CSE 250: Maze Exploration

Lecture 17

Oct 7, 2024

Reminders

- WA3 due Sun, Oct 13 at 11:59 PM

The Stack ADT

A stack of objects on top of one another.

- **Push**
  Put a new object on top of the stack.
- **Pop**
  Remove the object from the top of the stack.
- **Top**
  Peek at what's on top of the stack.

The Queue ADT

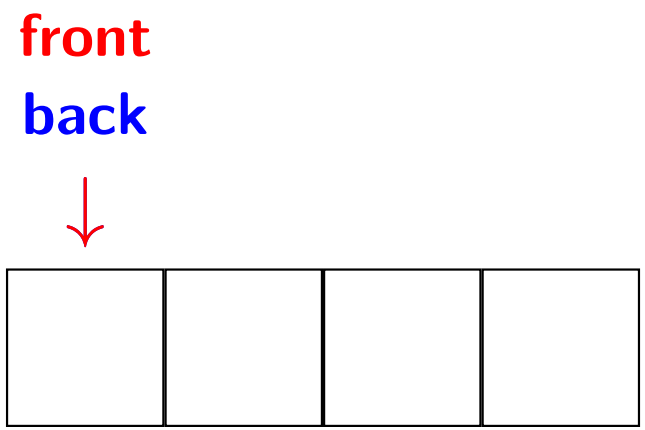
Outside of the US, "queueing" is lining up.

- **Enqueue** (`add(item)` or `offer(item)`)
  Put a new object at the end of the queue.
- **Dequeue** (`remove()` or `poll()`)
  Remove the object from the front of the queue.
- **Peek** (`element()` or `peek()`)
  Peek at what's at the front of the queue.

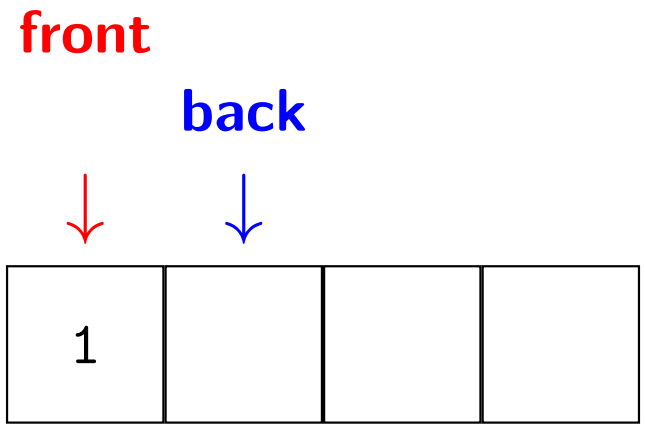
Queues vs Stacks

- **Queue**
  First in, First out (FIFO)
- **Stack**
  Last in, First out (LIFO, FILO)

Circular Buffer

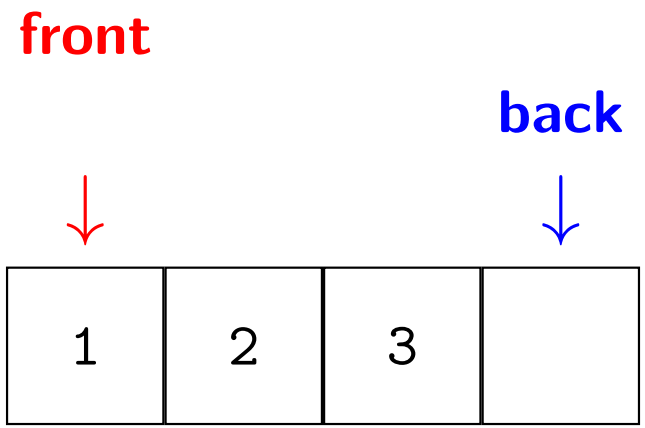


Circular Buffer



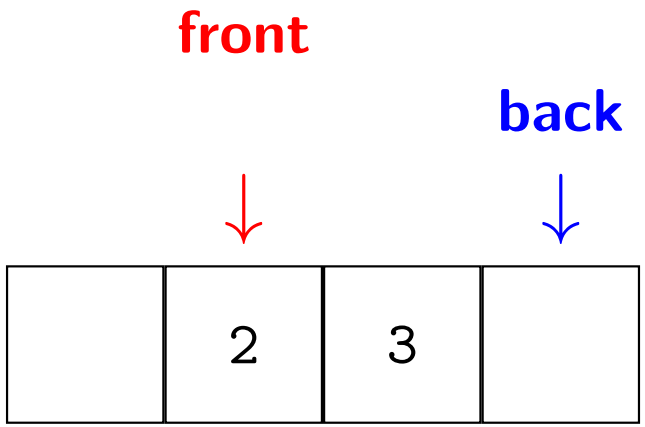

```
list.add(1)
```

Circular Buffer



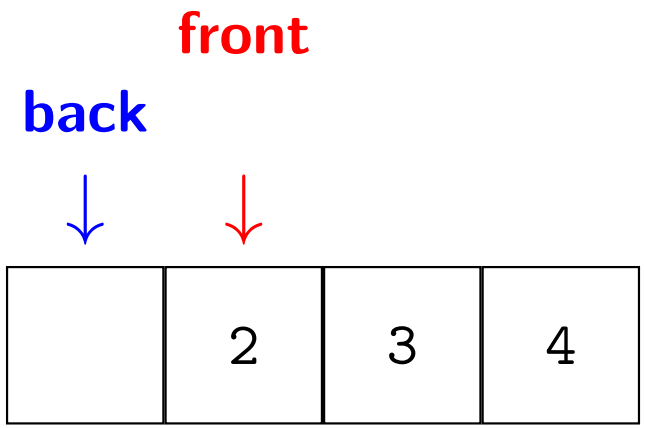

```
list.add(1)
list.add(2)
list.add(3)
```

Circular Buffer



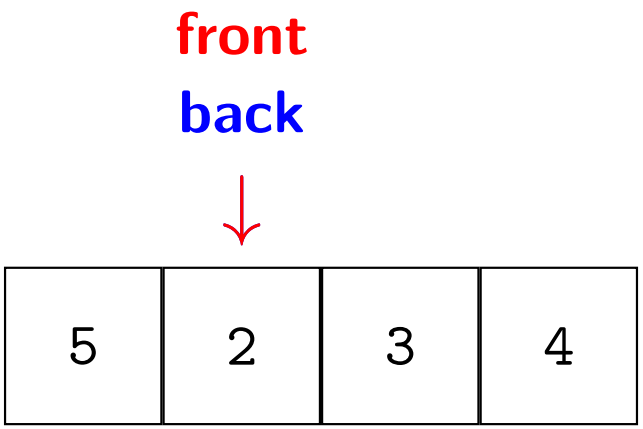

```
list.add(1)
list.add(2)
list.add(3)
list.remove() → 1
```

Circular Buffer



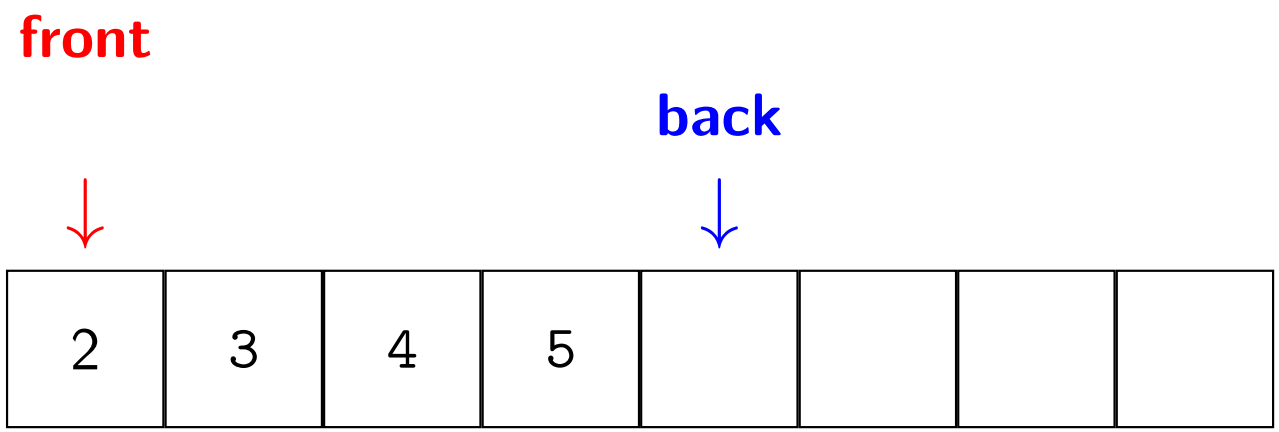

```
list.add(1)
list.add(2)
list.add(3)
list.remove() → 1
list.add(4)
```

Circular Buffer



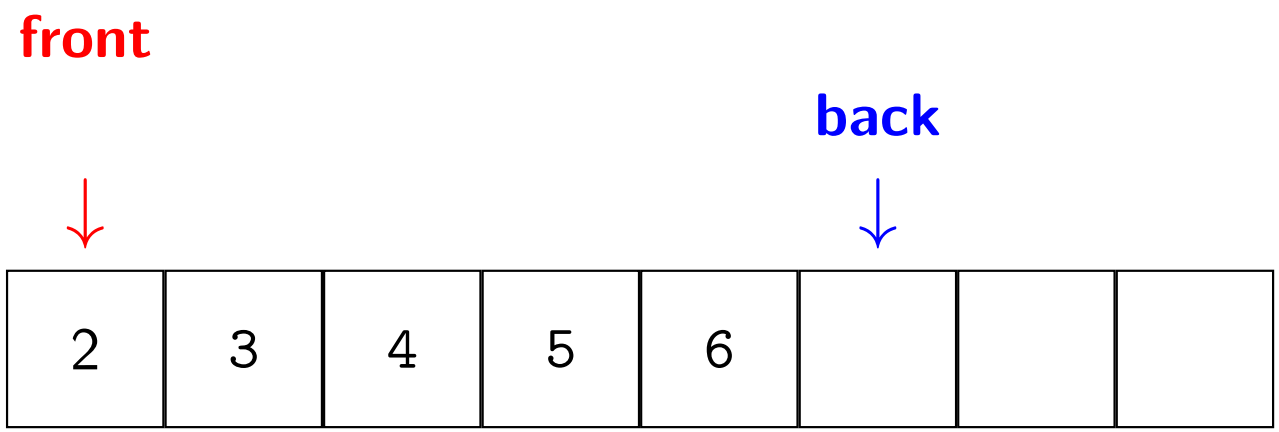

```
list.add(1)
list.add(2)
list.add(3)
list.remove() → 1
list.add(4)
list.add(5)
```

Circular Buffer



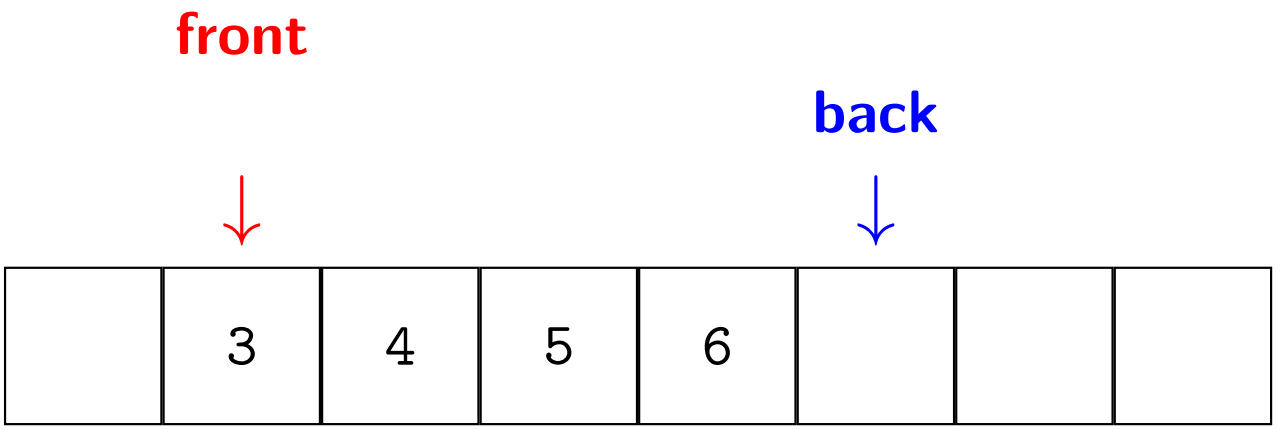

```
list.add(1)
list.add(2)
list.add(3)
list.remove() → 1
list.add(4)
list.add(5)
```

