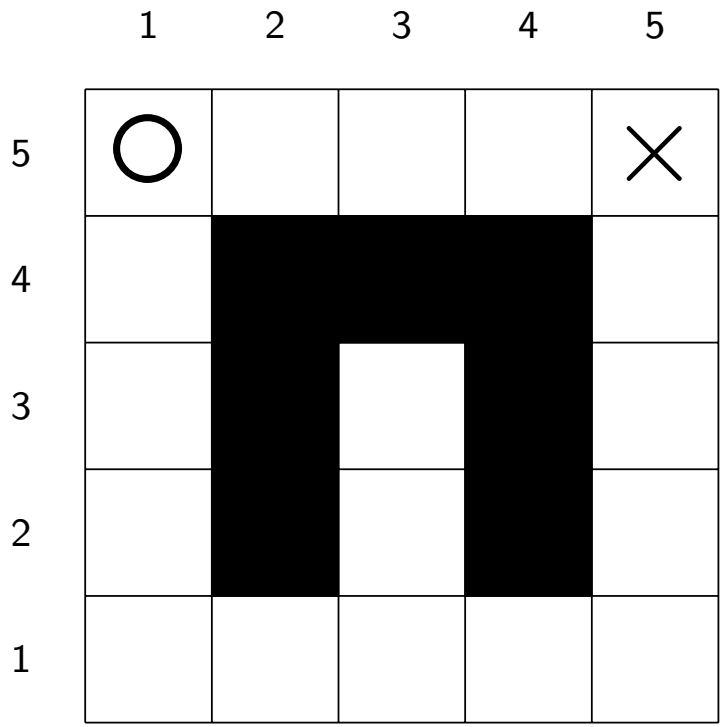
Insight: A path with a loop in it can't be shorter than one without the loop.

Mark nodes as visited so you don't visit them twice.

Formalizing Maze Solving

```
1   public int steps(Point pos, Point dest)
2   {
3     if(pos == dest){ return 0; }
4     else if(is_filled(pos){ return ∞; }
5     else if(is_visited(pos){ return ∞; }
6     else {
7       mark_visited(pos);
8       return 1 + Math.min(
9         steps(pos.moveRight, dest),
10        steps(pos.moveDown, dest),
11        steps(pos.moveLeft, dest),
12        steps(pos.moveUp, dest)
13      );
14    }
15  }
```

Formalizing Maze Solving



Formalizing Maze Solving

Problem: The first time you visit a node it may be via a longer path!

Formalizing Maze Solving

Problem: The first time you visit a node it may be via a longer path!

Unmark nodes as you leave them.

Formalizing Maze Solving

```
1   public int steps(Point pos, Point dest)
2   {
3     if(pos == dest){ return 0; }
4     else if(is_filled(pos){ return ∞; }
5     else if(is_visited(pos){ return ∞; }
6     else {
7       mark_visited(pos);
8       int stepCount = 1 + Math.min(
9         steps(pos.moveRight, dest),
10        /* ... */
11      );
12      unmark_visited(pos);
13      return stepCount;
14    }
15  }
```

Formalizing Maze Solving

Question: What path did we take?

Formalizing Maze Solving

Question: What path did we take?

Track the current path in a `Stack`

Formalizing Maze Solving

```
1   public Array[Point] steps(Point pos, Point dest, Stack visited)
2   {
3     if(pos == dest){ return visited.toArray(); }          ←
4     else if(is_filled(pos){ return null; }
5     else if(visited.contains(pos){ return null; }         ←
6     else {
7       visited.push(pos);                                  ←
8       int steps = shortest_array(
9         steps(pos.moveRight, dest),
10        /* ... */
11      );
12      visited.pop(pos);                                   ←
13    }
14  }
```