

CSE 250 Recitation

September 22 - 23: Asymptotic Analysis in Code



Analyzing Code/Algorithms

Remember the different types of control flow:

- Sequential (statements executed one after another)
 - Add the number of steps together
 - If I do A then B, the total cost is the cost to do A plus the cost to do B
- Selection (conditional execution of statements)
 - Our growth function will be a piecewise function
- Repetition (repeating execution of one or more statements)
 - Add up the total number of steps...with summations

Code Analysis

```
1 Structure ds = new Structure();
2
3 ds.initialize();
4
5 for (int i = 0; i < n; i++) {
6     Thing thing = createAThing(i, n);
7     ds.insertAThing(thing);
8 }
9
10 if (weFeelLikeIt())
11     ds.computeSomething();
```

Function	Growth Function
Structure()	$T_{\text{new}} =$
ds.initialize()	$T_{\text{init}} =$
createAThing()	$T_{\text{create}} =$
ds.insertAThing()	$T_{\text{insert}} =$
weFeelLikeIt()	$T_{\text{cond}} =$
ds.computeSomething()	$T_{\text{comp}} =$

Exercise: Write the growth function, $T(n)$, that represents the runtime of this code (start by writing it in terms of the other growth functions)

Growth Function

Here is the growth function you should have gotten

(If T_{cond} was pulled out of the piecewise that's fine too)

```
1 Structure ds = new Structure();
2
3 ds.initialize();
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5 for (int i = 0; i < n; i++) {
6     Thing thing = createAThing(i, n);
7     ds.insertAThing(thing);
8 }
9
10 if (weFeelLikeIt())
11     ds.computeSomething();
```

$$T(n) = T_{new} + T_{init} + \sum_{i=0}^{n-1} (T_{create} + T_{insert}) + \begin{cases} T_{cond} + T_{comp} & \text{if weFeelLikeIt()} \\ T_{cond} & \text{otherwise} \end{cases}$$

Finding Bounds

Exercise:

1. Update your growth function with the growth functions to the right
2. Find its closed form solution
3. Determine the \mathbf{O} , $\mathbf{\Omega}$, and $\mathbf{\Theta}$ bounds

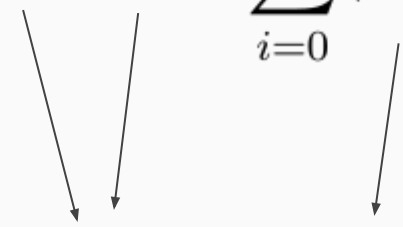
Note: $|x|$ is the notation for "size of x "

You may assume that `insertAThing` increases the size of the data structure by one, and nothing else changes its size

Function	Growth Function
<code>Structure()</code>	$T_{\text{new}} = 5$
<code>ds.initialize()</code>	$T_{\text{init}} = 23$
<code>createAThing()</code>	$T_{\text{create}} = 6$
<code>ds.insertAThing()</code>	$T_{\text{insert}} = 2 \cdot ds $
<code>weFeelLikeIt()</code>	$T_{\text{cond}} = 4n + 5$
<code>ds.computeSomthing()</code>	$T_{\text{comp}} = 3 \cdot ds ^3$

Getting the Closed Form Solution

$$T(n) = 5 + 23 + \sum_{i=0}^{n-1} (6 + 2i) + \begin{cases} 4n + 5 + 3n^3 & \text{if weFeelLikeIt()} \\ 4n + 5 & \text{otherwise} \end{cases}$$


$$T(n) = 28 + (5n + n^2) + \begin{cases} 4n + 5 + 3n^3 & \text{if weFeelLikeIt()} \\ 4n + 5 & \text{otherwise} \end{cases}$$

Finding Bounds (Big-O)

$$T(n) = 28 + (5n + n^2) + \begin{cases} 4n + 5 + 3n^3 & \text{if weFeelLikeIt()} \\ 4n + 5 & \text{otherwise} \end{cases}$$