Circular Buffer



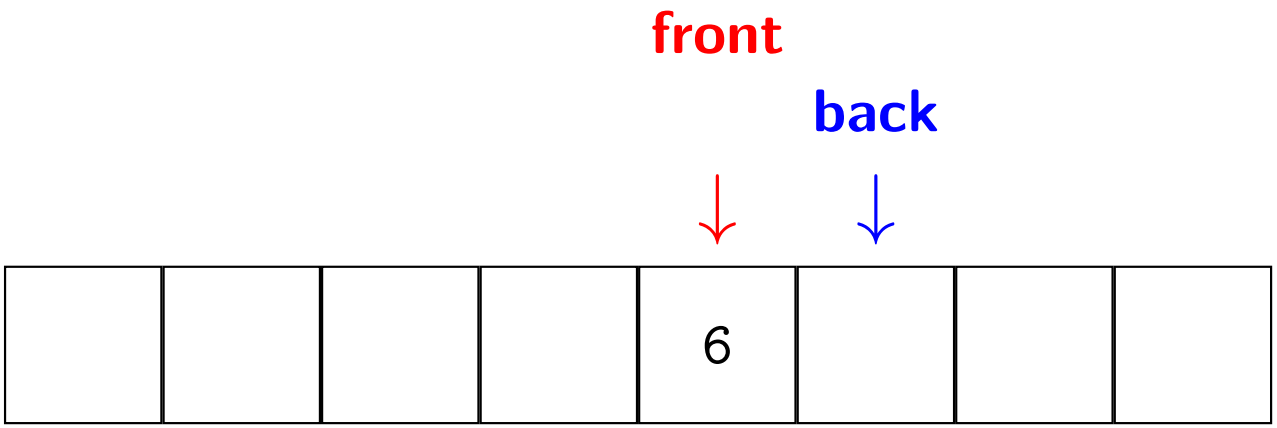

```
list.add(1)
list.add(2)
list.add(3)
list.remove() → 1
list.add(4)
list.add(6)
```

Circular Buffer



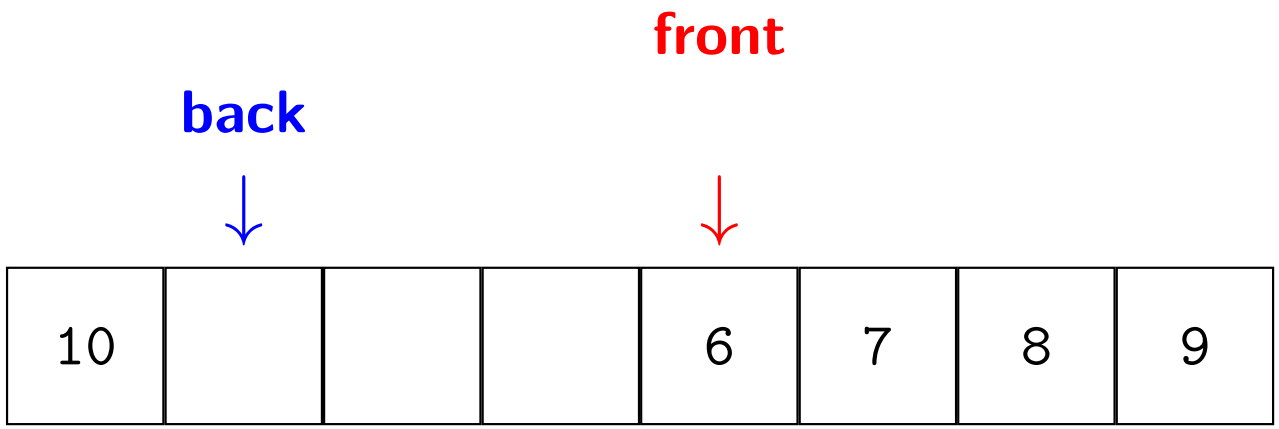

```
list.add(1)
list.add(2)
list.add(3)
list.remove() → 1
list.add(4)
list.add(6)
list.remove() → 2
```

Circular Buffer




```
list.add(1)
list.add(2)
list.add(3)
list.remove() → 1
list.add(4)
list.add(6)
list.remove() → 2
list.remove() → 3
list.remove() → 4
list.remove() → 5
list.add(7)
list.add(8)
list.add(9)
list.add(10)
```

Circular Buffer

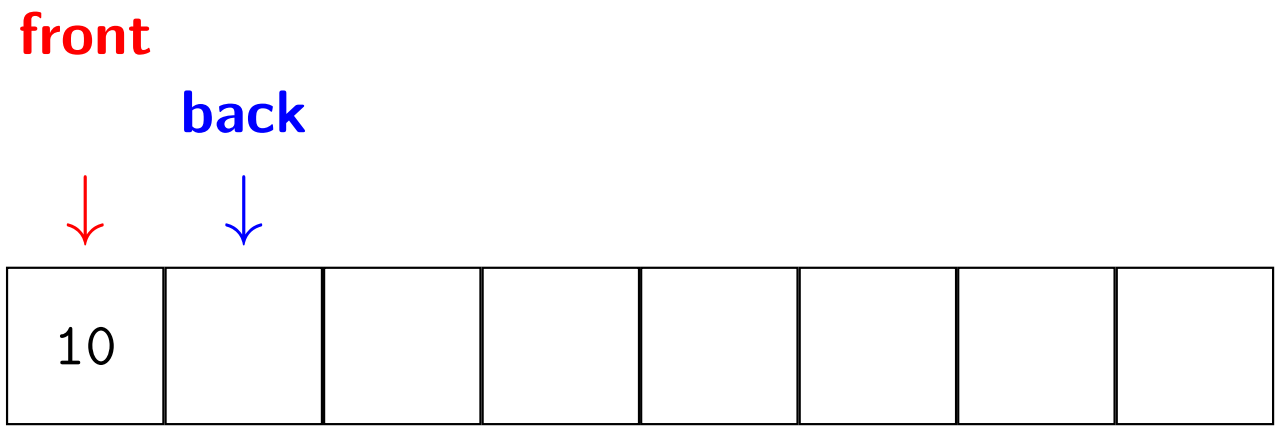



```
list.add(1)
list.add(2)
list.add(3)
list.remove() → 1
list.add(4)
list.add(6)
```

```
list.remove() → 2
list.remove() → 3
list.remove() → 4
list.remove() → 5
list.add(7)
list.add(8)
```

```
list.add(9)
list.add(10)
list.remove() → 6
list.remove() → 7
list.remove() → 8
list.remove() → 9
```

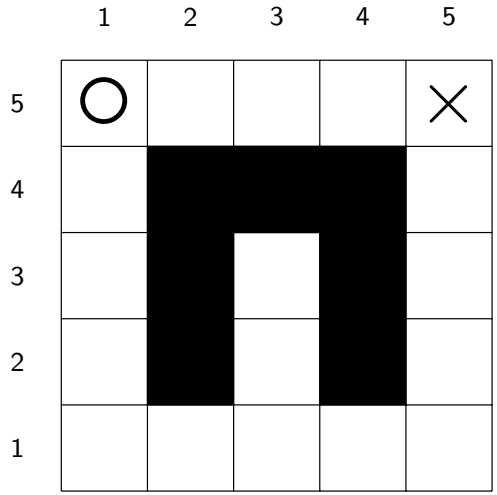
Circular Buffer




```
list.add(1)              list.remove() → 2   list.add(9)
list.add(2)              list.remove() → 3   list.add(10)
list.add(3)              list.remove() → 4   list.remove() → 6
list.remove() → 1        list.remove() → 5   list.remove() → 7
list.add(4)              list.add(7)         list.remove() → 8
list.add(6)              list.add(8)         list.remove() → 9
```

Mazes

- ○ is the start, × is the objective.
  - There may be multiple paths.
  - Generally, we want the shortest
- **Approach 1**: Take the first available route in one direction.
  - **Right, Down, Left, or Up**
  - **Down, Right, Up, or Left**



Mazes

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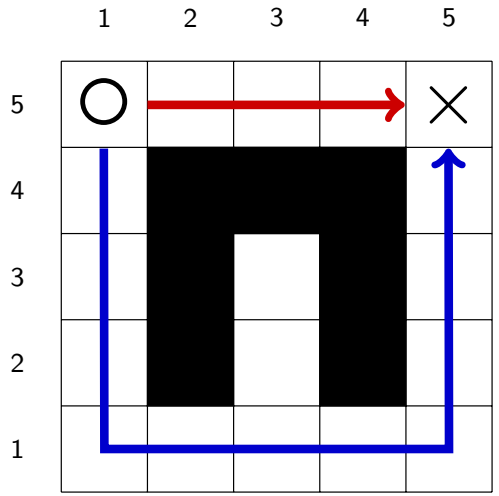
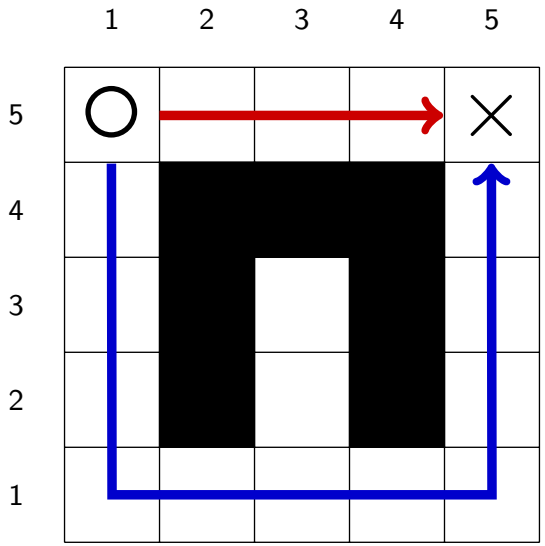
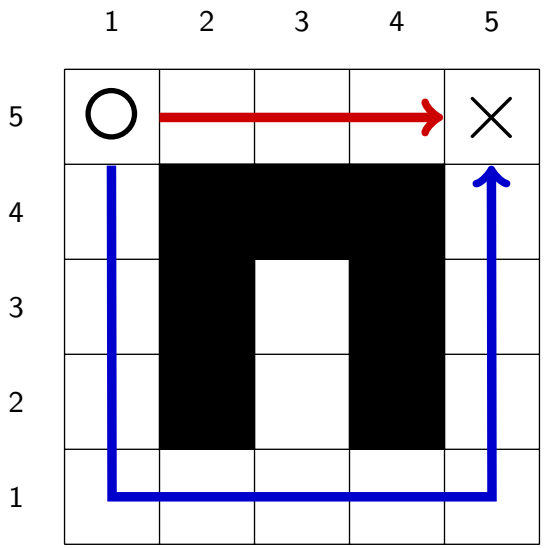


Mazes

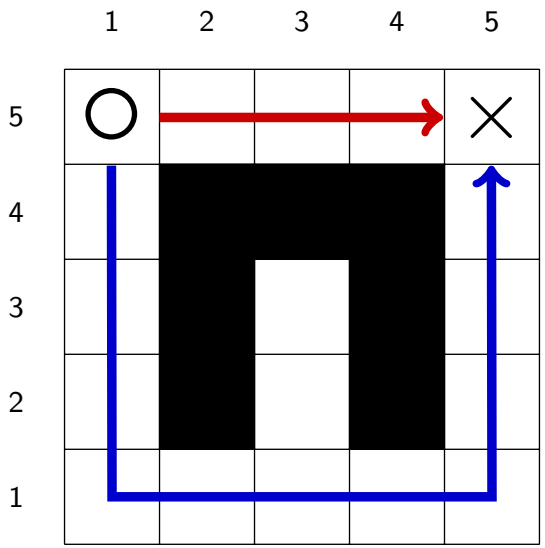


Mazes



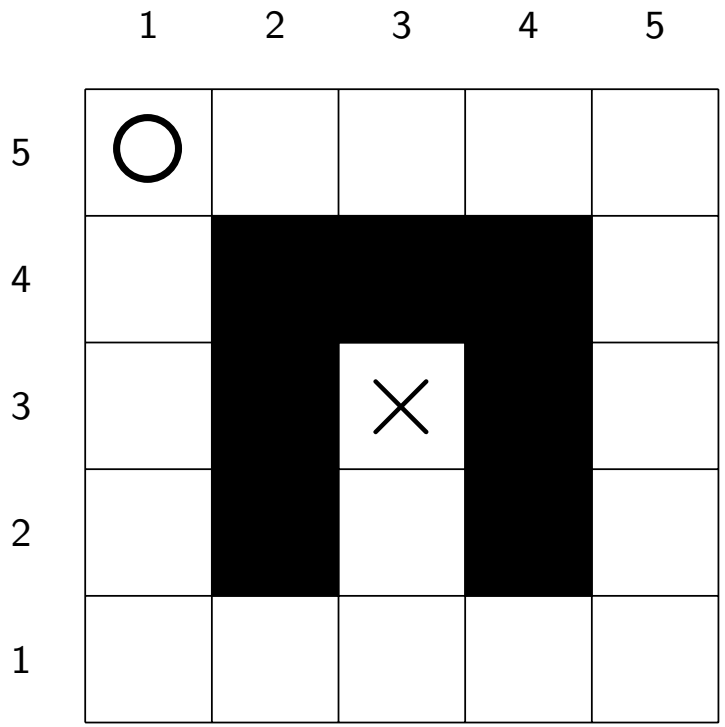
- How do we know which one is best?

