CSE 250

Data Structures

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Lec 19: Adjacency Lists and Matrices

Announcements

- WA3 due Sunday

Edge List Summary

Graph

vertices: LinkedList<Vertex>
edges: LinkedList<Edge>

Vertex

label: T
node: LinkedListNode

Edge

label: T
vertex: origin
vertex: destination
node: LinkedListNode

Storing the list nodes in the edges/vertices allows us to remove by reference in $\Theta(1)$ time

Edge List Summary

- `addEdge, addVertex`: $O(1)$
- `removeEdge`: $O(1)$
- `removeVertex`: $O(m)$
- `vertex.incidentEdges`: $O(m)$
- `vertex.edgeTo`: $O(m)$
- **Space Used:** $O(n) + O(m)$

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← Involves checking every edge in the graph

How can we improve?

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Idea: Store the in/out edges for each vertex!

(Called an adjacency list)

Adjacency List

```
public class Vertex<V,E> {
  public Node<Vertex> node;
  public List<Edge> inEdges = new CustomLinkedList<Edge>();
  public List<Edge> outEdges = new CustomLinkedList<Edge>();
  /*...*/
}
```

Each vertex stores a list of **inEdges** and **outEdges**, which are maintained as the graph is modified…

What functions need to change to maintain these lists?

Adjacency List

```java
public Edge addEdge(Vertex orig, Vertex dest, E label) {
  Edge e = new Edge(orig, dest, label);
  e.node = edges.add(e);
  orig.outEdges.add(e);
  dest.inEdges.add(e);
  return e;
}
```

← When we add an edge to the graph, also add it to the appropriate adjacency lists

What is the complexity of **addEdge** now?

Adjacency List

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public Edge addEdge(Vertex orig, Vertex dest, E label) {
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← When we add an edge to the graph, also add it to the appropriate adjacency lists

What is the complexity of **addEdge** now? Still $\Theta(1)$

Adjacency List

```java
public void removeEdge(Edge edge) {
  edges.remove(edge.node);
  edge.orig.outEdges.remove(edge);
  edge.dest.inEdges.remove(edge);
}
```

← When we remove an edge from the graph, also remove it from the adjacency lists

What is the complexity of **`removeEdge`** now?

Adjacency List

```
public void removeEdge(Edge edge) {
  edges.remove(edge.node);
  edge.orig.outEdges.remove(edge);
  edge.dest.inEdges.remove(edge);
}
```

← When we remove an edge from the graph, also remove it from the adjacency lists

What is the complexity of **`removeEdge`** now? ***O*(deg(orig) + deg(dest)) :(**

But how can we fix this?

Adjacency List

```java
public class Edge<V,E> {
  public Node<Edge> node;
  public Node<Edge> inNode;
  public Node<Edge> outNode;
  /*...*/
}
```

Each Edge now also stores a reference to the nodes in each adjacency list

Adjacency List

```java
public Edge addEdge(Vertex orig, Vertex dest, E label) {
  Edge e = new Edge(orig, dest, label);
  e.node = edges.add(e);
  e.outNode = orig.outEdges.add(e);
  e.inNode = dest.inEdges.add(e);
  return e;
}
```

← When we add an edge to the graph, also add it to the appropriate adjacency lists AND store the node refs in the Edge object

What is the complexity of **addEdge** now? Still $\Theta(1)$

Adjacency List

```java
public void removeEdge(Edge edge) {
  edges.remove(edge.node);
  edge.orig.outEdges.remove(edge.outNode);
  edge.dest.inEdges.remove(edge.inNode);
}
```

← When we remove an edge from the graph, also remove it from the adjacency lists (remove by reference)

What is the complexity of **`removeEdge`** now?

Adjacency List

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← When we remove an edge from the graph, also remove it from the adjacency lists (remove by reference)

What is the complexity of **`removeEdge`** now? $\Theta(1)$

Adjacency List

So, we are able to store and maintain adjacency lists in each vertex while still keeping a $\Theta(1)$ runtime for **`addVertex`**, **`addEdge`**, and **`removeEdge`**

How much extra space is used?

Adjacency List

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How much extra space is used? $\Theta(1)$ per edge

Each edge only appears in 3 lists:

- The edge list
- One vertices inList
- One vertices outList

Adjacency List

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How much extra space is used? $\Theta(1)$ per edge

Each edge only appears in 3 lists:

- The edge list
- One vertices inList
- One vertices outList

But now what have we gained?

