CSE 250: Depth- and Breadth-First Search

Lecture 20

Oct 16, 2024

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Reminders

- PA2 released
  - Implement Graph Traversal.
  - Due **Tuesday**, Oct 22
- Midterm Report out Today
  - Extrapolated from PA1, WA1, Midterm1
  - If you don't like your grade, have a chat (Fridays @ 10 AM)
- Midterm Grades
  - Out soon... (waiting on makeups)

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Edge List Summary

- `addEdge`, `addVertex`: $O(1)$
- `removeEdge`: $O(1)$
- `removeVertex`: $O(M)$
- `incidentEdges`: $O(M)$
- `hasEdgeTo`: $O(M)$

Space Used: $O(N + M)$
(constant space per vertex, edge)

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Adjacency List Summary

Starting with an edge list:

- Store a linked list of in/out edges with each vertex
- Store the linked list node for the in/out lists with each edge

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Edge List Summary

- `addEdge`, `addVertex`: $O(1)$
- `removeEdge`: $O(1)$
- `removeVertex`: $O(\texttt{deg}(v))$
- `incidentEdges`: $O(1) + O(1)$ per `next()`
- `hasEdgeTo`: $O(\texttt{deg}(v))$

Space Used: $O(N + M)$
(constant space per vertex, edge)

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hasEdgeTo

Can we cut `hasEdgeTo` down to $O(1)$?

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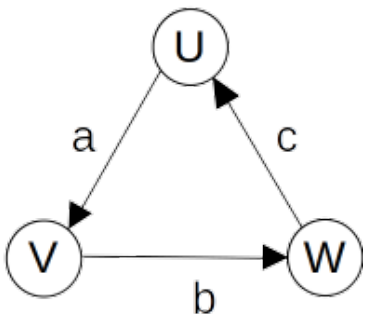
The Adjacency Matrix Data Structure

Origin \ Destination	U	V	W
U	-	a	-
V	-	-	b
W	c	-	-



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Space Used:

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A few more definitions

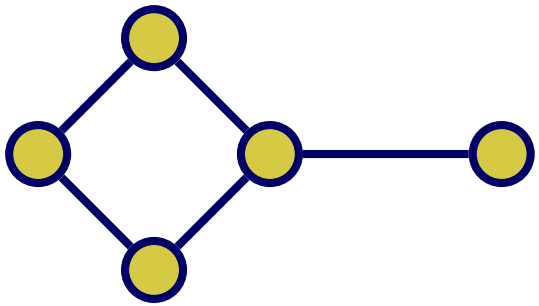
A graph is **connected** if...

- ...there is a path between every pair of vertices.

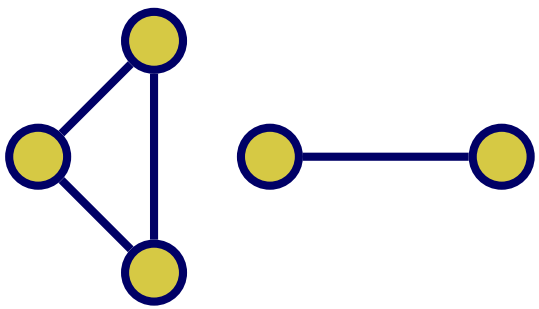
A **connected component** of G is a maximal, connected subgraph of G

- "maximal" means that adding any other vertices from G would break the connected property.
- Any subset of G's edges that makes the subgraph connected is fine.

Connected Graph



Disconnected Graph



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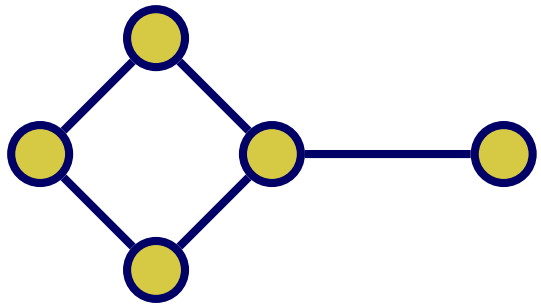

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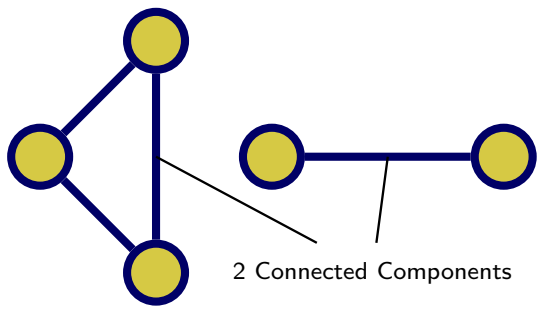

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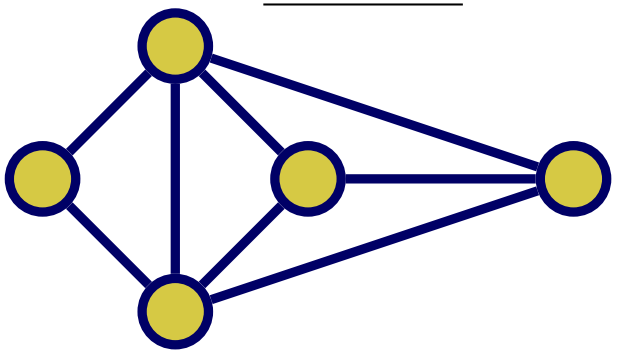
A few more definitions...

A **spanning tree** of a connected graph G is:

- A spanning subgraph of G
- A tree

Graph G



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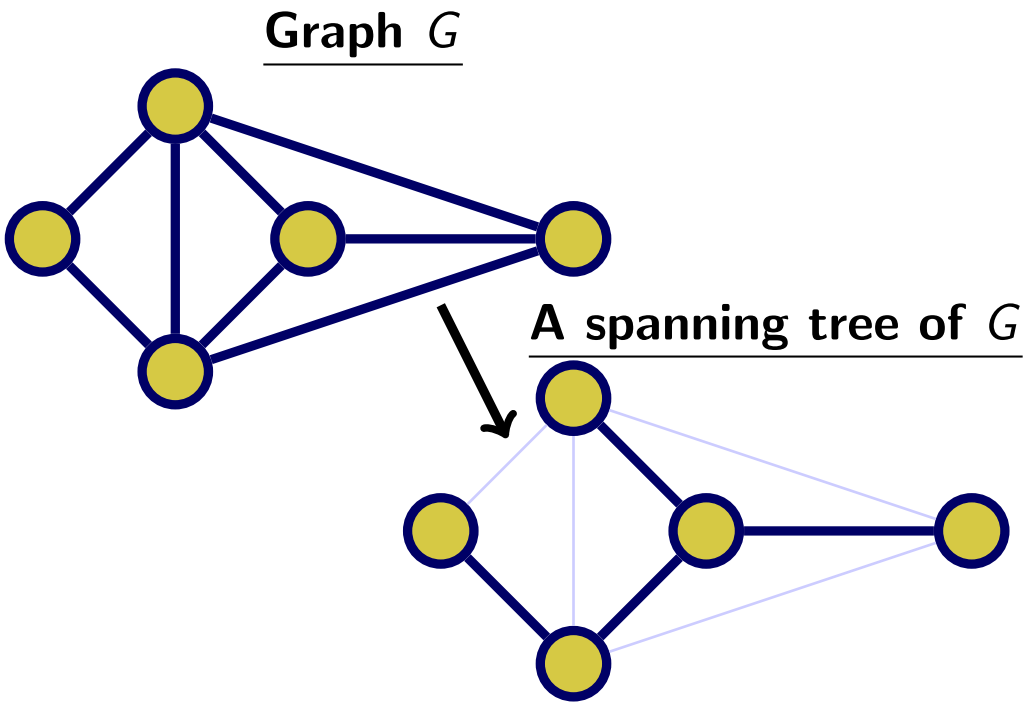

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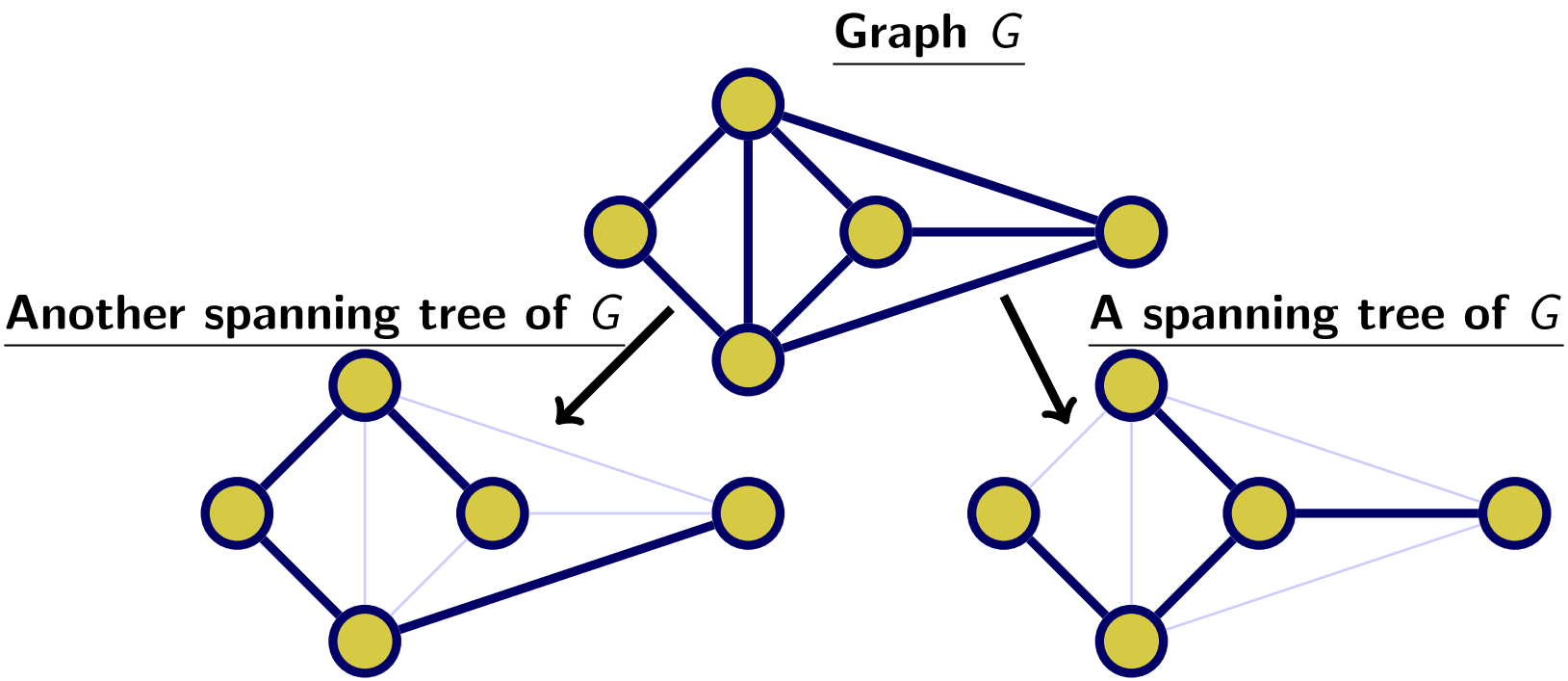