Mazes

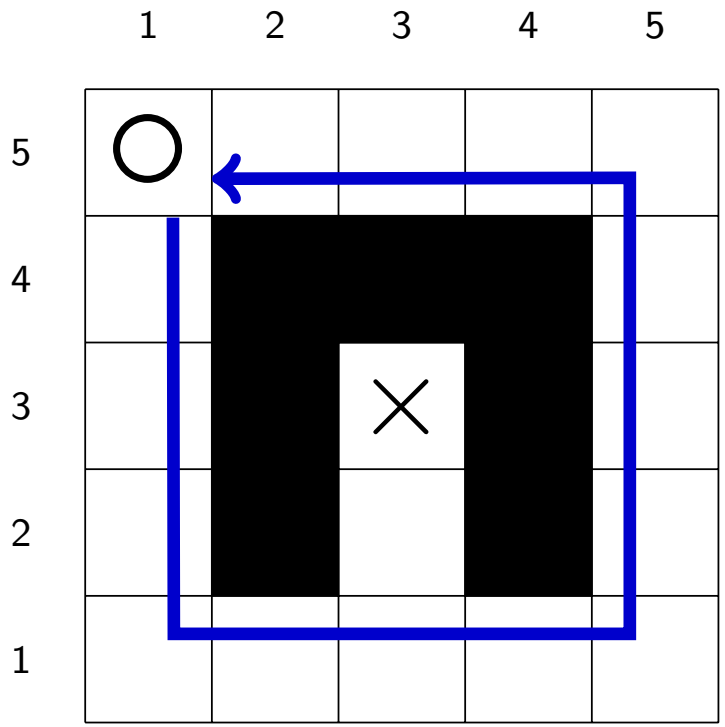


- How do we know which one is best?
- Are there any other problems?

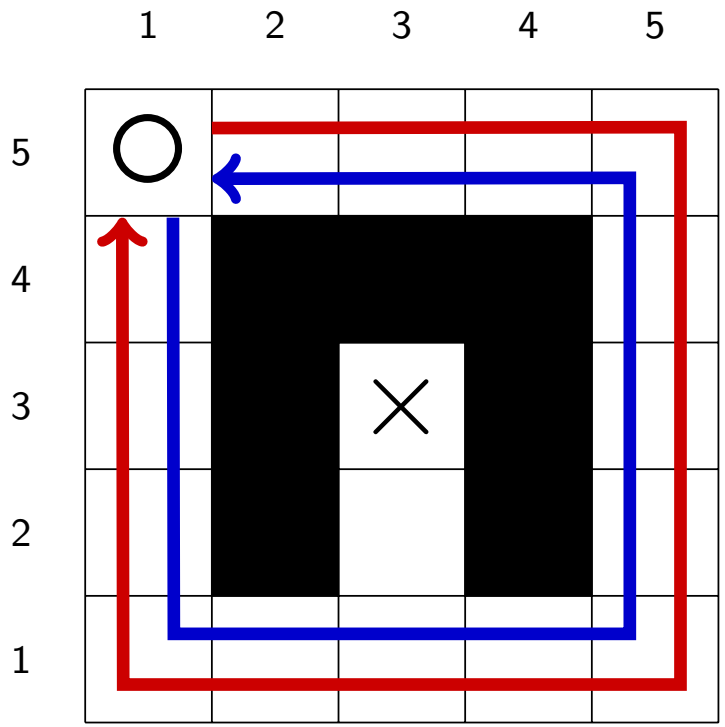
Mazes



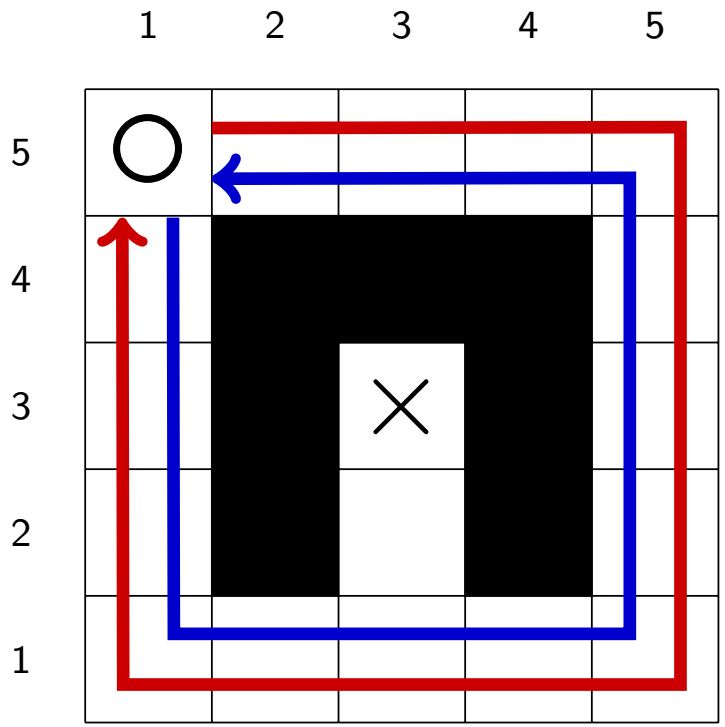
Mazes



Mazes



Mazes



- Priority order doesn't guarantee exploring the entire maze

Formalizing Maze Solving

- **Inputs**
  - The map: An $n \times m$ grid of filled/empty squares.
  - The $\circ$ is at position `start`
  - The $\times$ is at position `dest`
- **Goal**
  - Compute `steps`(`start`, `dest`), the minimum steps from start to end.

Formalizing Maze Solving

- **Inputs**
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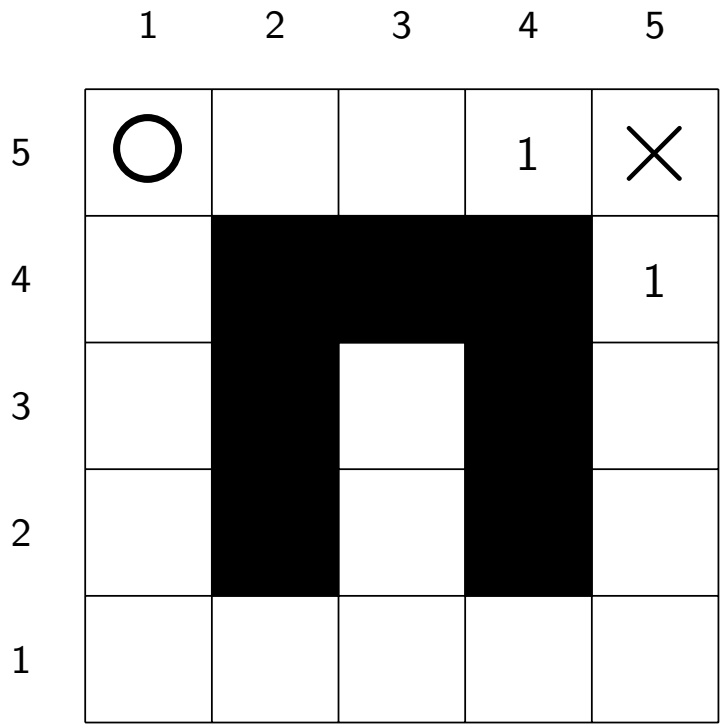
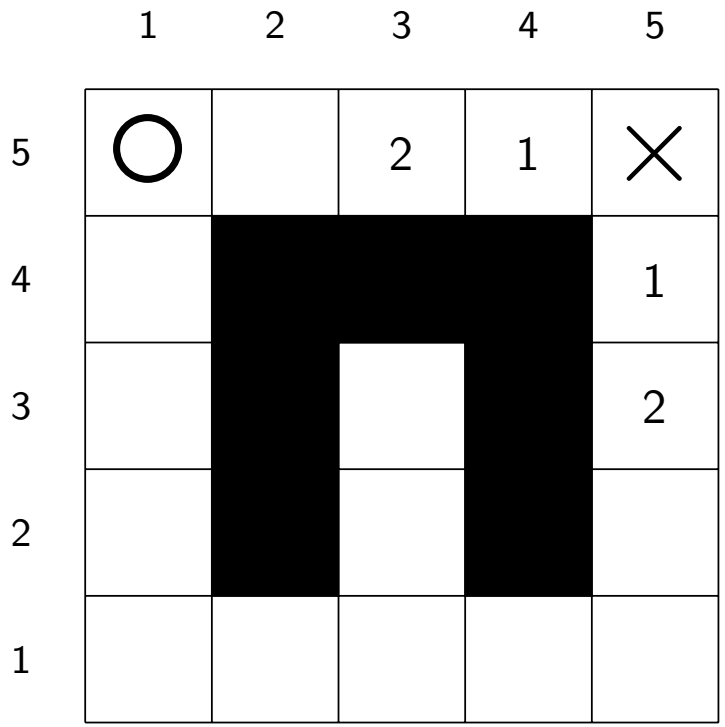
How do we define `steps`?

Formalizing Maze Solving



Formalizing Maze Solving



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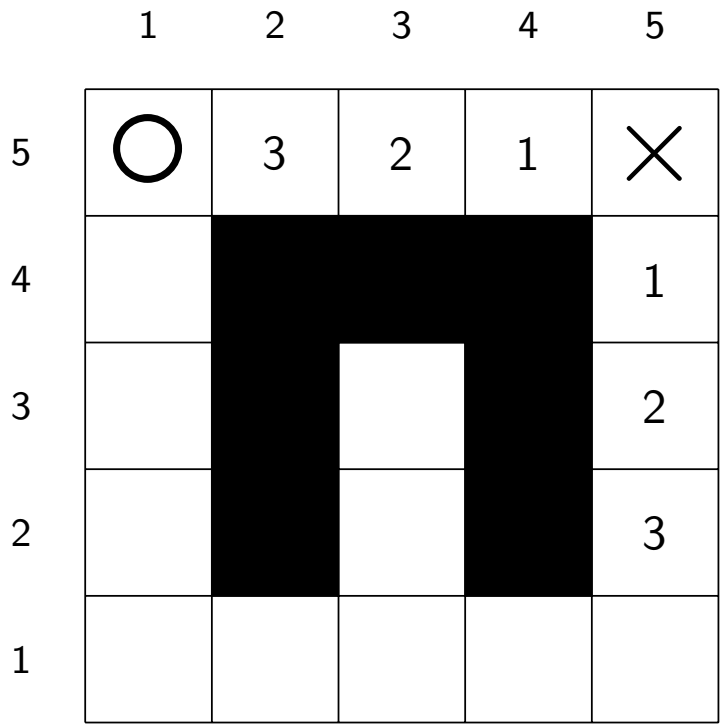

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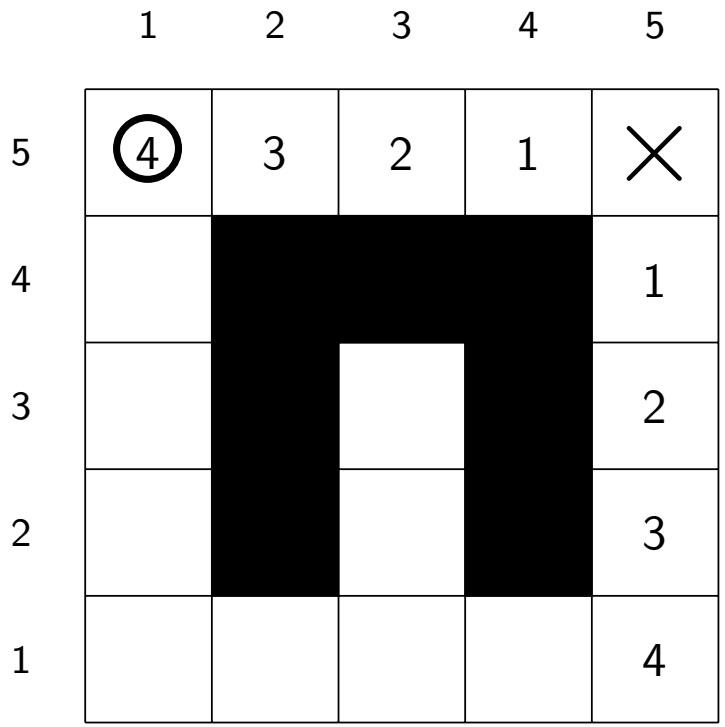
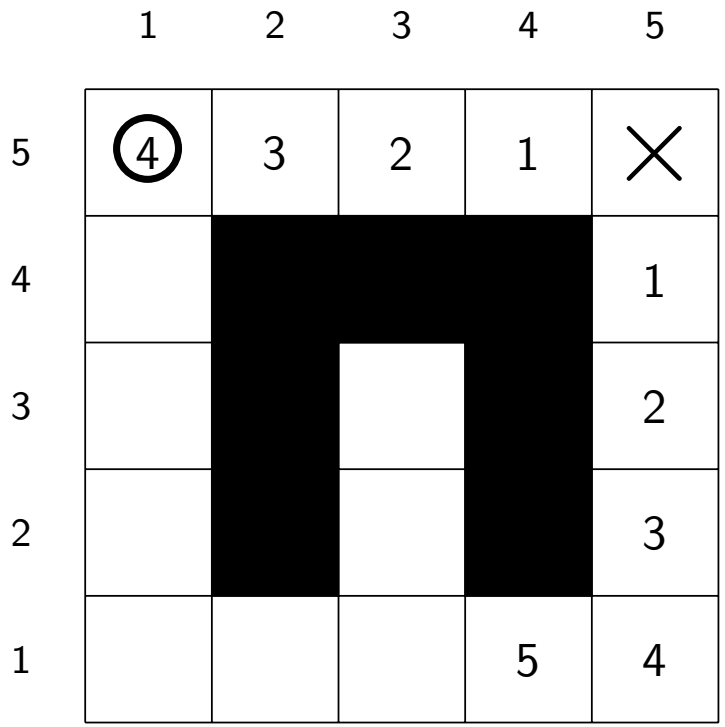
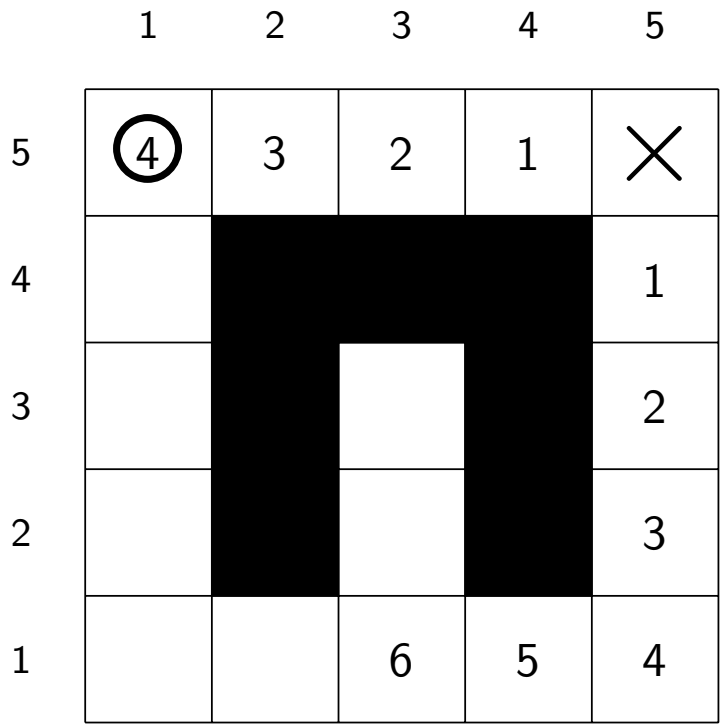