Adjacency List

```
public void removeVertex(Vertex v) {
  for(edge : v.getIncidentEdges()) {
    removeEdge(edge.node)
  }
  vertices.remove(v.node);
}
```

What is the complexity of **`removeVertex`** now?

Adjacency List

```
public void removeVertex(Vertex v) {
  for(edge : v.getIncidentEdges()) {
    Θ(1)
  }
  Θ(1)
}
```

What is the complexity of **`removeVertex`** now?

Adjacency List

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We now have a reference to the list of edges in $\Theta(1)$ time, and there are **deg(v)** edge in the list

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Adjacency List

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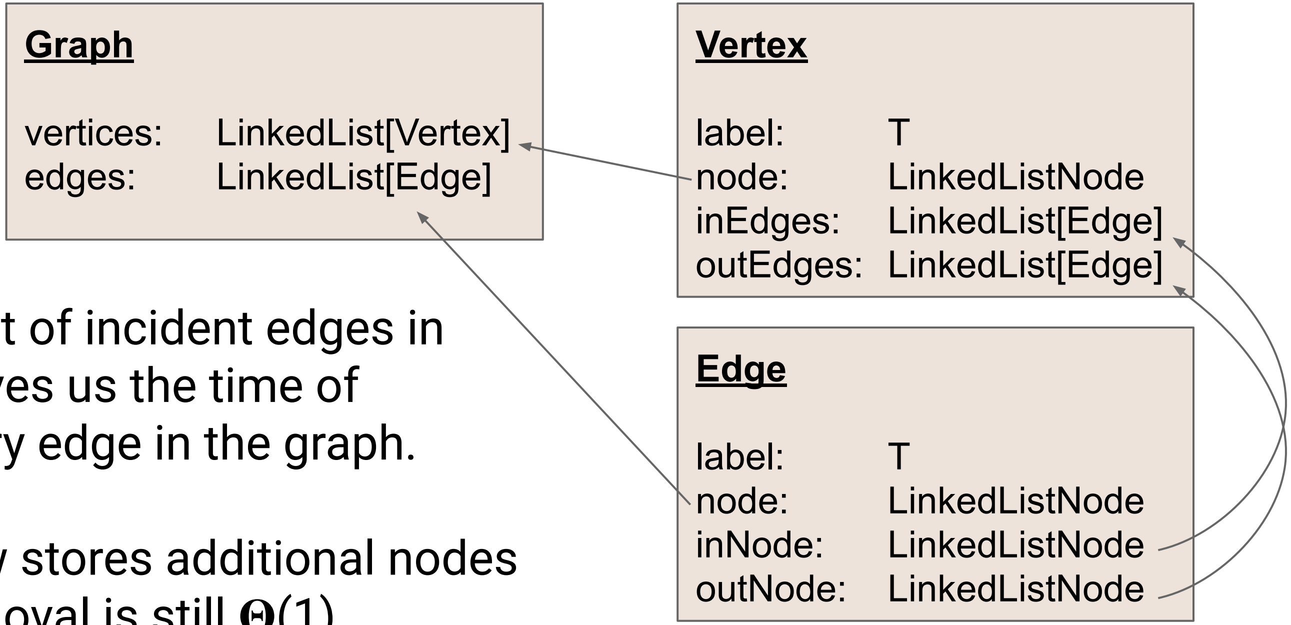
We now have a reference to the list of edges in $\Theta(1)$ time, and there are **deg(v)** edge in the list

What is the complexity of **`removeVertex`** now? $\boldsymbol{\Theta(\mathit{deg}(v))}$

Adjacency List Summary



Storing the list of incident edges in the vertex saves us the time of checking every edge in the graph.