$O(1) + O(n^2) + O(n^3) = O(1 + n^2 + n^3) = O(n^3)$

Finding Bounds (Big- Ω)

$$T(n) = 28 + (5n + n^2) + \begin{cases} 4n + 5 + 3n^3 & \text{if weFeelLikeIt()} \\ 4n + 5 & \text{otherwise} \end{cases}$$

$\Omega(1) + \Omega(n^2) + \Omega(n) = \Omega(1 + n^2 + n) = \Omega(n^2)$

Finding Bounds (Big- Θ)

$$T(n) = 28 + (5n + n^2) + \begin{cases} 4n + 5 + 3n^3 & \text{if weFeelLikeIt()} \\ 4n + 5 & \text{otherwise} \end{cases}$$

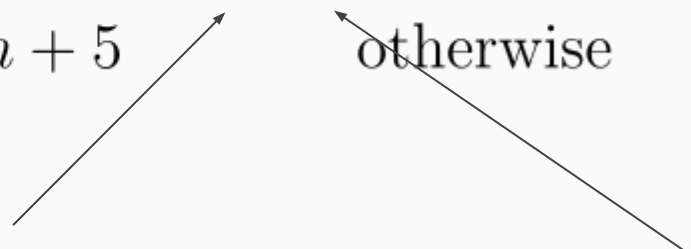
$$T(n) \in O(n^3)$$

$$T(n) \in \Omega(n^2)$$

Since both of these bounds are **tight** then there does not exist an $f(n)$ such that

$$T(n) \in \Theta(f(n))$$

Finding Bounds (Big- Θ)

$$T(n) = 28 + (5n + n^2) + \begin{cases} 4n + 5 + 3n^3 & \text{if weFeelLikeIt()} \\ 4n + 5 & \text{otherwise} \end{cases}$$


Follow Up: What if T_{comp} was $3n^2$ instead? What if it was $3n$?

Finding Bounds (Big- Θ)

$$T(n) = 28 + (5n + n^2) + \begin{cases} 4n + 5 + 3n^3 & \text{if weFeelLikeIt()} \\ 4n + 5 & \text{otherwise} \end{cases}$$

Follow Up: What if T_{comp} was $3n^2$ instead? What if it was $3n$?

In both cases, $T(n)$ would be in $O(n^2)$, $\Omega(n^2)$ and therefore $\Theta(n^2)$

Proving Bounds

Let $g(n) = 3n + n^2$. Prove that $g(n) \in O(n^2)$, $g(n) \in \Omega(n^2)$

First...what is the definition of big-O?

Proving Bounds

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First...what is the definition of big-O?

$$g(n) \leq c f(n), \text{ for all } n \geq n_0 \text{ for some } c > 0, \text{ and } n_0 \geq 0$$

What is the definition of big- Ω ?

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$$g(n) \geq c f(n), \text{ for all } n \geq n_0 \text{ for some } c > 0, \text{ and } n_0 \geq 0$$

Proving Bounds

Exercise: Prove the following

$$g(n) = 3n + n^2$$

1. $g(n) \in O(n^2)$

2. $g(n) \in \Omega(n^2)$

3. $g(n) \in \Theta(n^2)$

4. $T(n) \in O(n^3)$

5. $T(n) \in \Omega(n^2)$

$$T(n) = n^2 + 5n + 28 + \begin{cases} 3n^3 + 4n + 5 & \text{if true} \\ 4n + 5 & \text{otherwise} \end{cases}$$

Hint: For T, consider the fact that it is either:

$$3n^3 + n^2 + 9n + 33 \quad \text{OR} \quad n^2 + 9n + 33$$

More Examples

Prove the following:

$$12 \log(10 \times 2^n) \in O(n)$$

$$n^2 + n \log(n) \in O(2^n)$$

$$n^2 + 15n^3 \in \Omega(n)$$

$$\sum_{i=1}^n i \in \Omega(n^2)$$