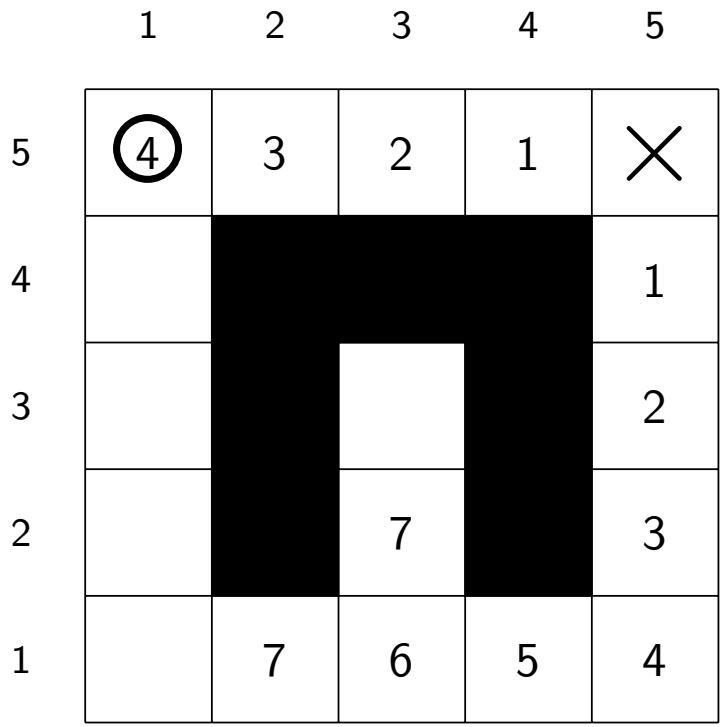
Formalizing Maze Solving



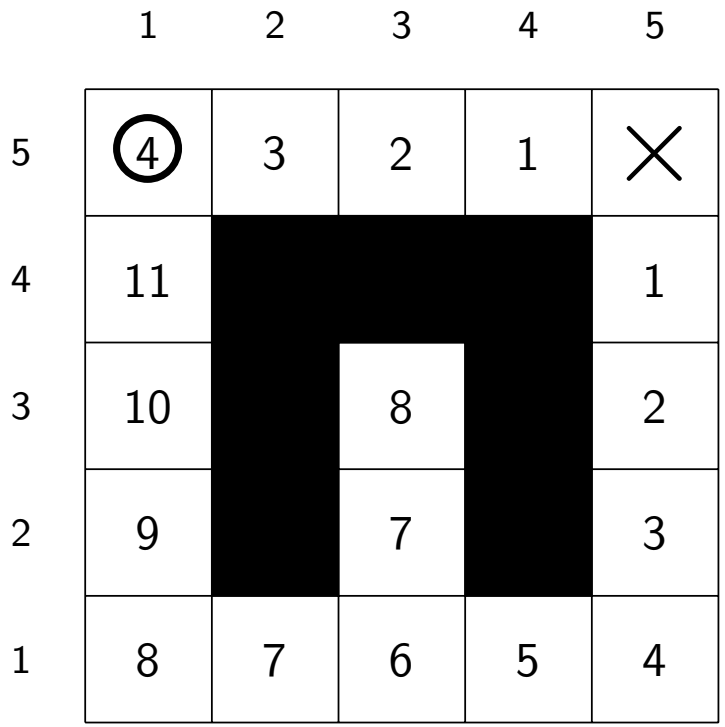
Formalizing Maze Solving



Formalizing Maze Solving



Formalizing Maze Solving



Formalizing Maze Solving

	1	2	3	4	5
5	(4)	3	2	1	✕
4	11	∞	∞	∞	1
3	10	∞	8	∞	2
2	9	∞	7	∞	3
1	8	7	6	5	4

Formalizing Maze Solving

$$\texttt{steps}(\texttt{pos}, \texttt{dest}) = \begin{cases} 0 & \textbf{if } \texttt{pos} = \texttt{dest} \end{cases}$$

Formalizing Maze Solving

$$\texttt{steps}(\texttt{pos}, \texttt{dest}) = \begin{cases} 0 & \textbf{if } \texttt{pos} = \texttt{dest} \\ \infty & \textbf{if } \texttt{pos} \textbf{ is filled} \\ \end{cases}$$

Formalizing Maze Solving

$$\texttt{steps}(\texttt{pos}, \texttt{dest}) = \begin{cases} 0 & \textbf{if } \texttt{pos} = \texttt{dest} \\ \infty & \textbf{if } \texttt{pos} \textbf{ is filled} \\ 1 + \texttt{min\_nearby}(\texttt{pos}, \texttt{dest}) & \textbf{otherwise} \end{cases}$$

Formalizing Maze Solving

$$\texttt{steps}(\texttt{pos}, \texttt{dest}) = \begin{cases} 0 & \textbf{if } \texttt{pos} = \texttt{dest} \\ \infty & \textbf{if } \texttt{pos} \textbf{ is filled} \\ 1 + \texttt{min\_nearby}(\texttt{pos}, \texttt{dest}) & \textbf{otherwise} \end{cases}$$

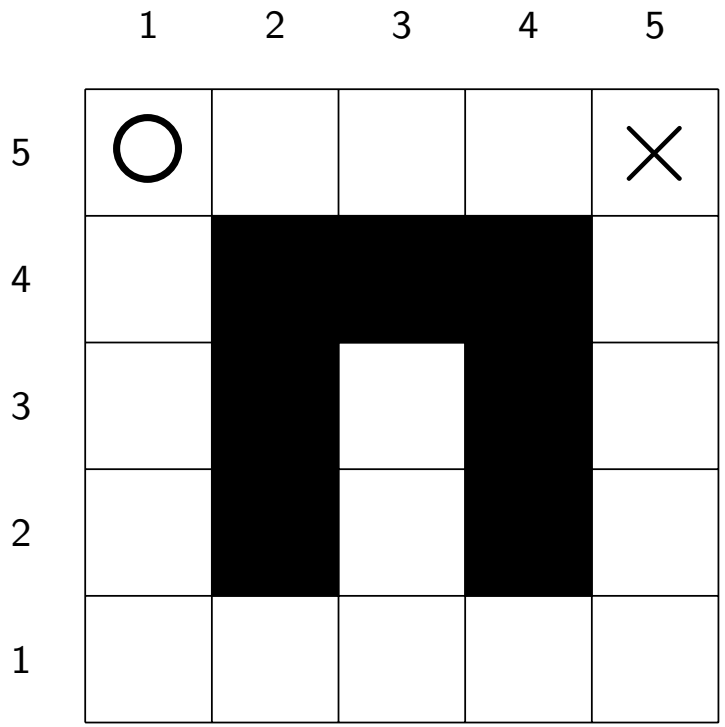
Where...

$$\texttt{min\_nearby}(\texttt{pos}, \texttt{dest}) = \min \begin{cases} \texttt{steps}(\texttt{moveDown}(\texttt{pos}), \texttt{dest}) \\ \texttt{steps}(\texttt{moveLeft}(\texttt{pos}), \texttt{dest}) \\ \texttt{steps}(\texttt{moveRight}(\texttt{pos}), \texttt{dest}) \\ \texttt{steps}(\texttt{moveUp}(\texttt{pos}), \texttt{dest}) \end{cases}$$

Formalizing Maze Solving

```
1   public int steps(Point pos, Point dest)
2   {
3     if(pos == dest){ return 0; }
4     else if(is_filled(pos){ return ∞; }
5     else {
6       return 1 + Math.min(
7         steps(pos.moveDown, dest),
8         steps(pos.moveLeft, dest),
9         steps(pos.moveRight, dest),
10        steps(pos.moveUp, dest)
11      );
12    }
13  }
```

Formalizing Maze Solving



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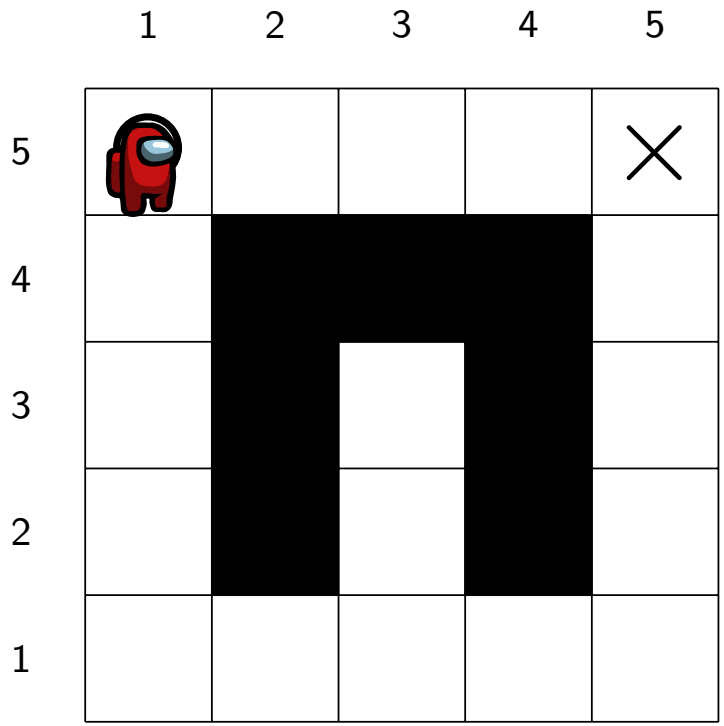

Shortest Path: ∞

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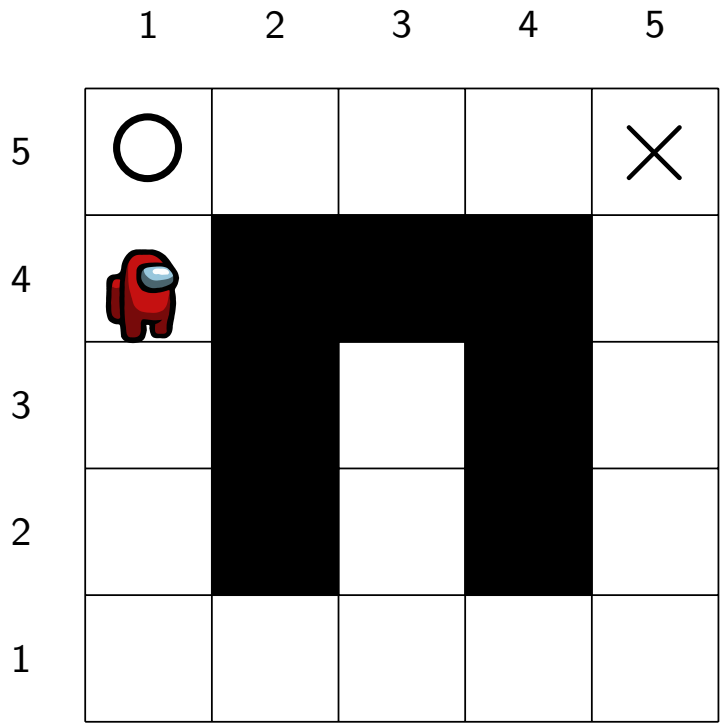

Shortest Path: ∞

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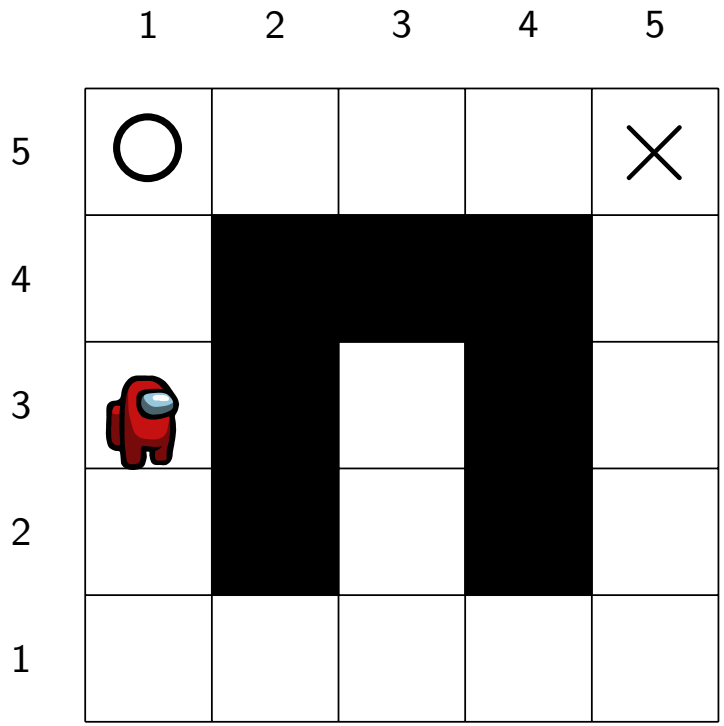