The edge now stores additional nodes to ensure removal is still $\Theta(1)$

Adjacency List Summary

- `addEdge, addVertex`: **Θ(1)**
- `removeEdge`: **Θ(1)**
- `removeVertex`: **Θ(deg(vertex))**
- `vertex.incidentEdges`: **Θ(deg(vertex))**
- `vertex.edgeTo`: **Θ(deg(vertex))**
- **Space Used: Θ(n) + Θ(m)**

Adjacency List Summary

- `addEdge, addVertex`: $\Theta(1)$
- `removeEdge`: $\Theta(1)$
- `removeVertex`: $\Theta(deg(vertex))$
- `vertex.incidentEdges`: $\Theta(deg(vertex))$
- `vertex.edgeTo`: $\Theta(deg(vertex))$
- **Space Used:** $\Theta(n) + \Theta(m)$

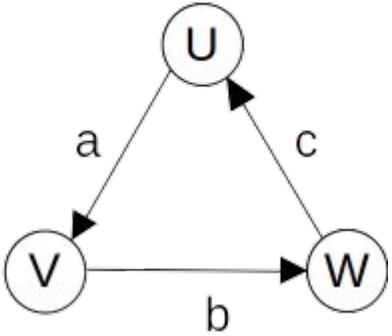
Now we already know what edges are incident without having to check them all

Adjacency Matrix

Destination

Origin

	U	V	W
U	*-*	***a***	*-*
V	*-*	*-*	***b***
W	***c***	*-*	*-*



Adjacency Matrix Summary

- `addEdge, removeEdge`:
- `addVertex, removeVertex`:
- `vertex.incidentEdges`:
- `vertex.edgeTo`:
- **Space Used:**

Adjacency Matrix Summary

Just change a single entry of the matrix

- `addEdge, removeEdge`: $\Theta(1)$
- `addVertex, removeVertex`:
- `vertex.incidentEdges`:
- `vertex.edgeTo`:
- **Space Used:**

Adjacency Matrix Summary

- `addEdge, removeEdge`: $\Theta(1)$
- `addVertex, removeVertex`: $\Theta(n^2)$
- `vertex.incidentEdges`:
- `vertex.edgeTo`:
- **Space Used:**

Resize and copy the whole matrix

Adjacency Matrix Summary

- `addEdge, removeEdge`: $\Theta(1)$
- `addVertex, removeVertex`: $\Theta(n^2)$
- `vertex.incidentEdges`: $\Theta(n)$
- `vertex.edgeTo`:
- **Space Used:**

Check the row and column for that vertex

Adjacency Matrix Summary

- `addEdge, removeEdge`: $\Theta(1)$
- `addVertex, removeVertex`: $\Theta(n^2)$
- `vertex.incidentEdges`: $\Theta(n)$
- `vertex.edgeTo`: $\Theta(1)$
- **Space Used:**

Check a single entry of the matrix

Adjacency Matrix Summary

- `addEdge, removeEdge`: $\Theta(1)$
- `addVertex, removeVertex`: $\Theta(n^2)$
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How does this relate to space of edge/adjacency lists?

Adjacency Matrix Summary

- `addEdge, removeEdge`: $\Theta(1)$
- `addVertex, removeVertex`: $\Theta(n^2)$
- `vertex.incidentEdges`: $\Theta(n)$
- `vertex.edgeTo`: $\Theta(1)$
- **Space Used:** $\Theta(n^2)$

How does this relate to space of edge/adjacency lists? **If the matrix is "dense" it's about the same**

So…what do we do with our graphs?

Connectivity Problems

Given graph ***G***:

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- Is vertex ***u*** **adjacent** to vertex ***v***?

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- Which vertices are **connected** to vertex ***v***?

Connectivity Problems

Given graph ***G***:

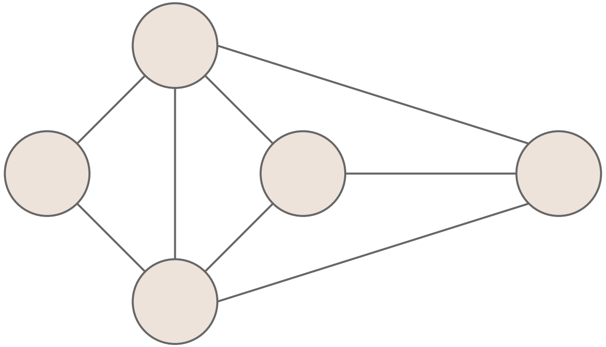
- Is vertex ***u*** **adjacent** to vertex ***v***?
- Is vertex ***u*** **connected** to vertex ***v*** via some path?
- Which vertices are **connected** to vertex ***v***?
- What is the **shortest path** from vertex ***u*** to vertex ***v***?