Note: A graph has multiple spanning trees (unless it is already a tree).

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Back to connectivity...

So how about connectivity?

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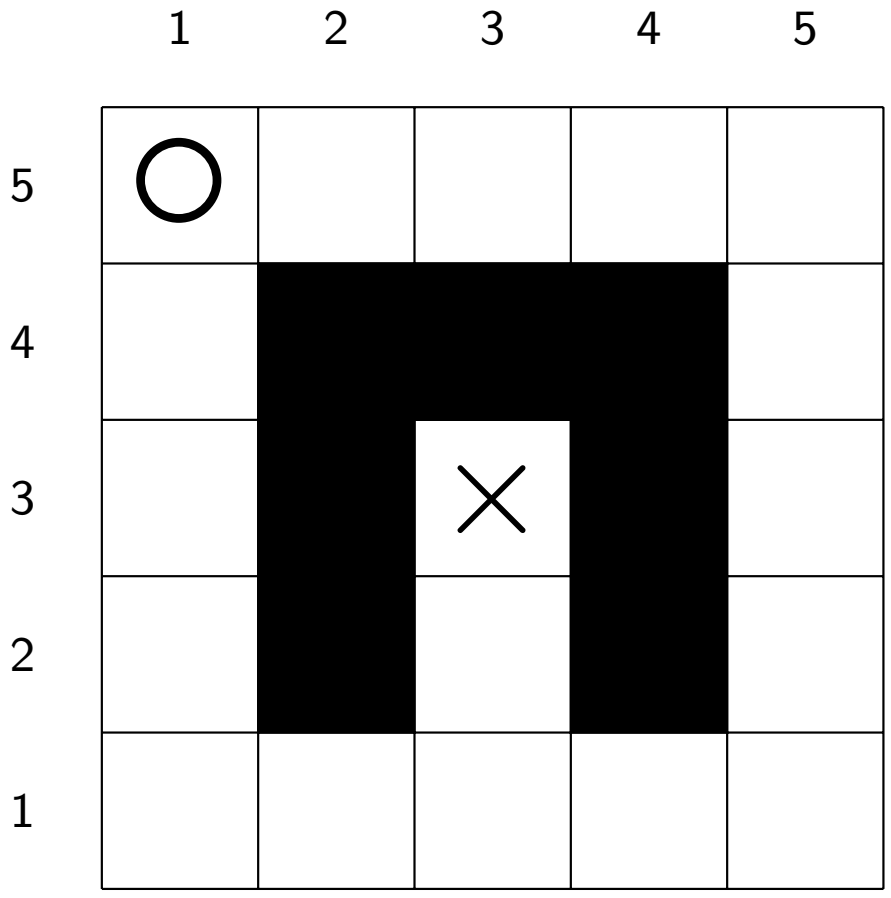
... or mazes?



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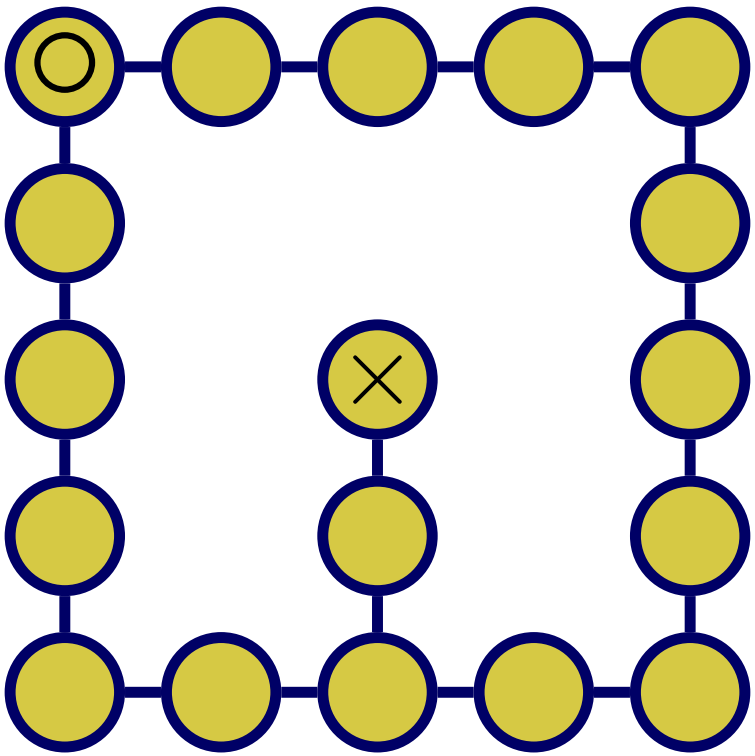

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Recall Mazes

- Start at a vertex
- If this is the target vertex, done!
- Push the next adjacent vertex (not filled, or already on the stack) onto the stack
- Explore from that vertex
- When that vertex is explored, pop it and move to the next adjacent vertex
- When everything is explored, return.

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This is called Depth-First Search

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This is called Depth-First Search Contrast with Breadth-First Search

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Depth First Search (DFS)

Primary Goals

- Visit every vertex in graph $G = (V, E)$.
- Construct a spanning tree for every connected component.

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- Visit every vertex in graph $G = (V, E)$.
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  - **Side Effect**: Compute connected components.

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Depth First Search (DFS)

Primary Goals

- Visit every vertex in graph $G = (V, E)$.
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  - **Side Effect**: Compute connected components.
  - **Side Effect**: Compute a path between all connected vertices.

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Depth First Search (DFS)

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- Visit every vertex in graph $G = (V, E)$.
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  - **Side Effect**: Compute a path between all connected vertices.
  - **Side Effect**: Determine if the graph is connected.

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  - **Side Effect**: Identify any cycles (if they exist).

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Depth First Search (DFS)

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- Visit every vertex in graph $G = (V, E)$.
- Construct a spanning tree for every connected component.
  - **Side Effect**: Compute connected components.
  - **Side Effect**: Compute a path between all connected vertices.
  - **Side Effect**: Determine if the graph is connected.
  - **Side Effect**: Identify any cycles (if they exist).
- Complete in time $O(N + M)$.

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Depth First Search (DFS)

`DFS(G)`

Input

- Graph $G = (V, E)$

Output

- Label every edge as a:
  - Spanning Edge: Part of the spanning tree
  - Back Edge: Part of a cycle

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Depth First Search (DFS)

`DFSOne(G, v)`

Input

- Graph $G = (V, E)$
- Start vertex $v \in V$

Output

- Label every edge in v's connected component as a:
  - Spanning Edge: Part of the spanning tree
  - Back Edge: Part of a cycle

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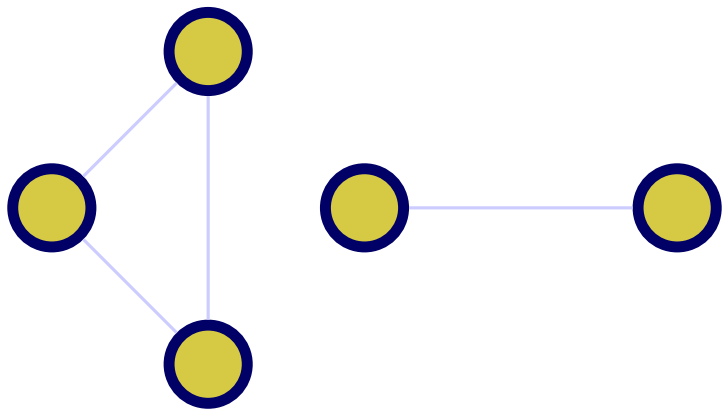
DFS Example