Shortest Path: ∞

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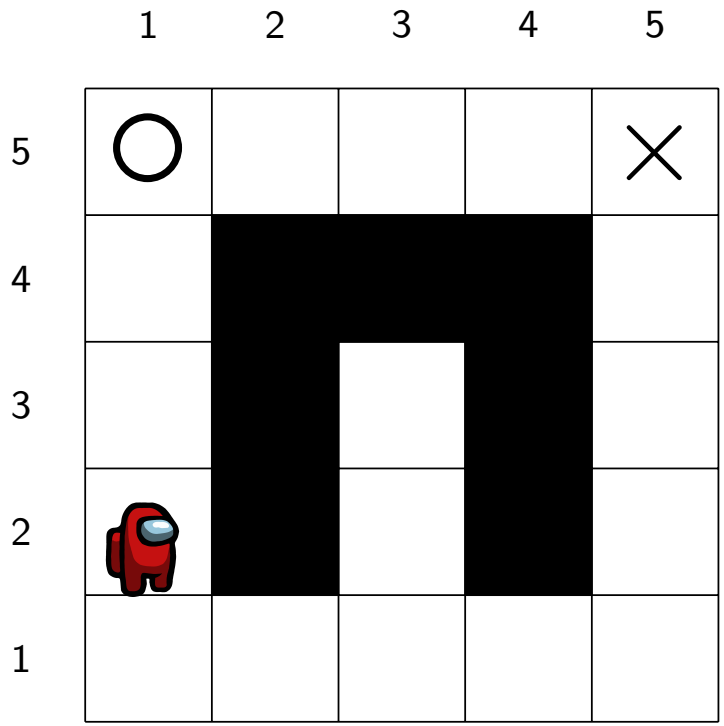

Shortest Path: ∞

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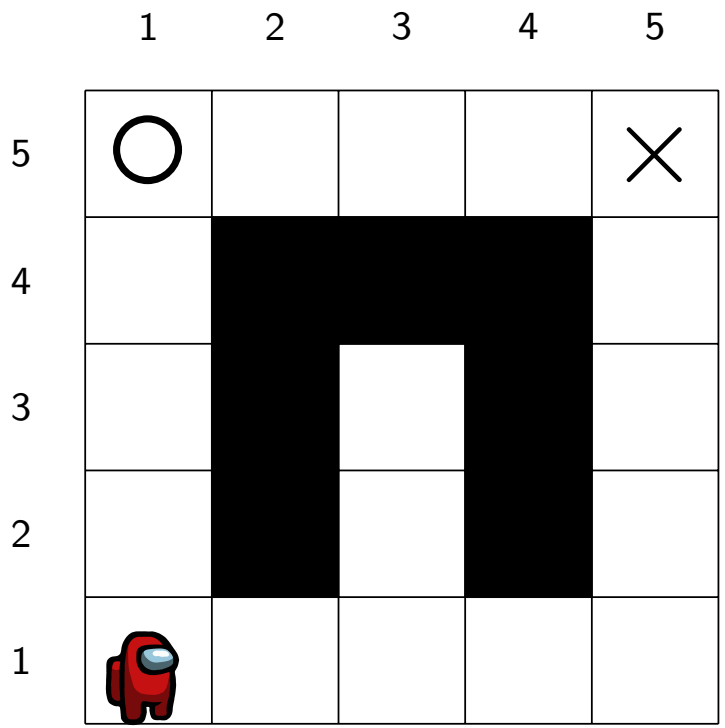

Shortest Path: ∞

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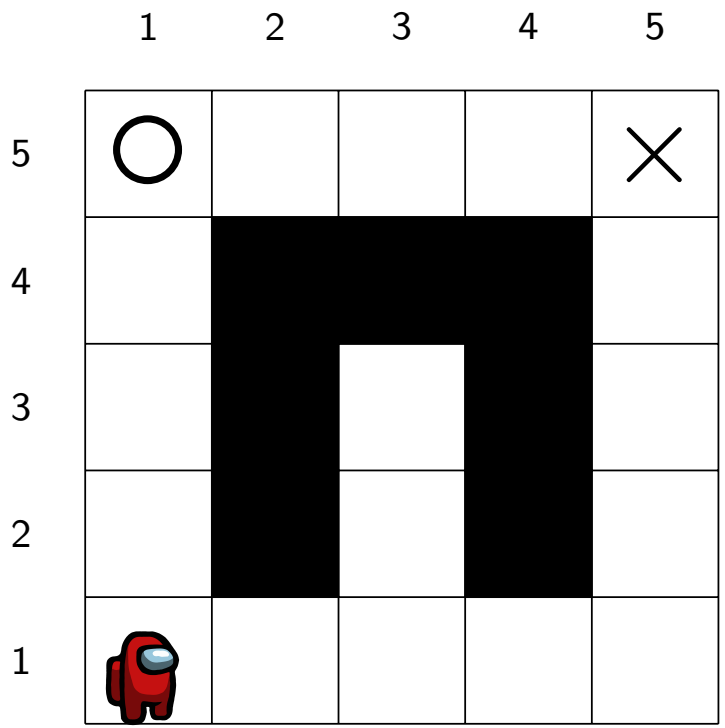

Shortest Path: ∞

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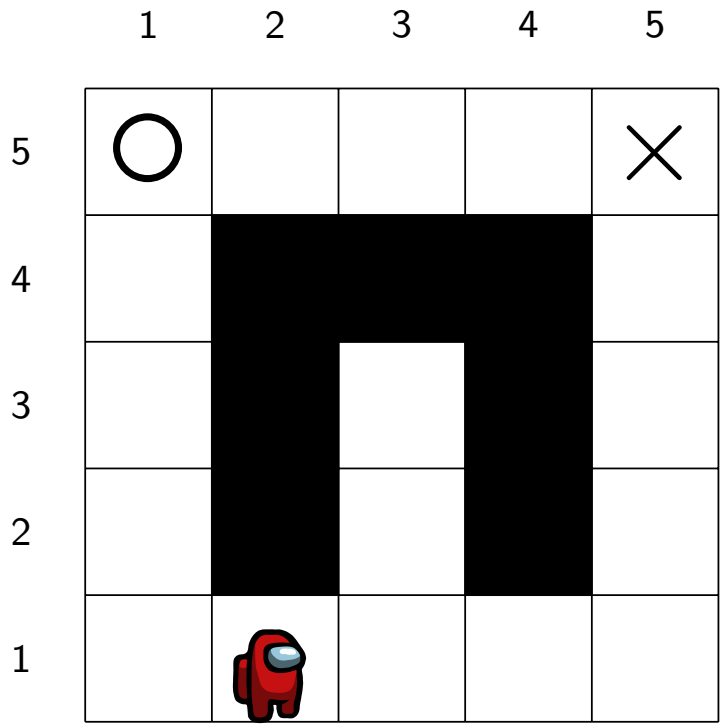

Shortest Path: ∞

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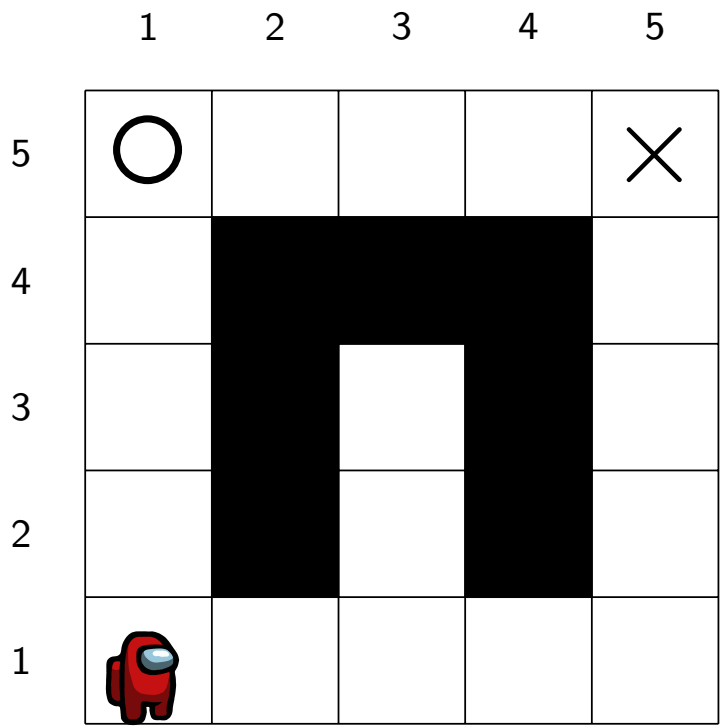

Shortest Path: ∞

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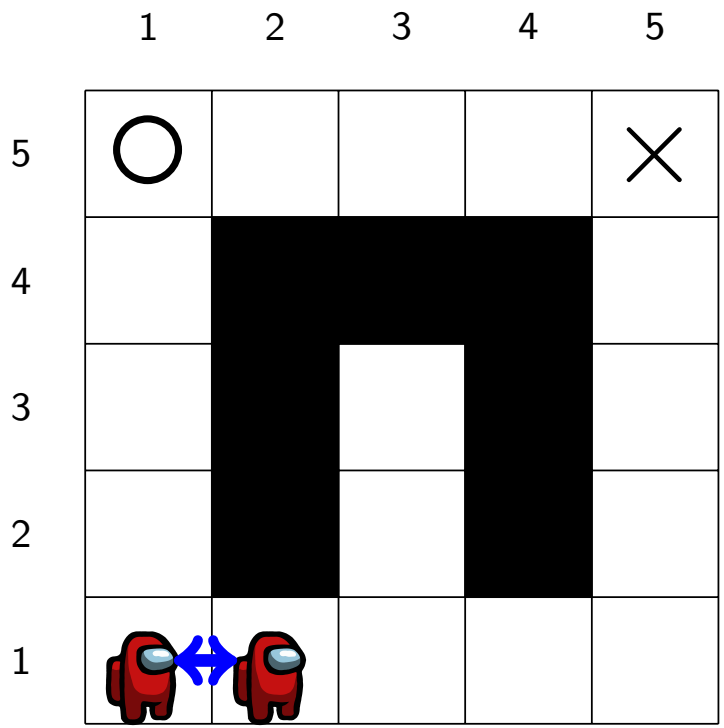

Shortest Path: ∞

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Formalizing Maze Solving

Problem: Infinite Loop

Formalizing Maze Solving

Problem: Infinite Loop

Insight: A path with a loop in it can't be shorter than one without the loop.

Formalizing Maze Solving

Problem: Infinite Loop

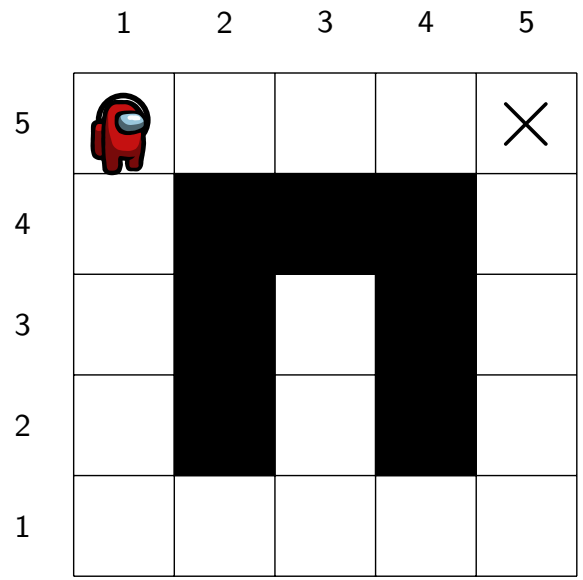
Insight: A path with a loop in it can't be shorter than one without the loop.

Mark nodes as visited so you don't visit them twice.