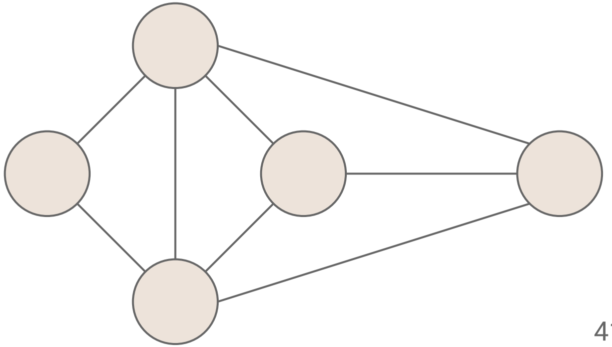
A few more definitions

A **subgraph, *S*,** of a graph ***G*** is a graph where:

S's vertices are a subset of ***G***'s vertices

S's edges are a subset of ***G***'s edges





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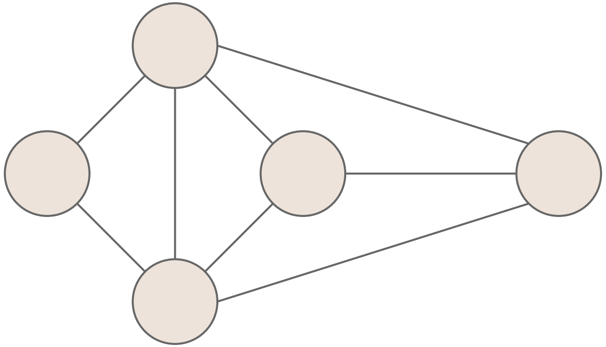
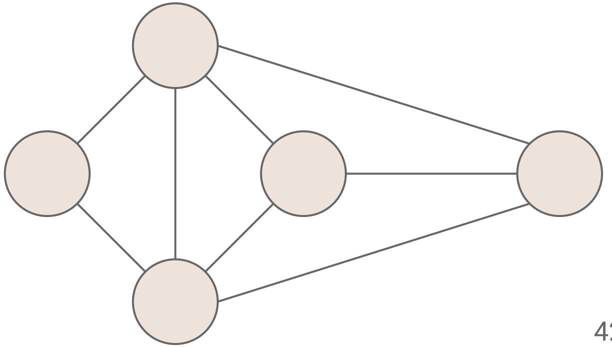


A **spanning subgraph** of ***G*…**

Is a subgraph of ***G***

Contains all of ***G***'s vertices



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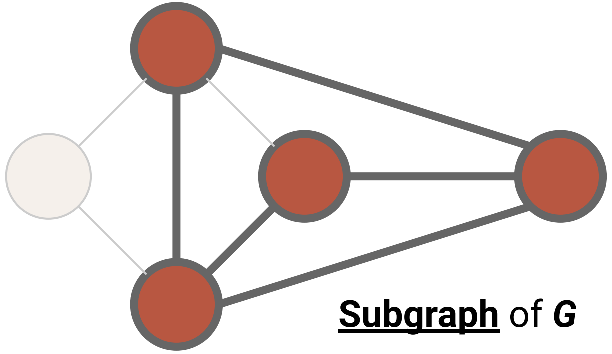

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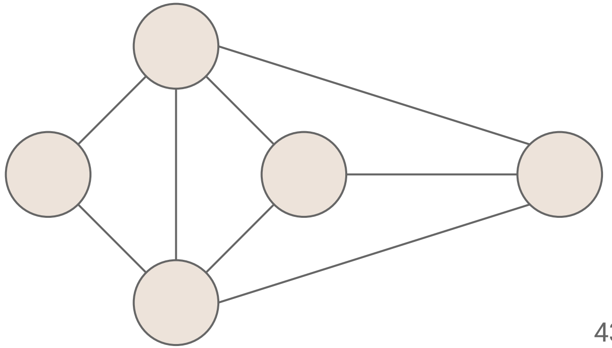

