

CSE 250

Data Structures

Dr. Eric Mikida
epmikida@buffalo.edu
208 Capen Hall

Lec 03: Math Refresher

Announcements and Feedback

- Make sure you are on Piazza and AutoLab
- Academic Integrity Quiz due 9/7 @ 11:59PM **(MUST GET 100%)**
- PA0 due 9/7 @ 11:59PM **(MUST GET 100%)**

Today's Topics

- Summations
- Logarithms

Summations

$$\sum_{i=j}^k f(i) = f(j) + f(j+1) + \dots + f(k)$$

Useful Tricks

If c is a constant (with respect to i)

$$\sum_{i=j}^k c = \underbrace{(c + c + \dots + c)}_{(k - j + 1) \text{ times}}$$

Useful Tricks

If c is a constant (with respect to i)

$$\sum_{i=j}^k c = (c + c + \dots + c)$$

$$= (k - j + 1) \cdot c$$

Useful Tricks

If c is a constant and $f(i)$ is a function of i :

$$\sum_{i=j}^k cf(i) = (cf(j) + cf(j+1) + \dots + cf(k))$$

Useful Tricks

If c is a constant and $f(i)$ is a function of i :

$$\begin{aligned}\sum_{i=j}^k cf(i) &= (cf(j) + cf(j+1) + \dots + cf(k)) \\ &= c(f(j) + f(j+1) + \dots + f(k))\end{aligned}$$

Useful Tricks

If c is a constant and $f(i)$ is a function of i :

$$\sum_{i=j}^k cf(i) = (cf(j) + cf(j+1) + \dots + cf(k))$$

$$= c(f(j) + f(j+1) + \dots + f(k))$$

$$= c \sum_{i=j}^k f(i)$$

Useful Tricks

If $f(i)$ and $g(i)$ are functions of i :

$$\sum_{i=j}^k f(i) + g(i) = (f(j) + g(j)) + (f(j+1) + g(j+1)) + \dots + (f(k) + g(k))$$

Useful Tricks

If $f(i)$ and $g(i)$ are functions of i :

$$\begin{aligned}\sum_{i=j}^k f(i) + g(i) &= (f(j) + g(j)) + (f(j+1) + g(j+1)) + \dots + (f(k) + g(k)) \\ &= (f(j) + f(j+1) + \dots + f(k)) + (g(j) + g(j+1) + \dots + g(k))\end{aligned}$$

Useful Tricks

If $f(i)$ and $g(i)$ are functions of i :

$$\begin{aligned}\sum_{i=j}^k f(i) + g(i) &= (f(j) + g(j)) + (f(j+1) + g(j+1)) + \dots + (f(k) + g(k)) \\ &= (f(j) + f(j+1) + \dots + f(k)) + (g(j) + g(j+1) + \dots + g(k)) \\ &= \left(\sum_{i=j}^k f(i) \right) + \left(\sum_{i=j}^k g(i) \right)\end{aligned}$$

Useful Tricks

If $j < l < k$:

$$\sum_{i=j}^k f(i) = f(j) + \dots + f(k)$$

Useful Tricks

If $j < l < k$:

$$\begin{aligned}\sum_{i=j}^k f(i) &= f(j) + \dots + f(k) \\ &= f(j) + \dots + f(l-1) + f(l) + \dots + f(k)\end{aligned}$$

Useful Tricks

If $j < l < k$:

$$\sum_{i=j}^k f(i) = f(j) + \dots + f(k)$$

$$= f(j) + \dots + f(l-1) + f(l) + \dots + f(k)$$

$$= \left(\sum_{i=j}^{l-1} f(i) \right) + \left(\sum_{i=l}^k f(i) \right)$$


Useful Tricks

If $j < l < k$:

$$\left(\sum_{i=j}^k f(i) \right) = \left(\sum_{i=j}^{l-1} f(i) \right) + \left(\sum_{i=l}^k f(i) \right)$$

Useful Tricks

If $j < l < k$:

$$\left(\sum_{i=j}^k f(i) \right) = \boxed{\left(\sum_{i=j}^{l-1} f(i) \right)} + \left(\sum_{i=l}^k f(i) \right)$$


Subtract to other side

Useful Tricks

If $j < l < k$:

$$\left(\sum_{i=j}^k f(i) \right) = \left(\sum_{i=j}^{l-1} f(i) \right) + \left(\sum_{i=l}^k f(i) \right)$$

$$\left(\sum_{i=j}^k f(i) \right) - \left(\sum_{i=j}^{l-1} f(i) \right) = \left(\sum_{i=l}^k f(i) \right)$$

Series

Some common closed form solutions:

$$\sum_{i=1}^k i = \frac{k(k+1)}{2}$$

$$\sum_{i=0}^k 2^i = 2^{k+1} - 1$$

Summary

- The previous rules will always be provided on WAs and exams
- Usually the goal will be to reduce some complicated summation to a simpler form without a summation
 - Some of the rules get rid of summations
 - Some allow you to manipulate summations/bounds so that you can apply rules that get rid of summations
- Be cognizant of what variables are constant ***with respect to the summation variable*** and which one aren't

Logarithms

$$a \cdot n = \underbrace{a + a + \dots + a}_{\text{a added together } n \text{ times}}$$

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The inverse operation is division

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The inverse operation is ???