Formalizing Maze Solving

```
1   public int steps(Point pos, Point dest)
2   {
3     if(pos == dest){ return 0; }
4     else if(is_filled(pos){ return ∞; }
5     else if(is_visited(pos){ return ∞; }
6     else {
7       mark_visited(pos);
8       return 1 + Math.min(
9         steps(pos.moveDown, dest),
10        steps(pos.moveLeft, dest),
11        steps(pos.moveRight, dest),
12        steps(pos.moveUp, dest)
13      );
14    }
15  }
```

Formalizing Maze Solving



Shortest: $1 + \min(?, ?, \infty, \infty) = ?$

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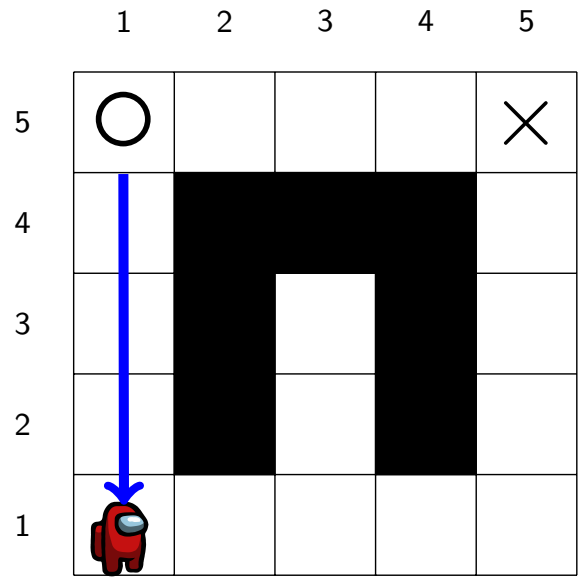
Formalizing Maze Solving



Shortest: $1 + \min(\infty, ?, \infty, \infty) = ?$

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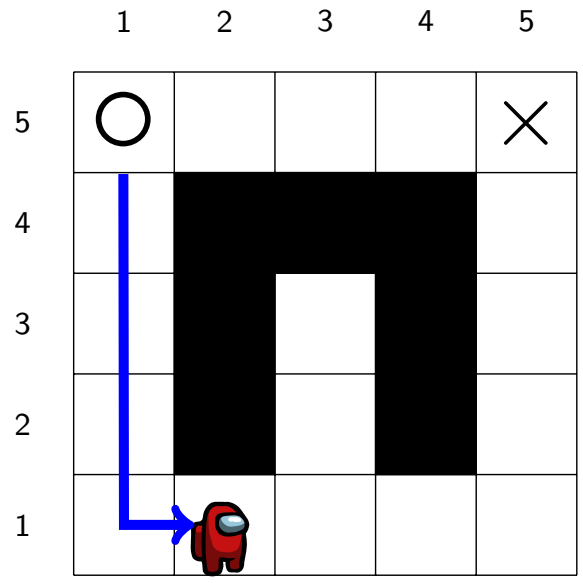
Formalizing Maze Solving



Shortest: $1 + \min(\infty, ?, \infty, \infty) = ?$

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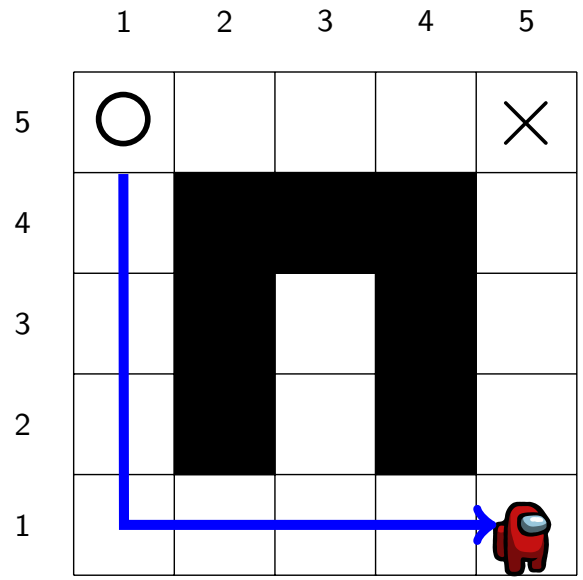
Formalizing Maze Solving



Shortest: $1 + \min(\infty, \infty, \infty, ?) = ?$

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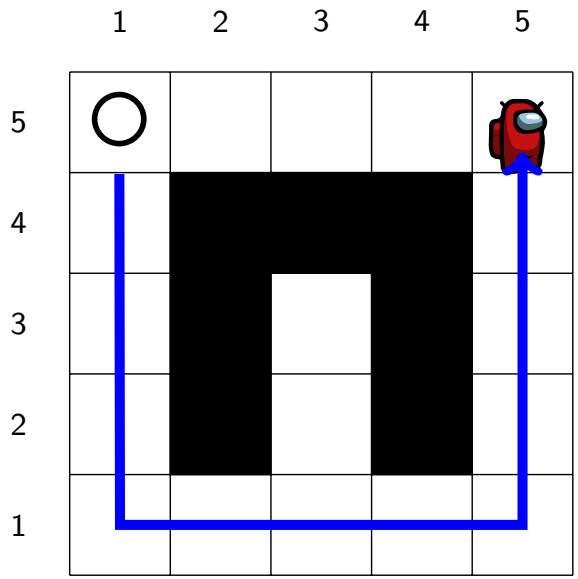

Shortest: 0

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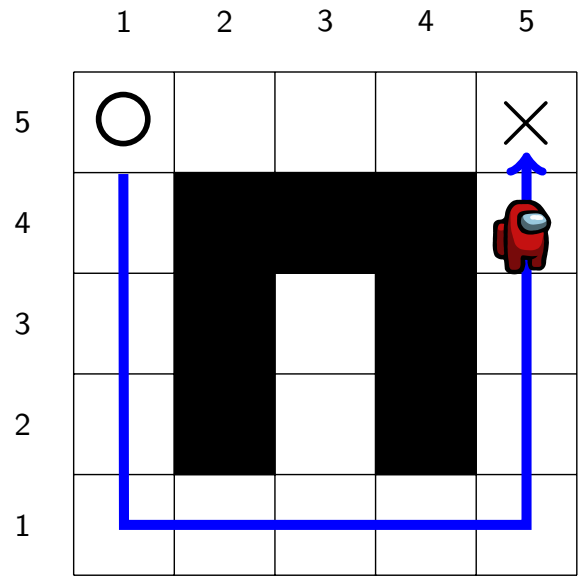

Shortest: $1 + \min(\infty, \infty, \infty, 0) = 1$

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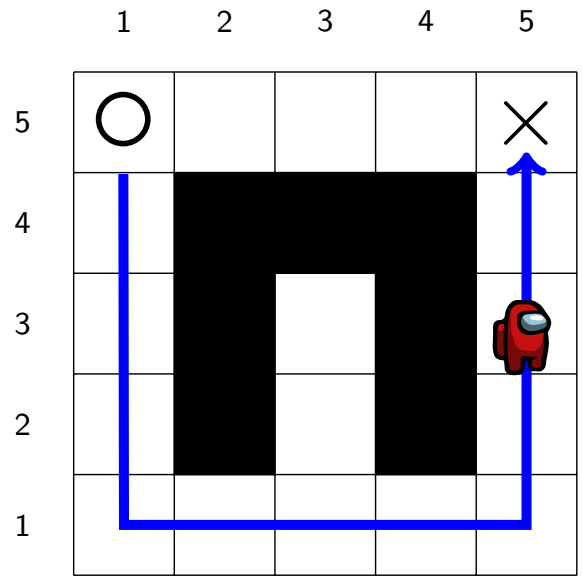

Shortest: $1 + \min(\infty, \infty, \infty, 1) = 2$

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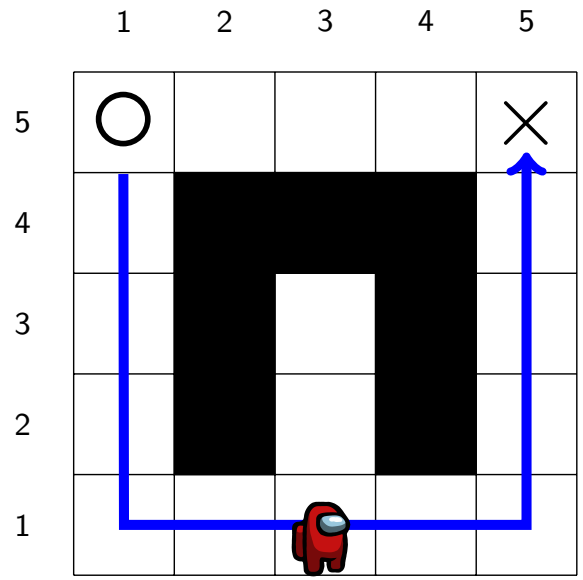
Formalizing Maze Solving



Shortest: $1 + \min(\infty, 5, \infty, ?) = ?$

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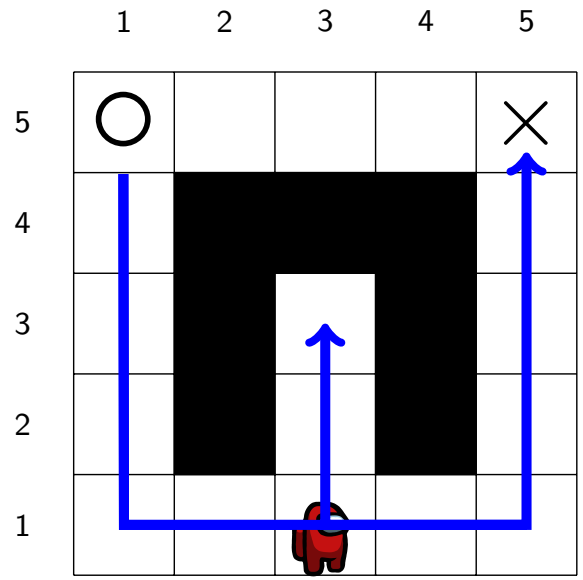
Formalizing Maze Solving



Shortest: $1 + \min(\infty, 5, \infty, \infty) = 6$

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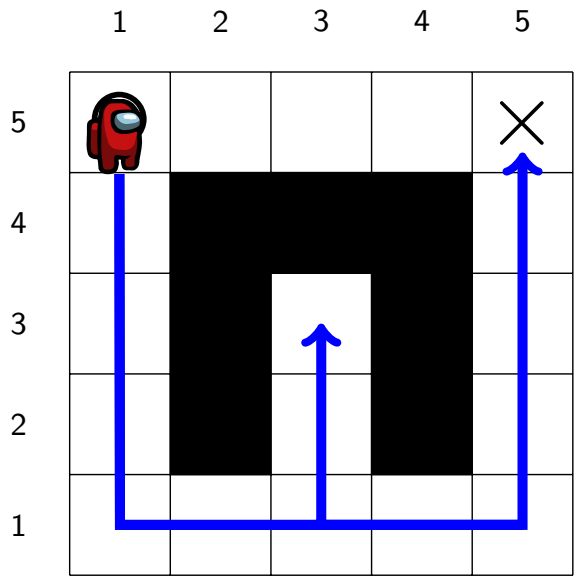
Formalizing Maze Solving



Shortest: $1 + \min(11, ?, \infty, \infty) = ?$

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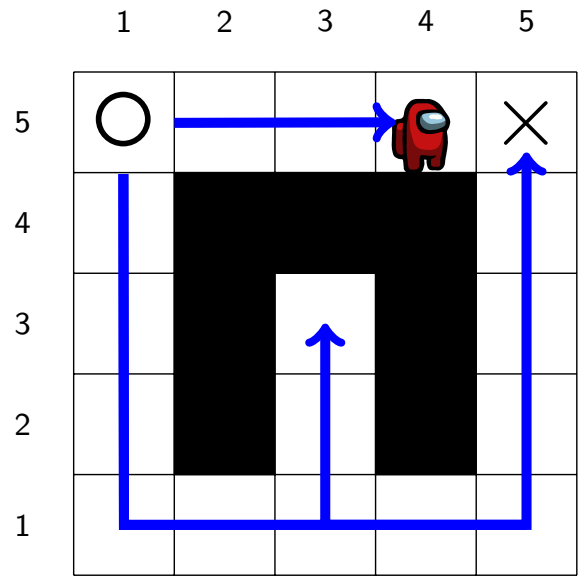
Formalizing Maze Solving



Shortest: $1 + \min(\infty, ?, \infty, \infty) = ?$

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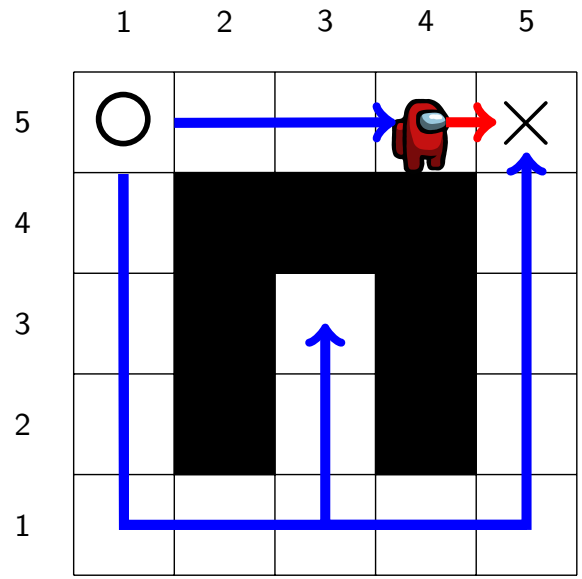
Formalizing Maze Solving



Shortest: $1 + \min(\infty, \infty, \infty, \infty) = \infty$

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Formalizing Maze Solving

Problem: The first time you visit a node it may be via a longer path!

Formalizing Maze Solving

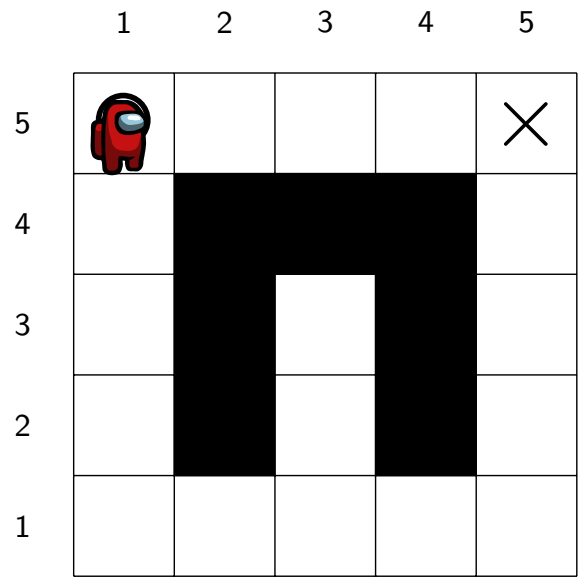
Problem: The first time you visit a node it may be via a longer path!