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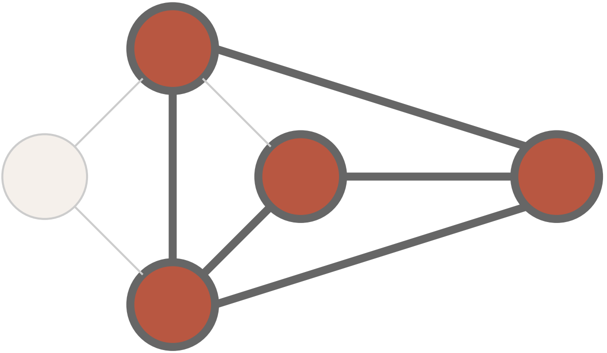

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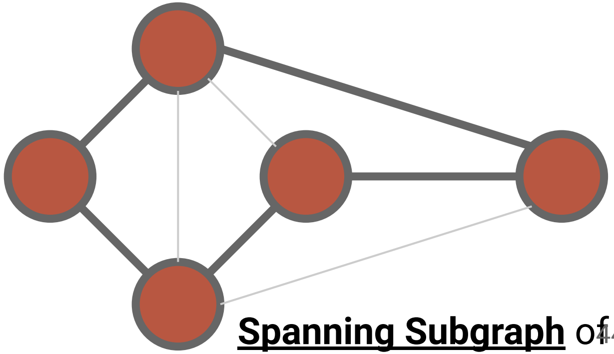

Spanning Subgraph of ***G***

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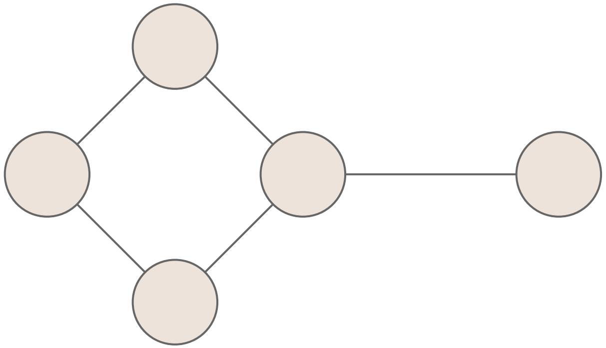
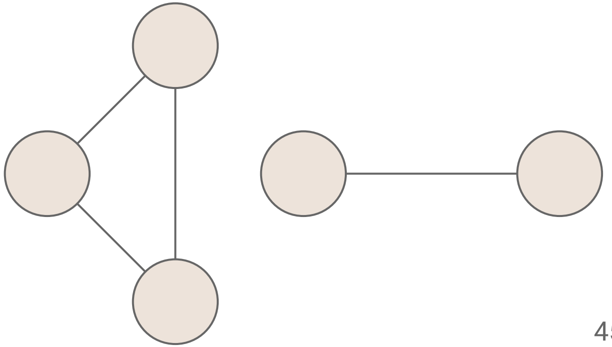
A graph is **connected**…

If there is a path between every pair of vertices





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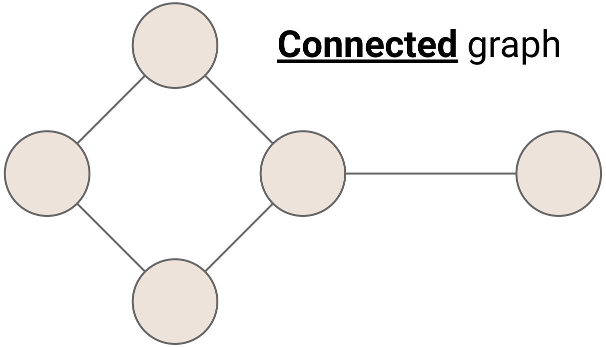

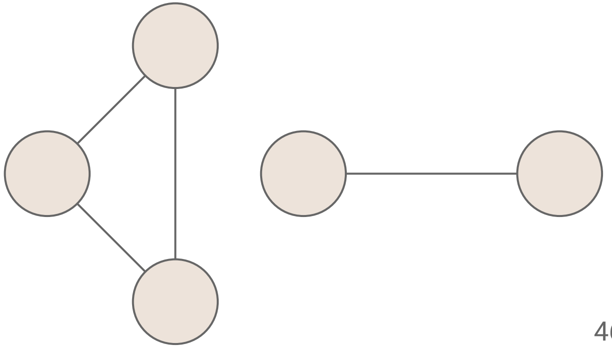

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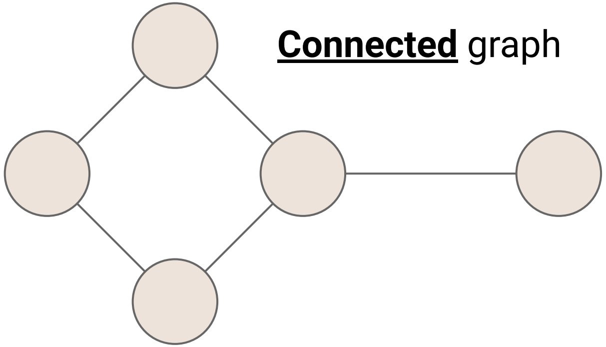