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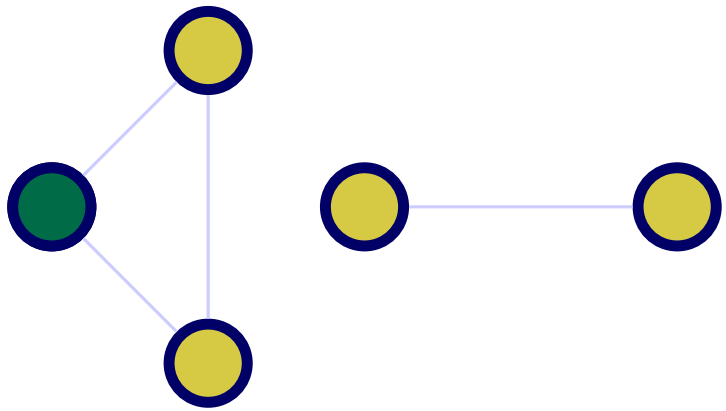

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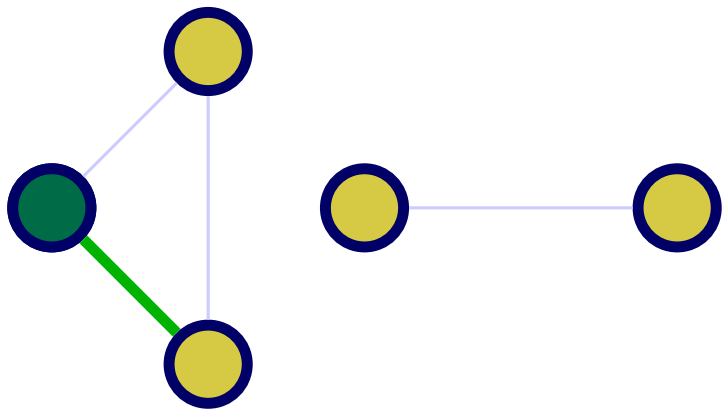

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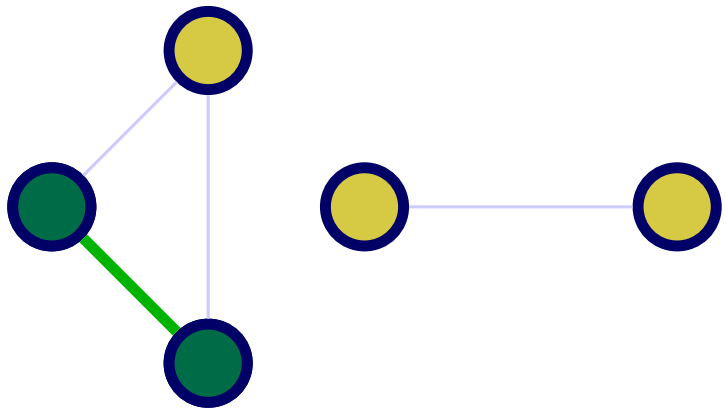

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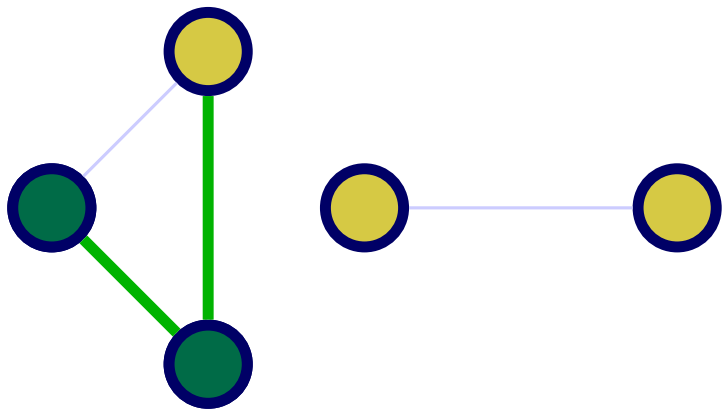

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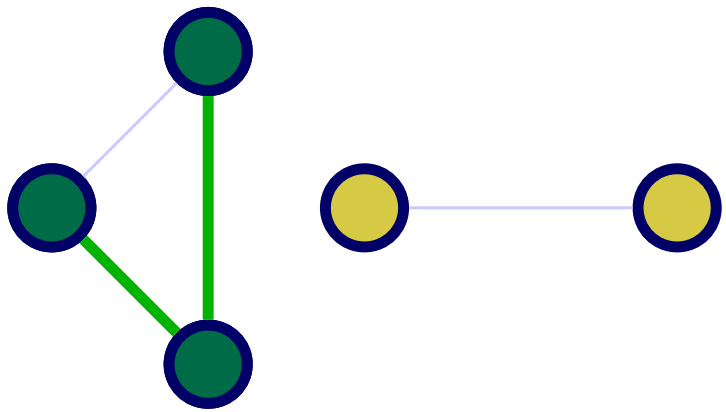

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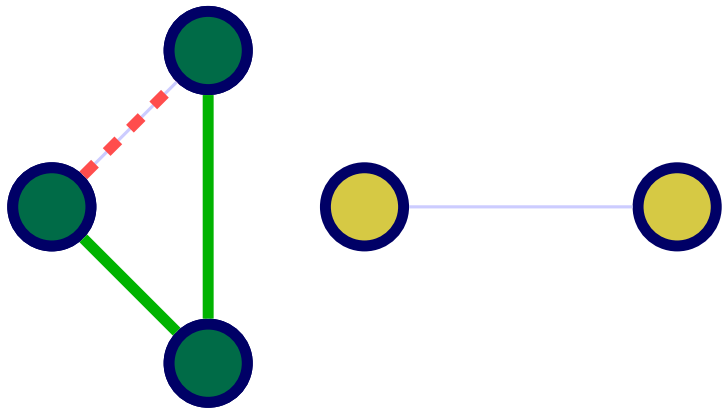

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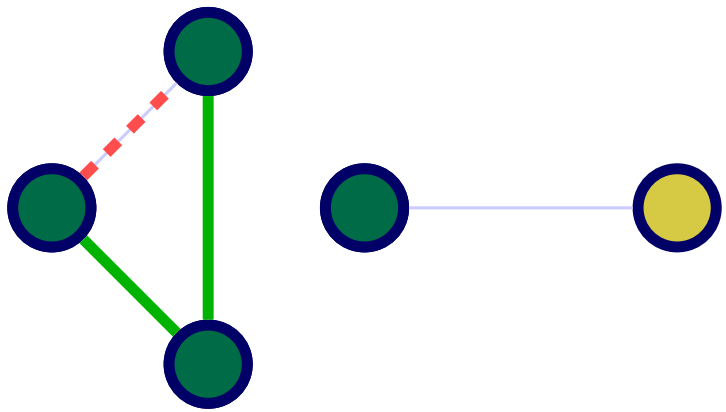

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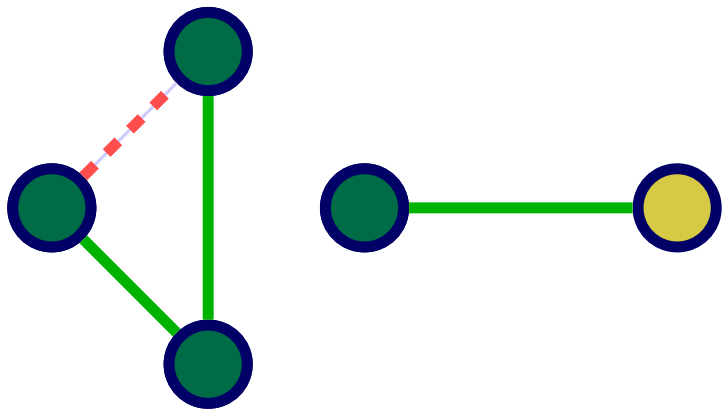

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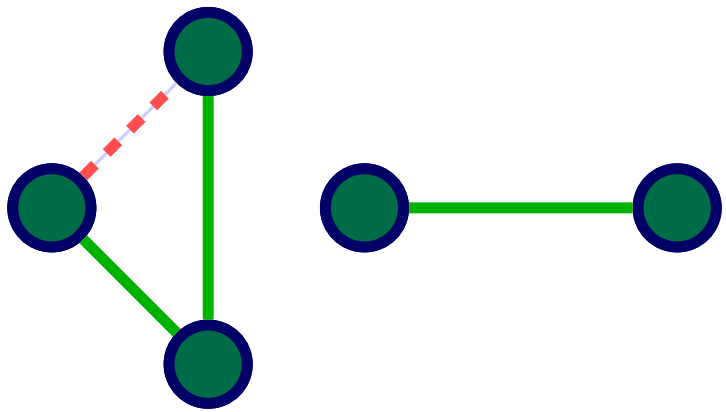

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Depth First Search (DFS)

```
1  public enum VertexLabel { UNEXPLORED, VISITED }
2  public enum EdgeLabel { UNEXPLORED, SPANNING, BACK }
3
4  public void DFS(Graph<VertexLabel,EdgeLabel> graph)
5  {
6    for(Vertex v : graph.vertices)
7                  { v.setLabel(VertexLabel.UNEXPLORED); }
8    for(Edge e   : graph.edges)
9                  { e.setLabel(EdgeLabel.UNEXPLORED); }
10   for(Vertex v : graph.vertices)
11   {
12     if(v.getLabel() == VertexLabel.UNEXPLORED)
13     {
14       DFSOne(graph, v)
15     }
16   }
17 }
```

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DFSOne

```
1   public void DFSOne(Graph<...> graph, Vertex start)
2   {
3     start.setLabel(VertexLabel.VISITED)
4
5     for( edge : start.getIncident() ) {
6       if(edge.getLabel == EdgeLabel.UNEXPLORED){
7         Vertex otherVertex = edge.getOppositeVertex(start);
8
9         if(otherVertex.getLabel == VertexLabel.UNEXPLORED){
10          edge.setLabel(EdgeLabel.SPANNING);
11          DFSOne(graph, otherVertex);
12        } else {
13          edge.setLabel(EdgeLabel.BACK);
14        }
15      }
16    }
17  }
```