Unmark nodes as you leave them.

Formalizing Maze Solving

```
1   public int steps(Point pos, Point dest)
2   {
3     if(pos == dest){ return 0; }
4     else if(is_filled(pos){ return ∞; }
5     else if(is_visited(pos){ return ∞; }
6     else {
7       mark_visited(pos);
8       int stepCount = 1 + Math.min(
9         steps(pos.moveRight, dest),
10        /* ... */
11      );
12      unmark_visited(pos);
13      return stepCount;
14    }
15  }
```

Formalizing Maze Solving



Shortest: $1 + \min(?, ?, \infty, \infty) = ?$

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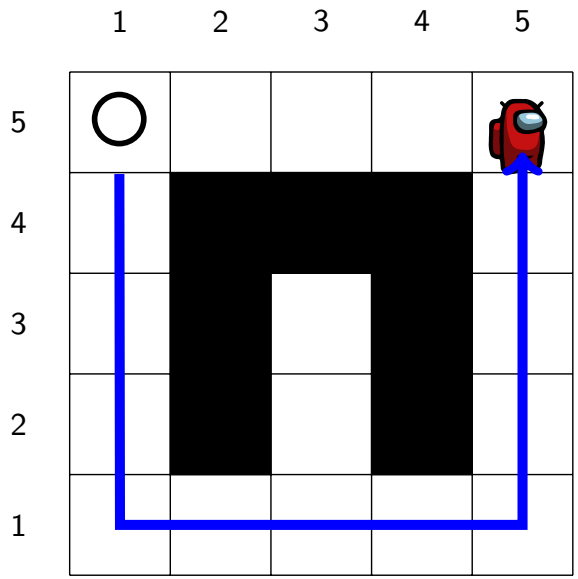
Formalizing Maze Solving



Shortest: 0

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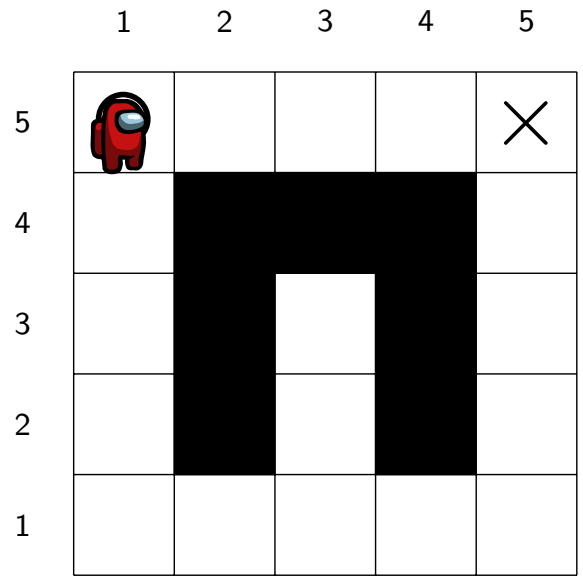

Shortest: $1 + \min(11, ?, \infty, \infty) = ?$

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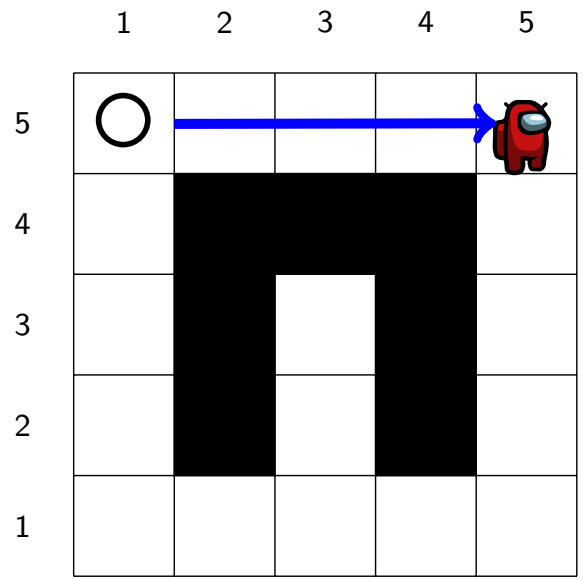
Formalizing Maze Solving



Shortest: 0

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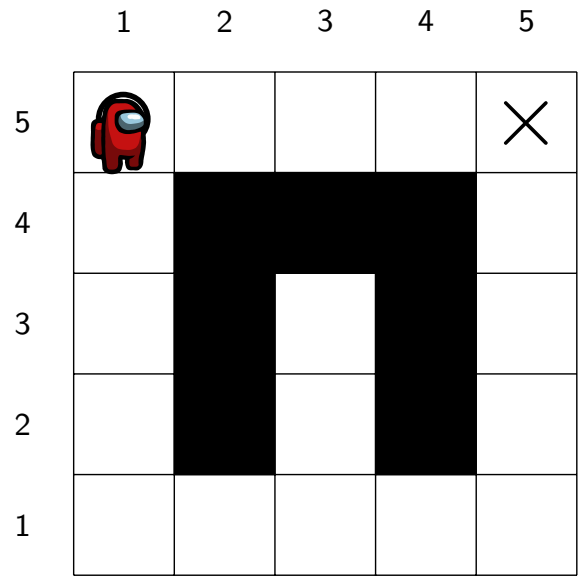
Formalizing Maze Solving



Shortest: $1 + \min(11, 3, \infty, \infty) = ?$

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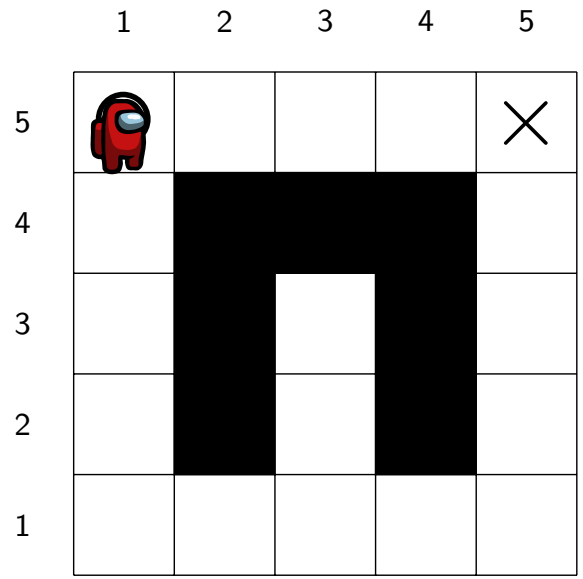

Shortest: $1 + \min(11, 3, \infty, \infty) = 4$

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Formalizing Maze Solving

Question: What path did we take?

Formalizing Maze Solving

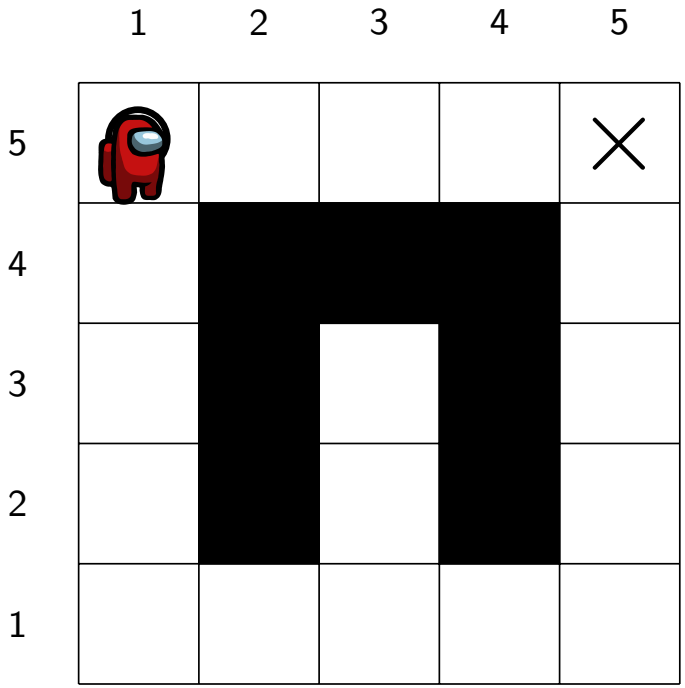
Question: What path did we take?

Track the current path in a `Stack`

Formalizing Maze Solving

```
1   public Array[Point] steps(Point pos, Point dest, Stack visited)
2   {
3     if(pos == dest){ return visited.toArray(); }          ←
4     else if(is_filled(pos){ return null; }
5     else if(visited.contains(pos){ return null; }         ←
6     else {
7       visited.push(pos);                                  ←
8       int steps = shortest_array(
9         steps(pos.moveRight, dest),
10        /* ... */
11      );
12      visited.pop(pos);                                   ←
13    }
14  }
```

Formalizing Maze Solving



(1,5)

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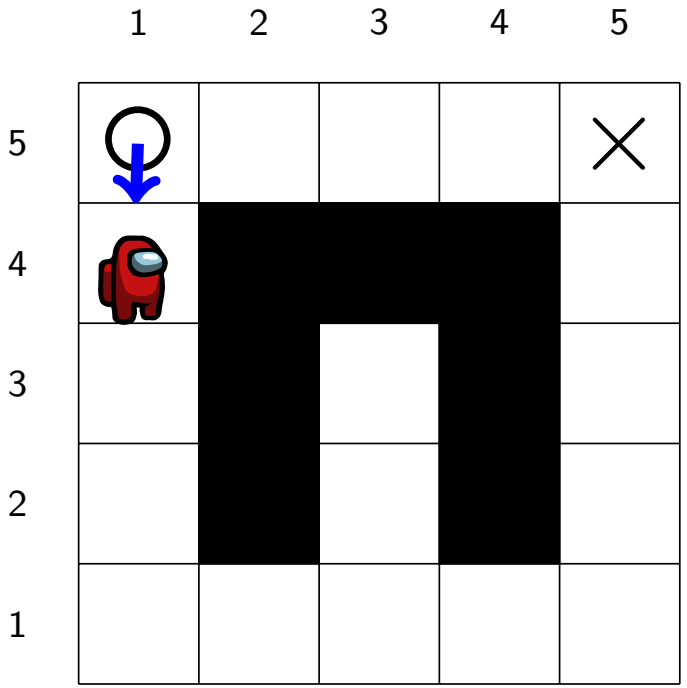
Formalizing Maze Solving



(1,5)
(1,4)

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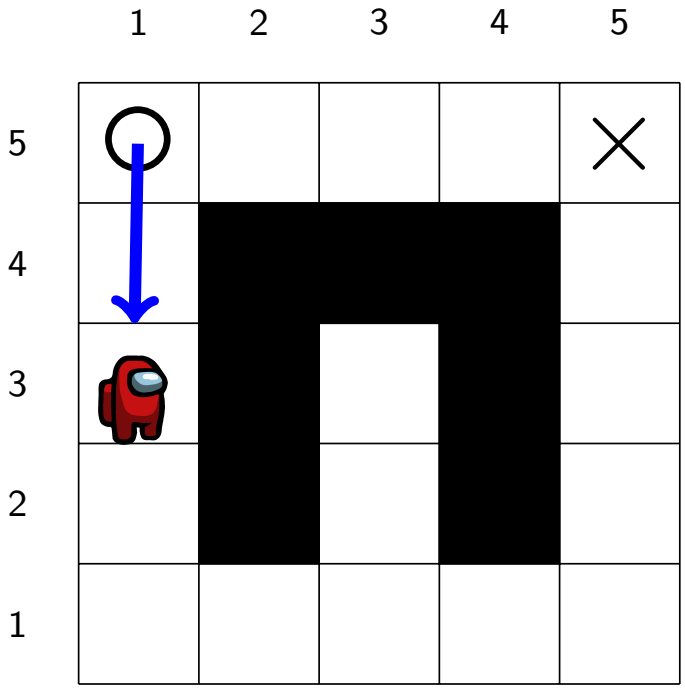
Formalizing Maze Solving



(1,5)
(1,4)
(1,3)

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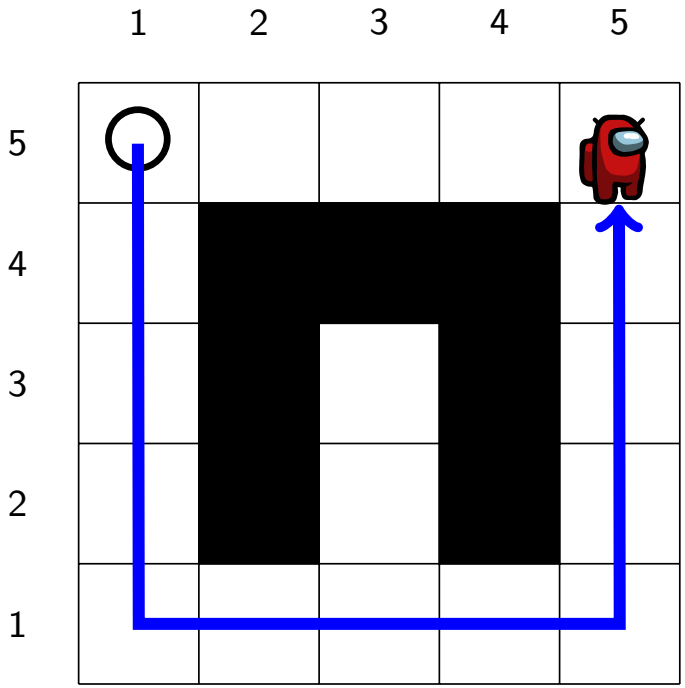
Formalizing Maze Solving