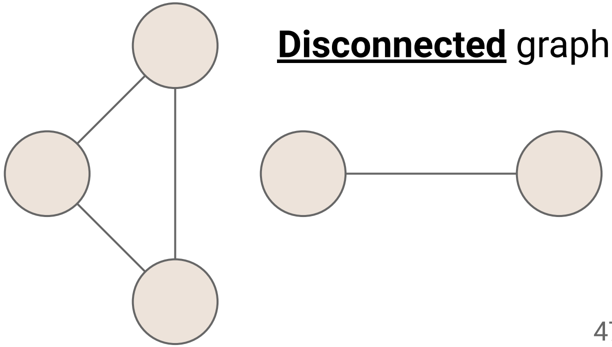

A few more definitions

A graph is **connected**…

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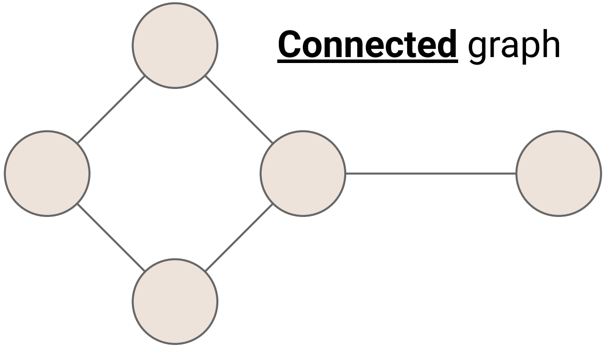
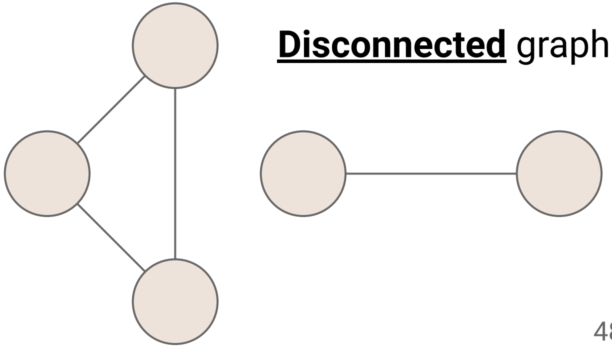
A **connected component** of ***G*…**

Is a maximal connected subgraph of ***G***

- "maximal" means you can't add a new vertex without breaking the property
- Any subset of ***G***'s edges that connect the subgraph are fine





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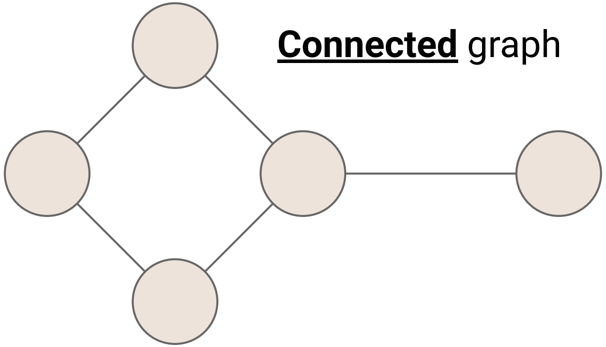

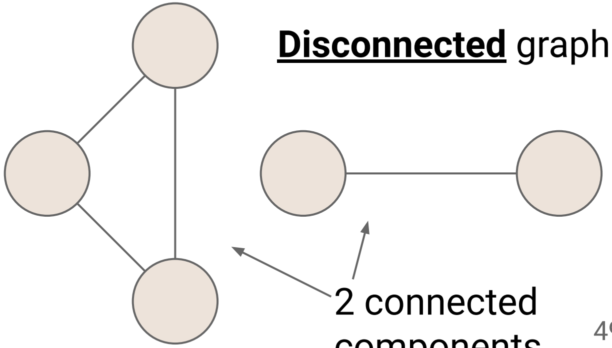

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There is exactly one simple path between any two nodes