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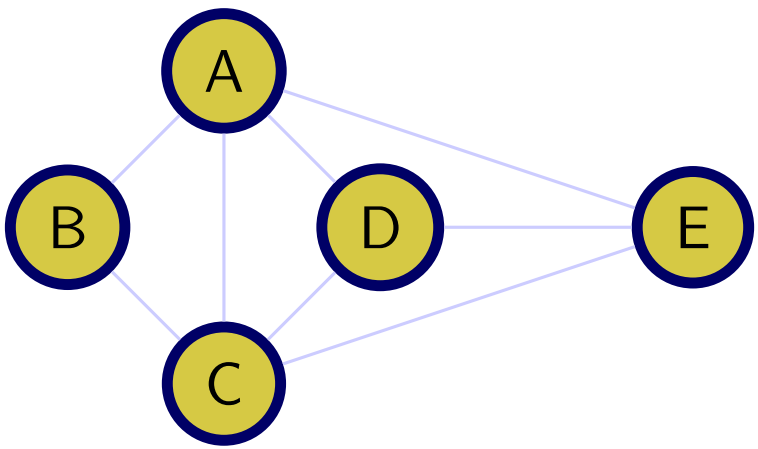
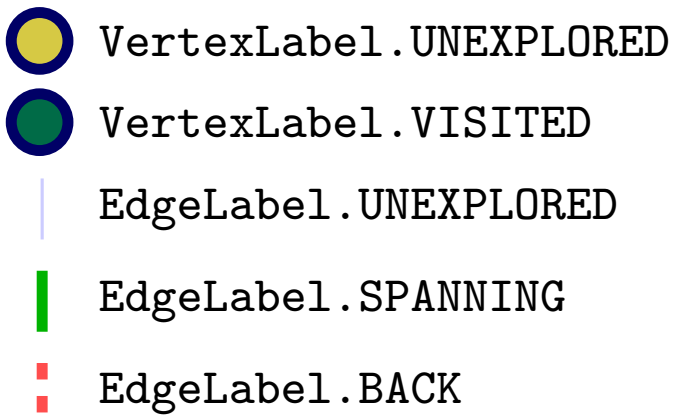
Depth First Search (DFS)



Call Stack



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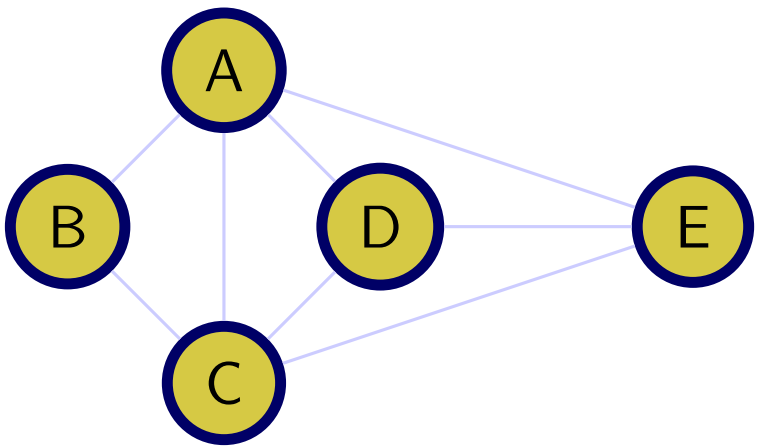

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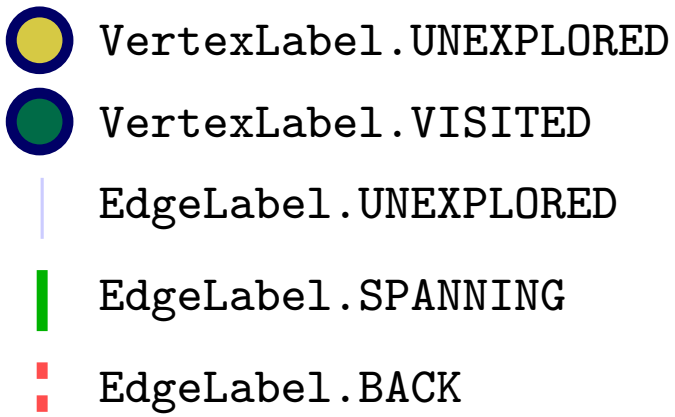
`DFS(G)`



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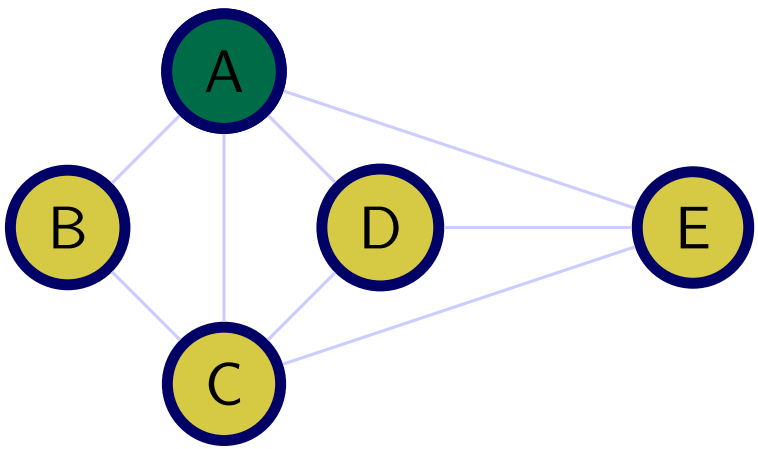

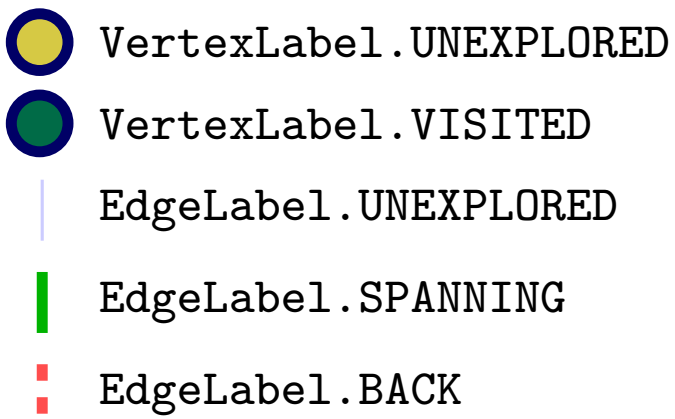

Call Stack

`DFS(G)`

`DFSOne(G, A)` ($\rightarrow$ B, C, D, E)

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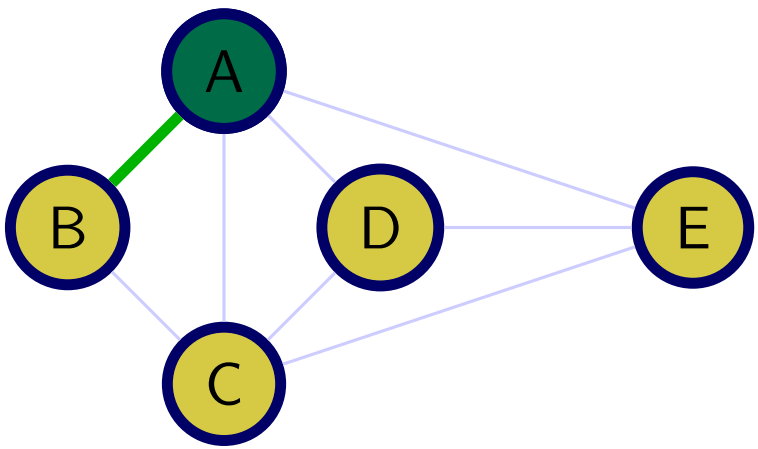

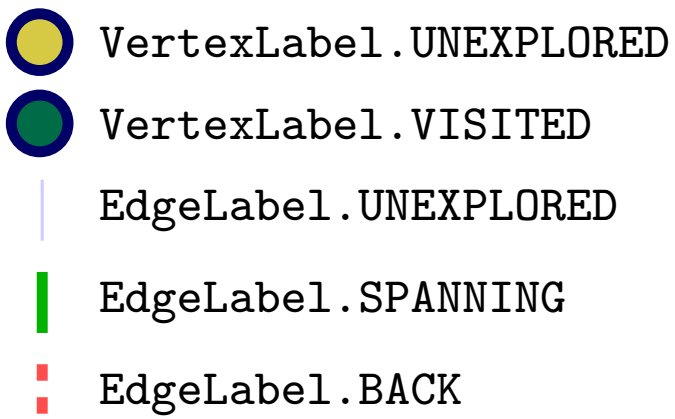

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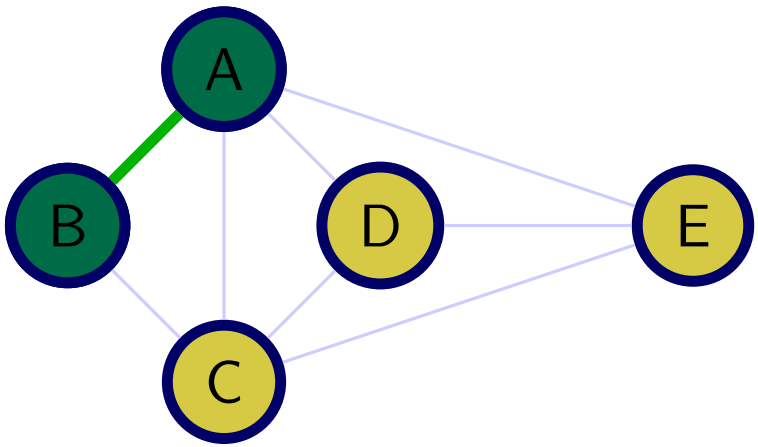