(1,5)
(1,4)
(1,3)
(1,2)
(1,1)
(2,1)
(3,1)
(4,1)
(5,1)
(5,2)
(5,3)
(5,4)
(5,5)

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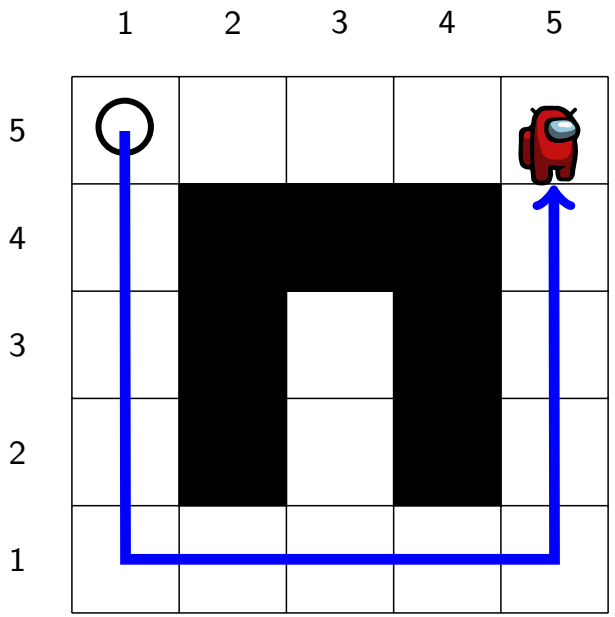

(1,5) (1,5)
(1,4) (1,4)
(1,3) (1,3)
(1,2) (1,2)
(1,1) (1,1)
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(5,1) (5,1)
(5,2) (5,2)
(5,3) (5,3)
(5,4) (5,4)
(5,5) (5,5)

✓

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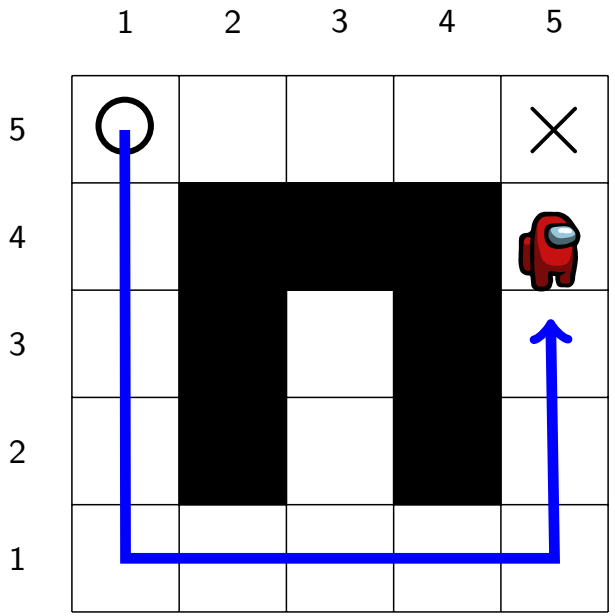

(1,5) (1,5)
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✓

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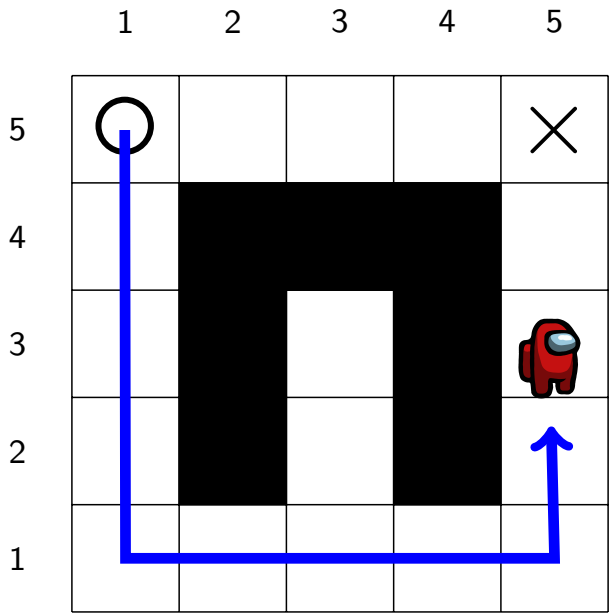

(1,5) (1,5)
(1,4) (1,4)
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(1,1) (1,1)
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(3,1) (3,1)
(4,1) (4,1)
(5,1) (5,1)
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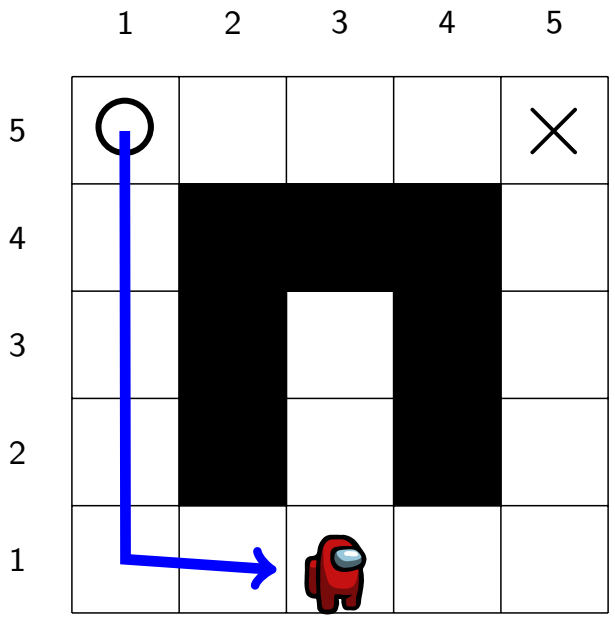
Formalizing Maze Solving



(1,5) (1,5)
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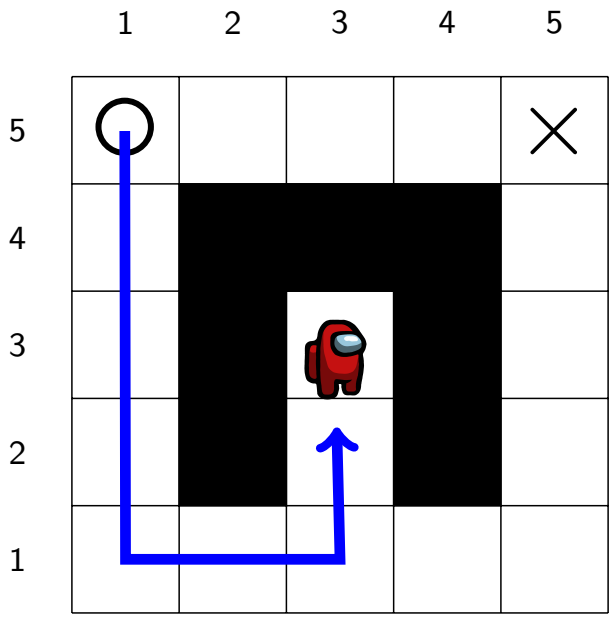

(1,5)	(1,5)
(1,4)	(1,4)
(1,3)	(1,3)
(1,2)	(1,2)
(1,1)	(1,1)
(2,1)	(2,1)
(3,1)	(3,1)
(4,1)	(3,2)
(5,1)	(3,3)
(5,2)	✗
(5,3)	
(5,4)	
(5,5)	
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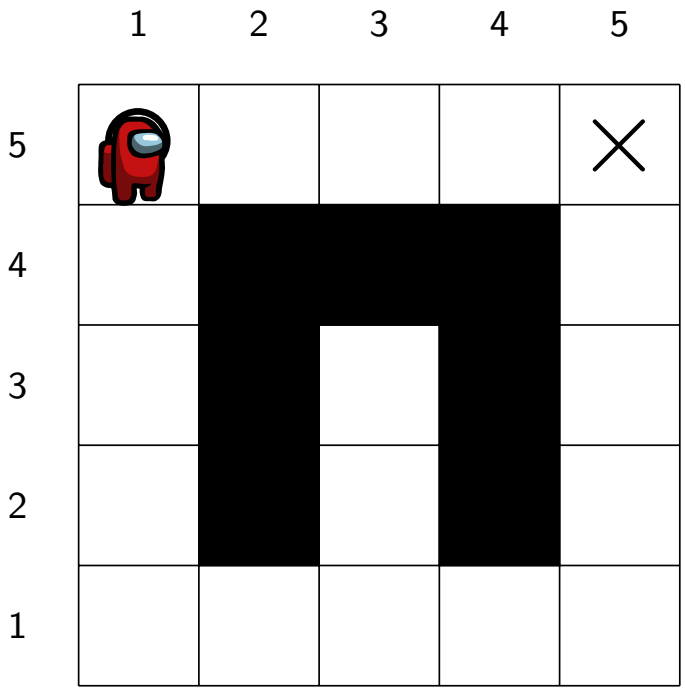

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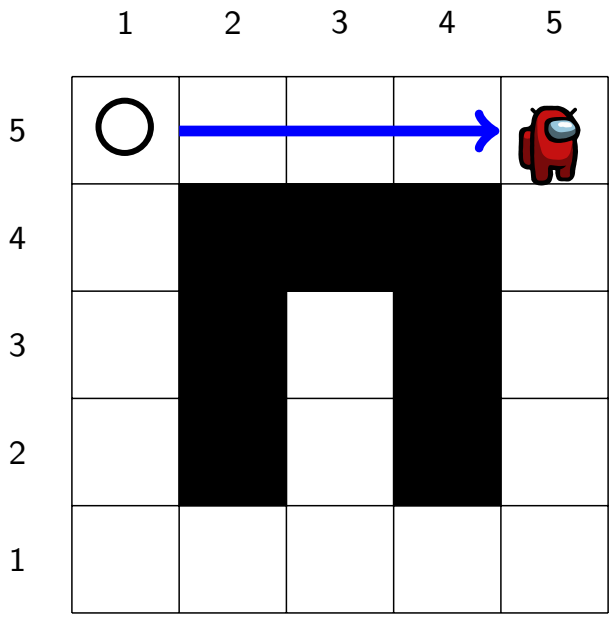

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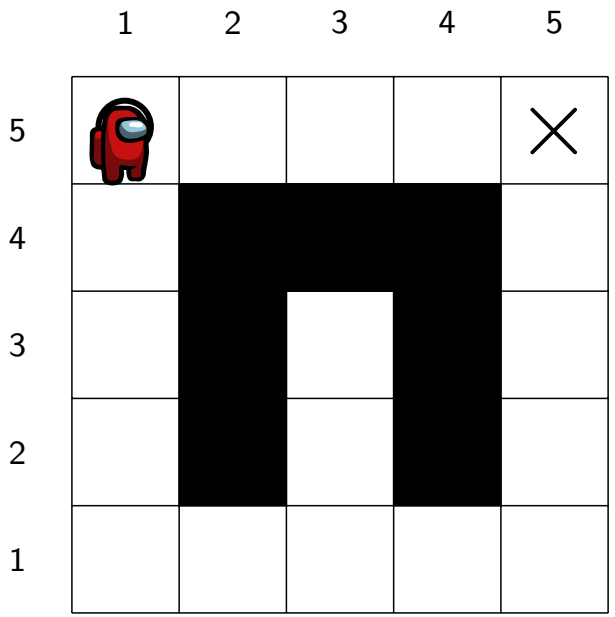

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(5,2)
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(5,5)
✓

(1,5)
(2,5)
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(4,5)
(5,5)
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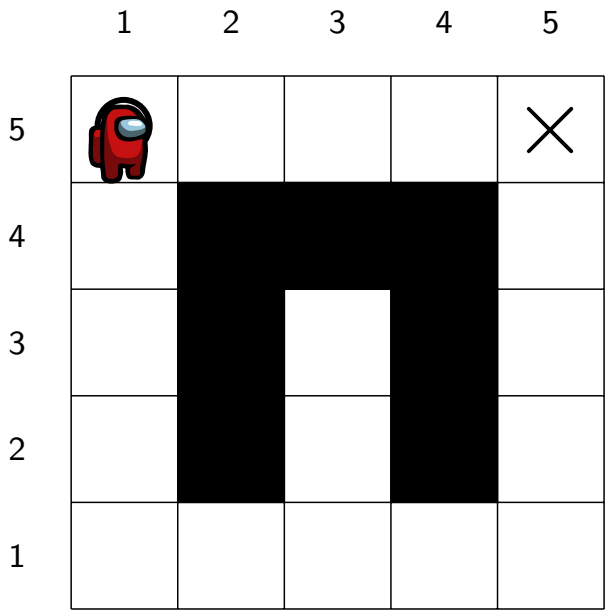

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(5,5)
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(3,5)
(4,5)
(5,5)
✓

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Queues

Thought Question: How would we need to change the algorithm to use a queue instead of a stack?