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One vertex of ***T*** is the **root**

There is exactly one simple path from the root to every other vertex in the graph

A (free/rooted) **forest** is a graph ***F*** such that…

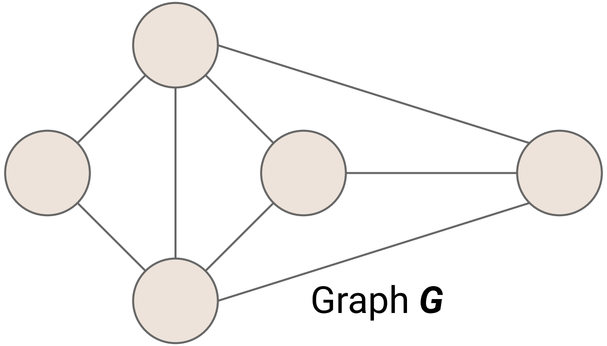
Every connected component is a tree

A few more definitions

A **spanning tree** of a connected graph…

…Is a spanning subgraph that is a tree

…It is not unique unless the graph is a tree



Graph ***G***

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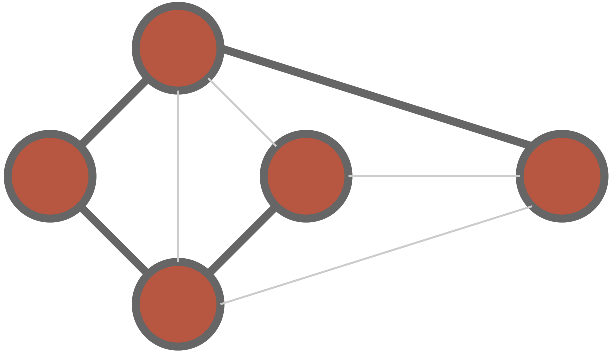
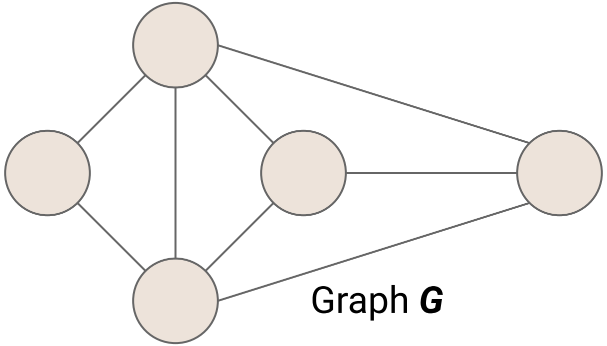
A **Spanning Tree** of ***G***





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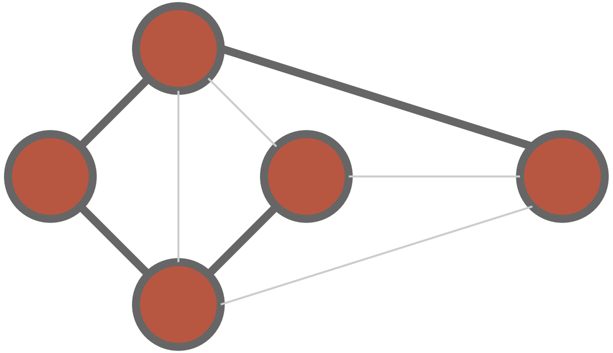

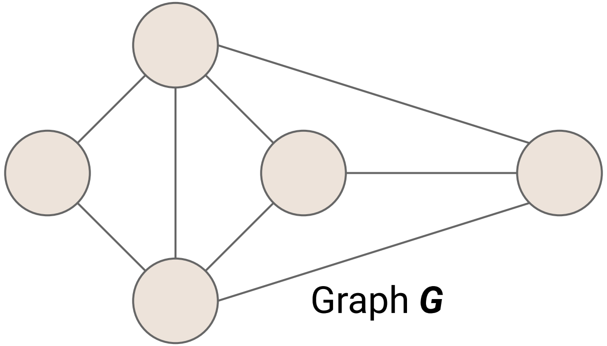

Graph ***G***

Another **Spanning Tree** of ***G***