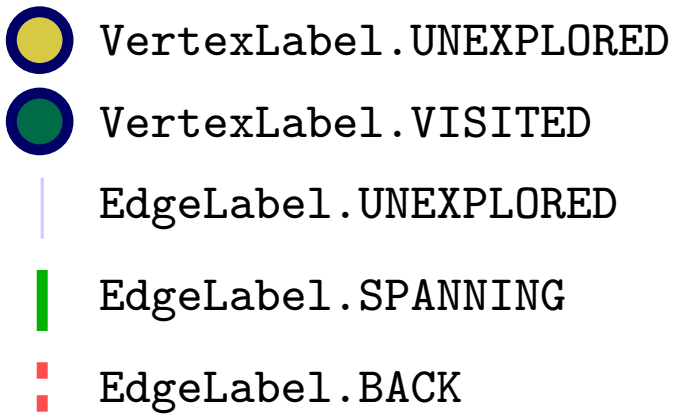

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`DFSOne(G, B)` ($\rightarrow$ A, C)

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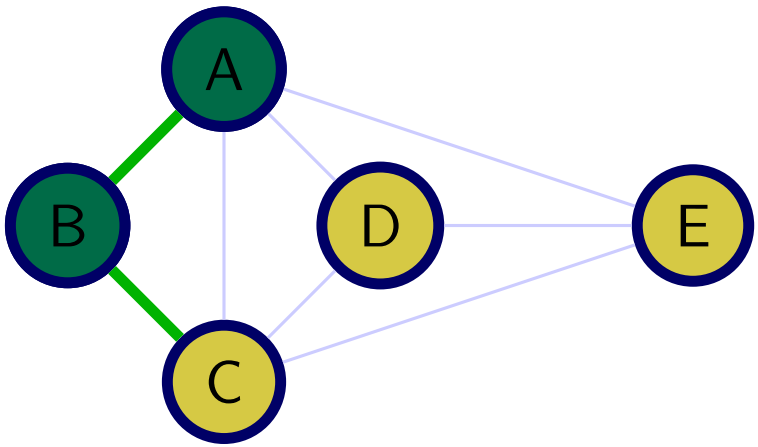

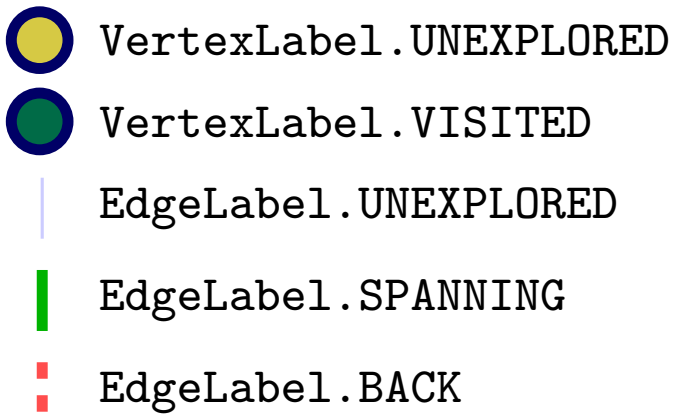

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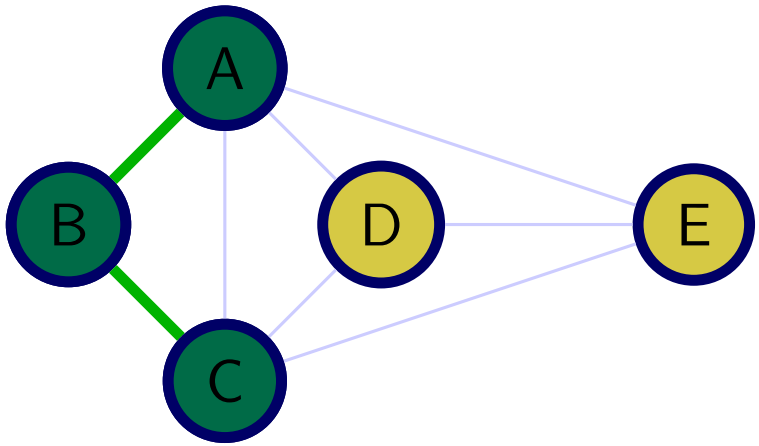
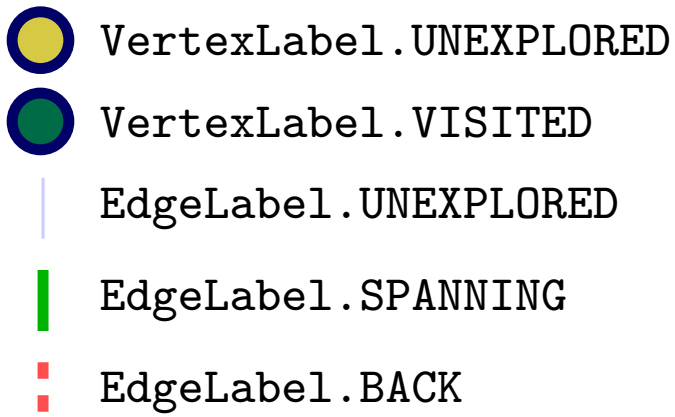
Depth First Search (DFS)





Call Stack

`DFS(G)`

`DFSOne(G, A)` ($\rightarrow$ B, C, D, E)

`DFSOne(G, B)` ($\rightarrow$ A, C)

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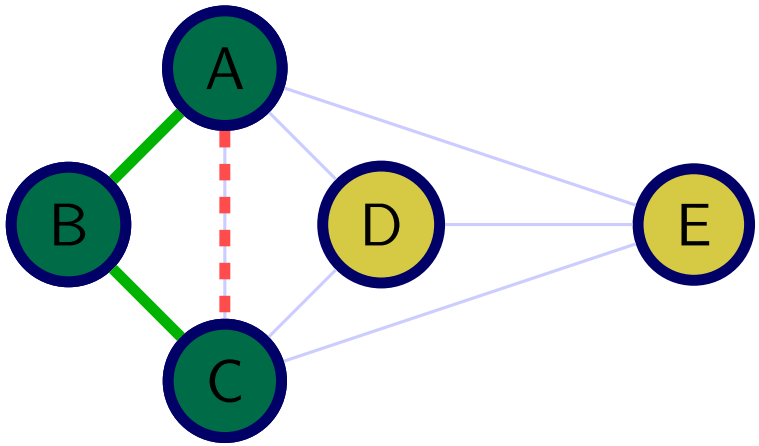

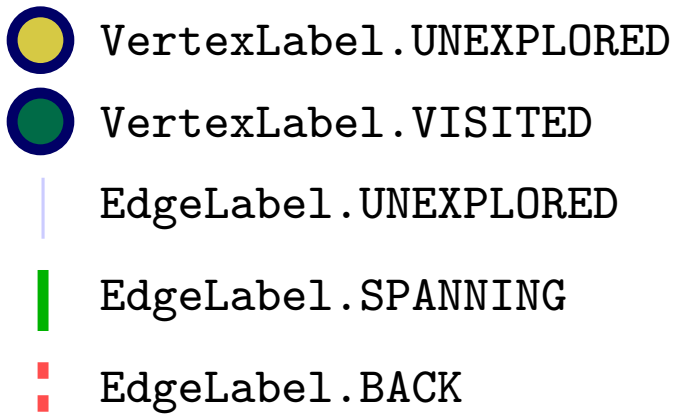

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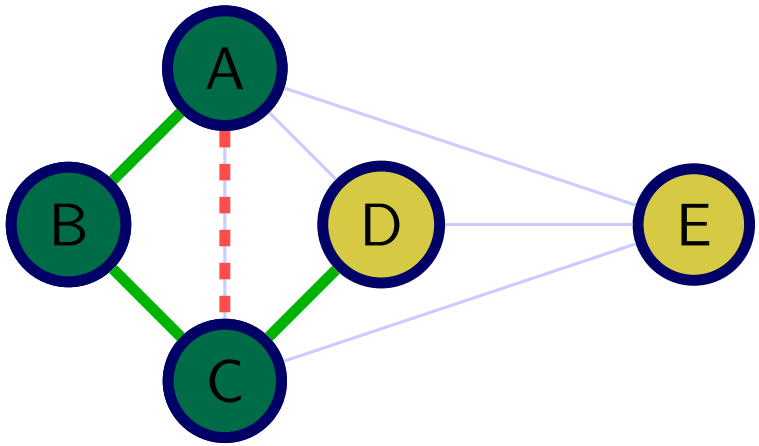

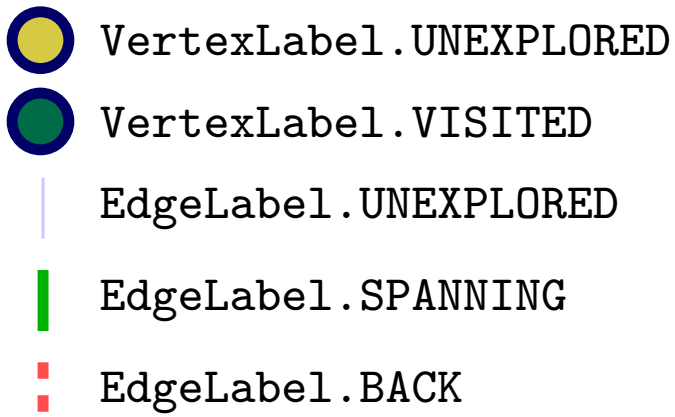

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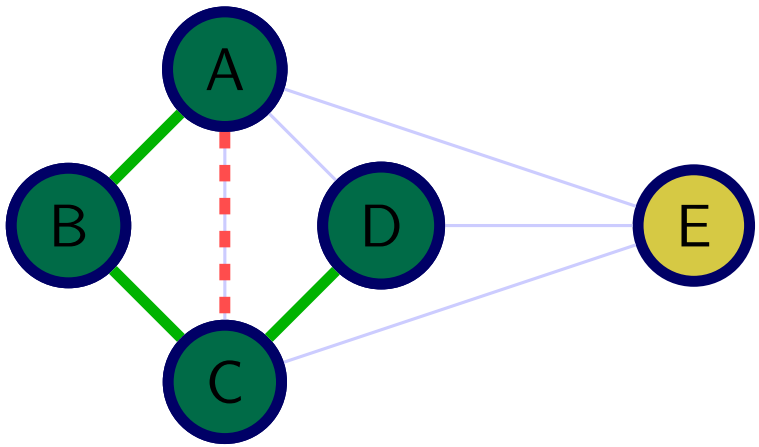