Logarithms

$$a \cdot n = \underbrace{a + a + \dots + a}_{a \text{ added together } n \text{ times}}$$

The inverse operation is division

$$a^n = \underbrace{a \cdot a \cdot \dots \cdot a}_{a \text{ multiplied together } n \text{ times}}$$

The inverse operation is \log_a

Logarithms

$\log_a(b)$ = the number of times you multiply a together to get b

$$\log_2(32) = 5$$

$$\log_3(27) = 3$$

$$\log_2\left(\frac{1}{8}\right) = -3$$

$$\log_2(2^{10}) = 10$$

Logarithms

Logarithm is the inverse exponent

$$b^{\log_b(n)} = n = \log_b(b^n)$$

Product Rule

Let's say $n = a \cdot b$

How are $\log_2(n)$, $\log_2(a)$, and $\log_2(b)$ related?

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$$a = \underbrace{2 \cdot \dots \cdot 2}_{\log_2(a) \text{ times}}$$

$$b = \underbrace{2 \cdot \dots \cdot 2}_{\log_2(b) \text{ times}}$$

Product Rule

Let's say $n = a \cdot b$

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$$a = 2 \cdot \dots \cdot 2$$

$$b = 2 \cdot \dots \cdot 2$$

$$n = \underbrace{2 \cdot \dots \cdot 2}_{\log_2(n) \text{ times}} = \underbrace{2 \cdot \dots \cdot 2}_{\log_2(a) \text{ times}} \cdot \underbrace{2 \cdot \dots \cdot 2}_{\log_2(b) \text{ times}}$$

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$$\log_2(n) = \log_2(ab) = \log_2(a) + \log_2(b)$$

Exponent Rule

$$\log_2(a^n) = \log_2(a \cdot \dots \cdot a)$$

Exponent Rule

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$$\begin{aligned}\log_2(a^n) &= \log_2(\overbrace{a \cdot \dots \cdot a}^{n \text{ times}}) \\ &= \log_2(a) + \log_2(\overbrace{a \cdot \dots \cdot a}^{n-1 \text{ times}})\end{aligned}$$

Exponent Rule

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Exponent Rule

$$\begin{aligned}\log_2(a^n) &= \log_2(\overbrace{a \cdot \dots \cdot a}^{n \text{ times}}) \\ &= \log_2(a) + \log_2(\overbrace{a \cdot \dots \cdot a}^{n-1 \text{ times}}) \\ &= \log_2(a) + \log_2(a) + \log_2(\overbrace{a \cdot \dots \cdot a}^{n-2 \text{ times}}) \\ &= \underbrace{\log_2(a) + \dots + \log_2(a)}_{n \text{ times}}\end{aligned}$$

Exponent Rule

$$\begin{aligned}\log_2(a^n) &= \log_2(a \cdot \dots \cdot a) \\ &= \log_2(a) + \log_2(a \cdot \dots \cdot a) \\ &= \log_2(a) + \log_2(a) + \log_2(a \cdot \dots \cdot a) \\ &= \log_2(a) + \dots + \log_2(a) \\ &= n \cdot \log_2(a)\end{aligned}$$

Division Rule

$$\log_2\left(\frac{a}{b}\right) = \log_2\left(a \cdot \frac{1}{b}\right)$$

Division Rule

$$\begin{aligned}\log_2\left(\frac{a}{b}\right) &= \log_2\left(a \cdot \frac{1}{b}\right) \\ &= \log_2(a) + \log_2\left(\frac{1}{b}\right)\end{aligned}$$

Division Rule

$$\begin{aligned}\log_2\left(\frac{a}{b}\right) &= \log_2\left(a \cdot \frac{1}{b}\right) \\ &= \log_2(a) + \log_2\left(\frac{1}{b}\right) \\ &= \log_2(a) + \log_2(b^{-1})\end{aligned}$$

Division Rule

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Change of Base

$$b^m = n$$

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$$\log_b(n) = \frac{\log_c(n)}{\log_c(b)}$$

Summary

Exponent Rule	$\log(n^a) = a \log(n)$
Product Rule	$\log(ab) = \log(a) + \log(b)$
Division Rule	$\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$
Change of Base	$\log_b(n) = \frac{\log_c(n)}{\log_c(b)}$
Inverse	$b^{\log_b(n)} = \log_b(b^n) = n$

** for this class, always assume base 2 unless otherwise stated **