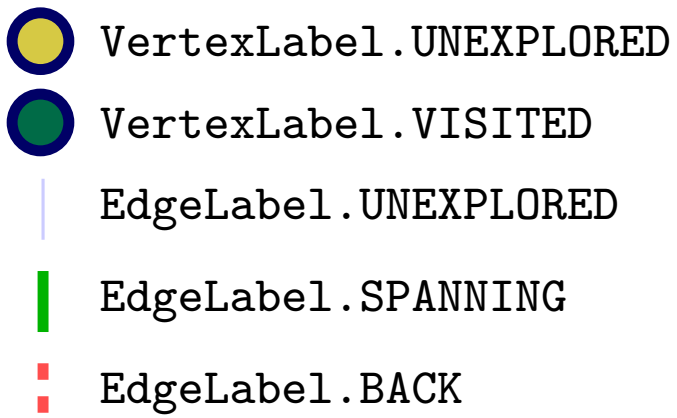

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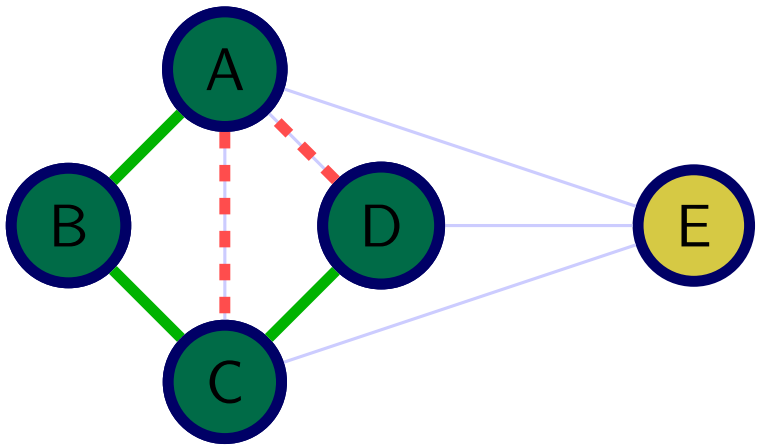

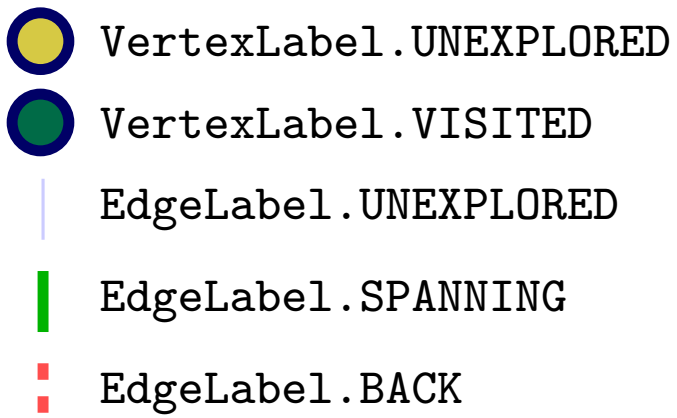

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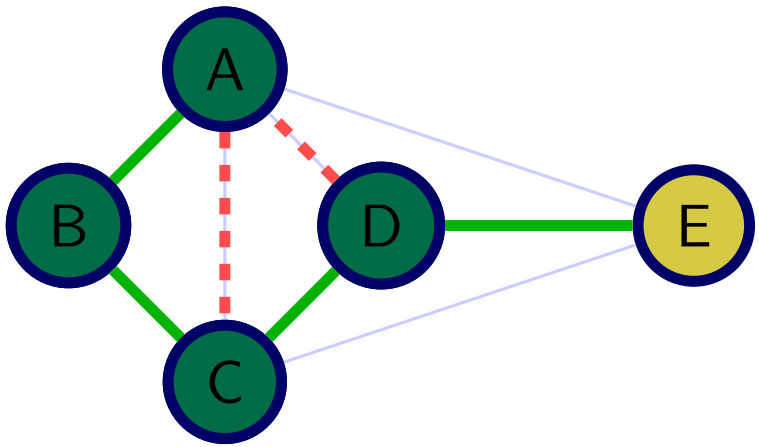

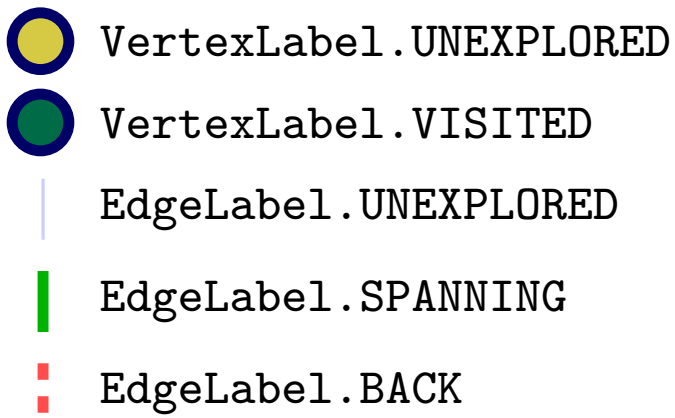

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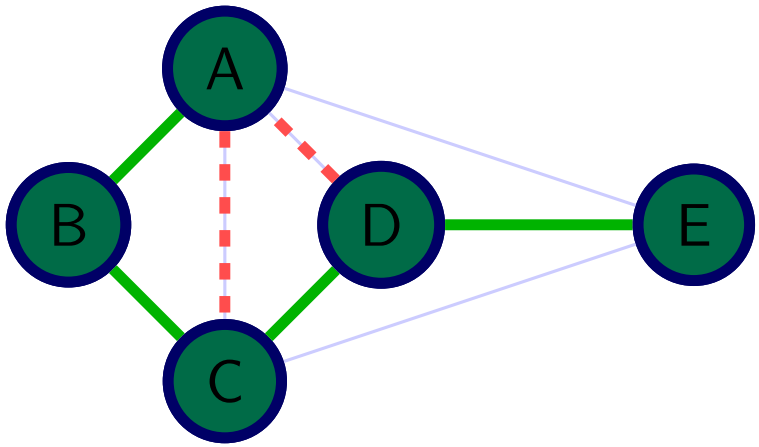

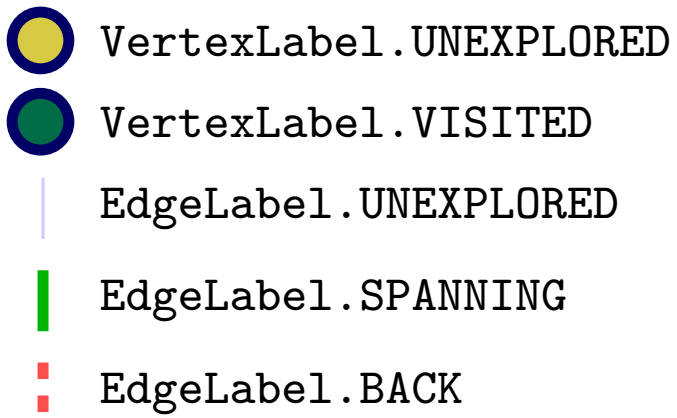

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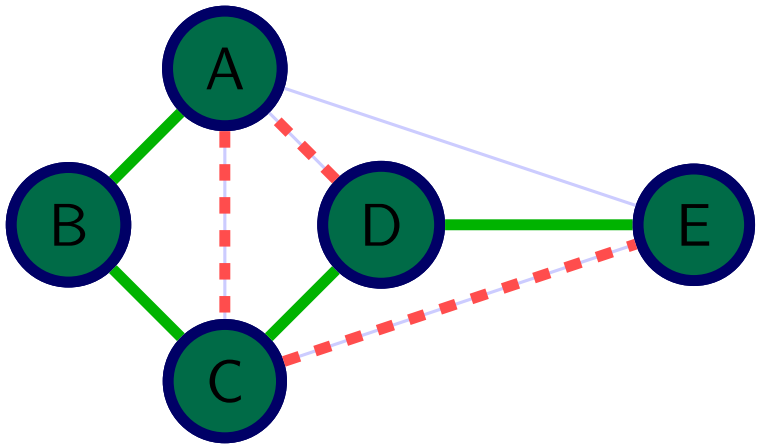

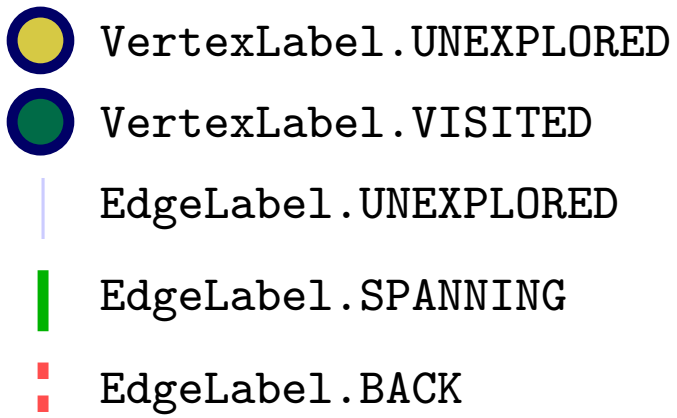

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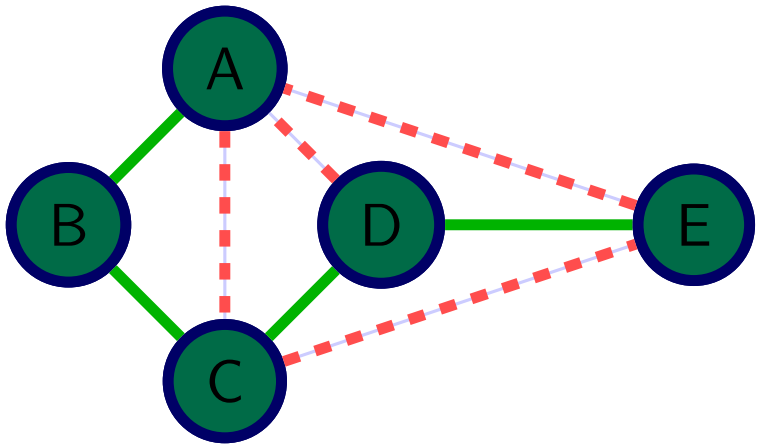

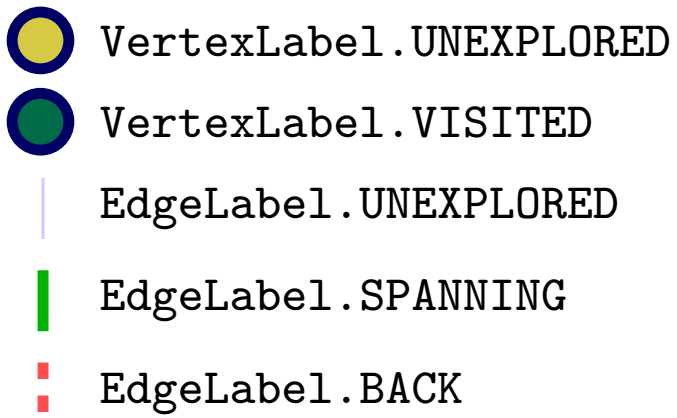

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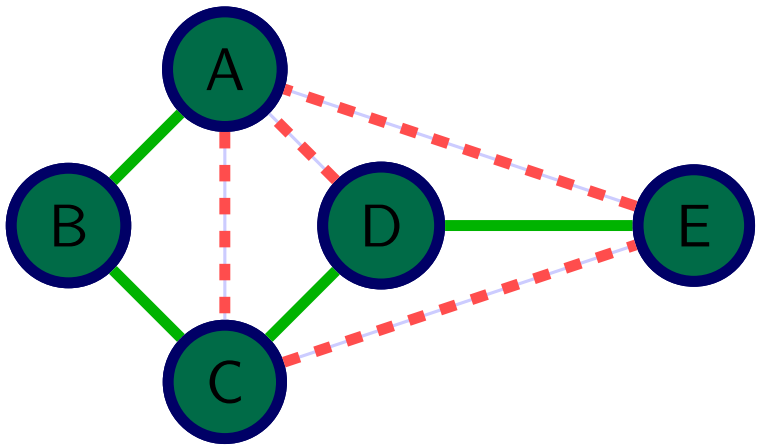

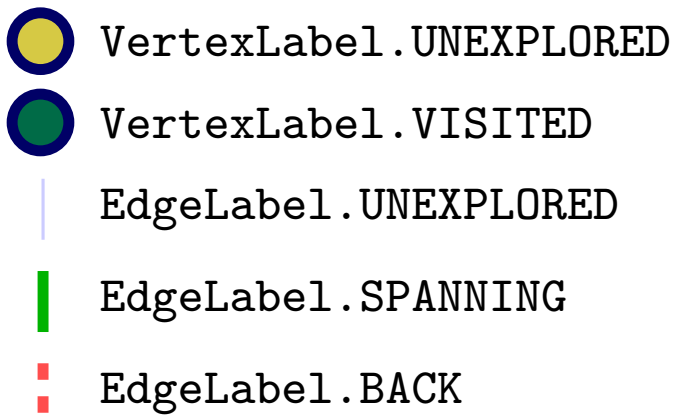

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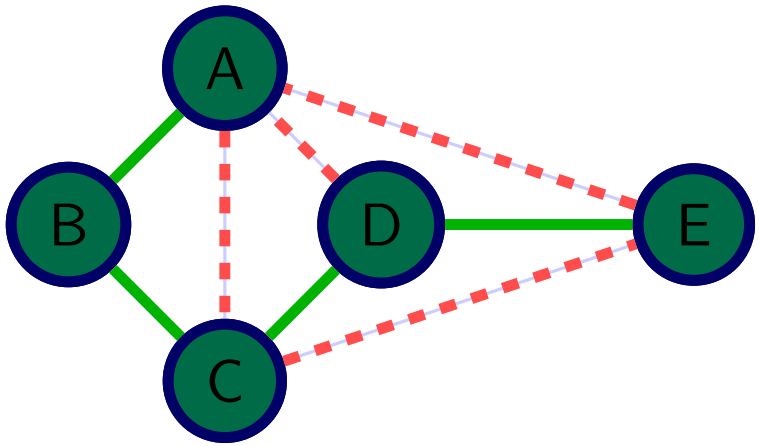

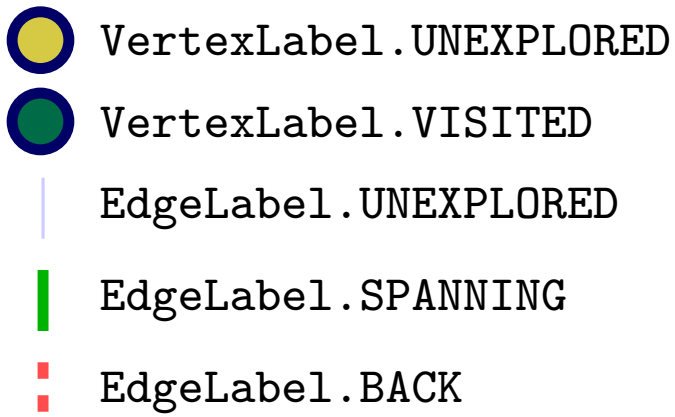

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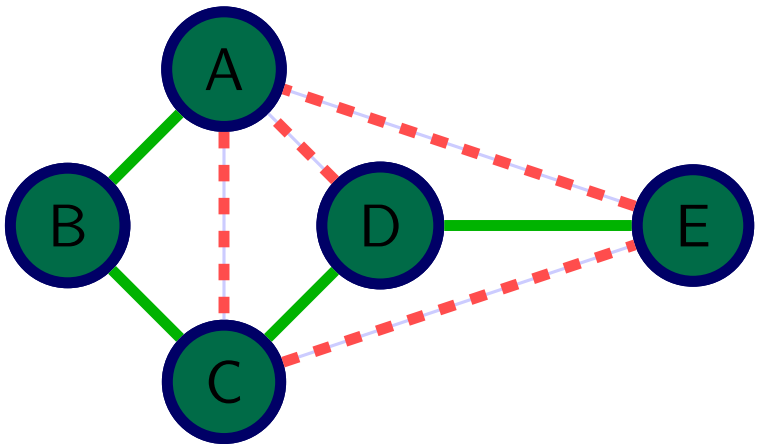

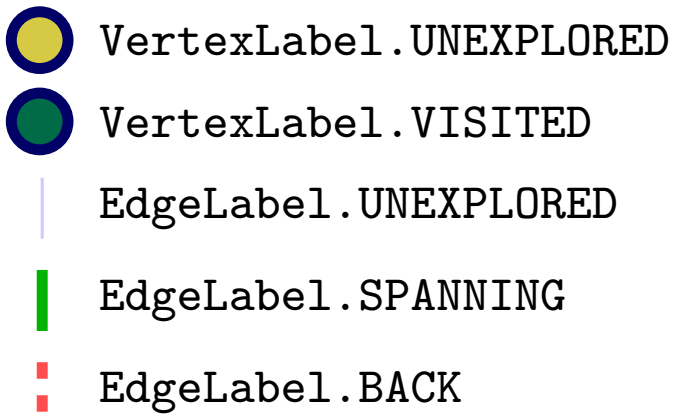

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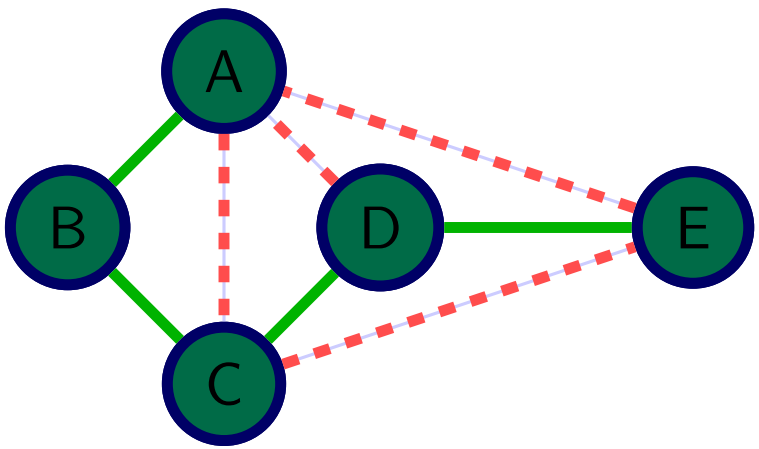

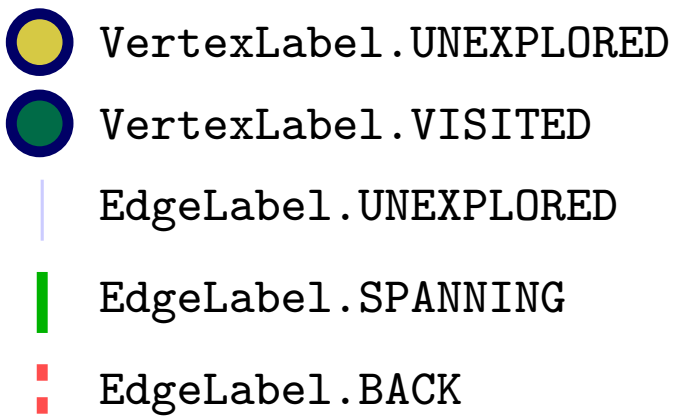

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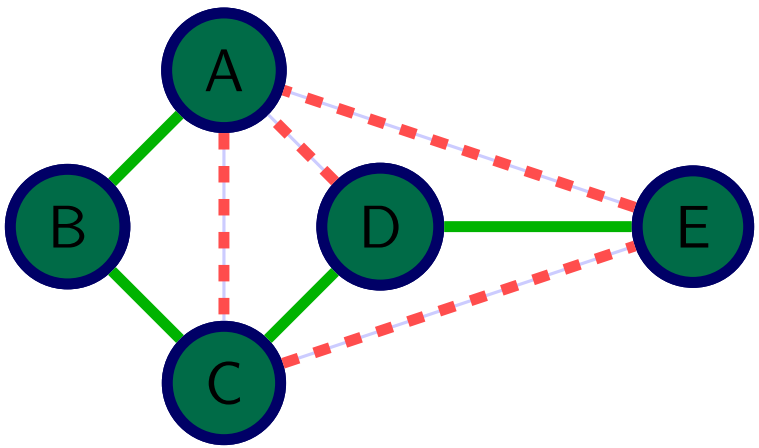

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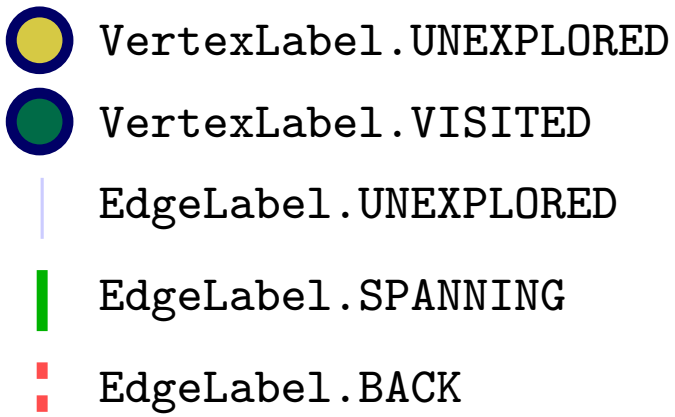

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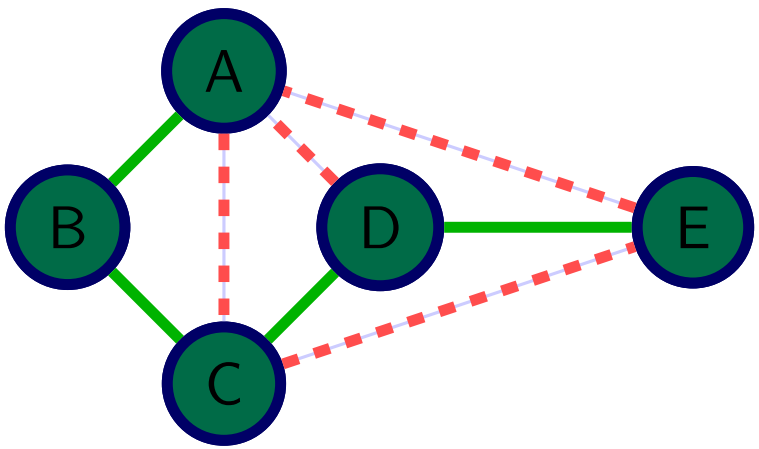

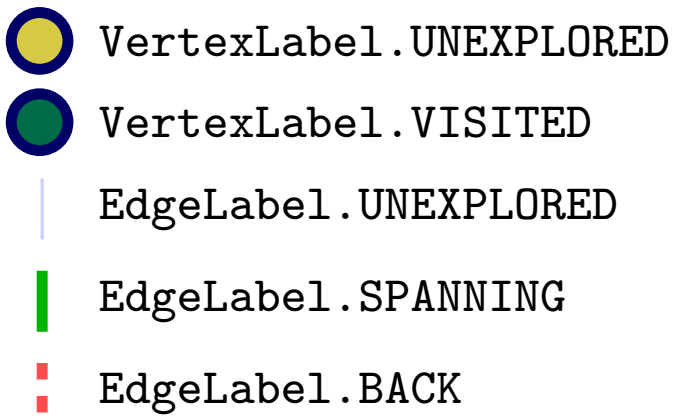

Call Stack

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DFS on Mazes

The DFS algorithm is like our stack-based maze solver

- Mark each grid square with `VISITED`
- Mark each path with `SPANNING`
- Only visit each vertex once.

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DFS on Mazes

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DFS won't necessarily find the shortest paths.

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DFS

What's the complexity?

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Depth First Search (DFS)

```
1  public enum VertexLabel { UNEXPLORED, VISITED }
2  public enum EdgeLabel { UNEXPLORED, SPANNING, BACK }
3
4  public void DFS(Graph<VertexLabel,EdgeLabel> graph)
5  {
6    for(Vertex v : graph.vertices)
7                  { v.setLabel(VertexLabel.UNEXPLORED); }
8    for(Edge e   : graph.edges)
9                  { e.setLabel(EdgeLabel.UNEXPLORED); }
10   for(Vertex v : graph.vertices)
11   {
12     if(v.getLabel() == VertexLabel.UNEXPLORED)
13     {
14       DFSOne(graph, v)
15     }
16   }
17 }
```

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3
4   public void DFS(Graph<VertexLabel,EdgeLabel> graph)
5   {
6     O(N)
7
8     for(Edge e    : graph.edges)
9                    { e.setLabel(EdgeLabel.UNEXPLORED); }
10    for(Vertex v : graph.vertices)
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8    O(M)
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7
8     O(M)
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10    O(N) times
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4   public void DFS(Graph<VertexLabel,EdgeLabel> graph)
5   {
6     O(N)
7
8     O(M)
9
10    O(N) times
11    {
12      O(1)
13      {
14        DFSOne(graph, v)
15      } else {
16        O(1)
17      }
18    }
19  }
```

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Depth First Search (DFS)

Observation: `DFSOne` is called on each vertex at most once.

- If `v`'s label is `VISITED`, both `DFSOne` and `DFS` skip it

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$O(N)$ calls to `DFSOne`

What's the runtime of `DFSOne` **excluding recursive calls**?

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DFSOne

```
1   public void DFSOne(Graph<...> graph, Vertex start)
2   {
3     start.setLabel(VertexLabel.VISITED)
4
5     for( edge : start.getIncident() ) {
6       if(edge.getLabel == EdgeLabel.UNEXPLORED){
7         Vertex otherVertex = edge.getOppositeVertex(start);
8
9         if(otherVertex.getLabel == VertexLabel.UNEXPLORED){
10          edge.setLabel(EdgeLabel.SPANNING);
11          DFSOne(graph, otherVertex);
12        } else {
13          edge.setLabel(EdgeLabel.BACK);
14        }
15      }
16    }
17  }
```

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DFSOne

```
1  public void DFSOne(Graph<...> graph, Vertex start)
2  {
3    O(1)
4
5    O(deg(start)) times {
6      O(1) {
7        O(1)
8
9        O(1) {
10         O(1)
11         DFSOne(graph, otherVertex);
12       } else {
13         O(1)
14       }
15     }
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```

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Depth First Search (DFS)

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Depth First Search (DFS)

What's the runtime of `DFSOne` **excluding recursive calls**?

$$O(\texttt{deg}(\texttt{start}))$$

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Depth First Search (DFS)

What's the sum over all calls to `DFSOne`

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Depth First Search (DFS)

What's the sum over all calls to `DFSOne`

$$\sum_{v \in V} O(\deg(v))$$

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Depth First Search (DFS)

What's the sum over all calls to `DFSOne`

$$\sum_{v \in V} O(\deg(v))$$

$$= O\left(\sum_{v \in V} \deg(v)\right)$$

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Depth First Search (DFS)

What's the sum over all calls to `DFSOne`

$$\sum_{v \in V} O(\deg(v))$$

$$= O\left(\sum_{v \in V} \deg(v)\right)$$

$$= O(2M) = O(M)$$

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To summarize...

Mark Vertices `UNVISITED`
Mark Edges `UNVISITED`
`DFS` Vertex Loop
All calls to `DFSOne`

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Mark Vertices `UNVISITED`	$O(N)$
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To summarize...

Mark Vertices `UNVISITED`	$O(N)$
Mark Edges `UNVISITED`	$O(M)$
`DFS` Vertex Loop	$O(N)$
All calls to `DFSOne`	$O(M)$
	$O(N+M)$

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