0 Overview

Instructions

Due Date: Sunday Feb 2 @ 11:59PM

Total points: 50

Your written solution may be either handwritten and scanned, or typeset. Either way, you must produce a PDF that is legible and displays reasonably on a typical PDF reader. This PDF should be submitted via autolab as WA1. You should view your submission after you upload it to make sure that it is not corrupted or malformed. Submissions that are rotated, upside down, or that do not load will not receive credit. Illegible submissions may also lose credit depending on what can be read. Ensure that your final submission contains all pages.

You are responsible for making sure your submission went through successfully.

Written submissions may be turned in up to one day late for a 50% penalty.

No grace day usage is allowed.

1 Questions

Simplify each of the following equations, $f_i(n)$. Your final result should be a sum-free equation in terms of n. Show your work as a sequence of steps. For each step, indicate the specific rule (see the provided cheatsheet below) that relates the current step to the previous one. All logarithms are base-2 and should be simplifed when possible.

$$f_1(n) = \sum_{i=1}^{n} \frac{3i}{4} \tag{1}$$

$$f_2(n) = \sum_{i=1}^{2n} 6 + 3i \tag{2}$$

$$f_3(n) = \sum_{i=n}^{2n} 2^i \tag{3}$$

$$f_4(n) = \sum_{i=0}^{\log n} 2^i \tag{4}$$

$$f_5(n) = \sum_{i=0}^{3} 3i^2 \tag{5}$$

$$f_6(n) = \sum_{i=1}^{n^2} i + n \tag{6}$$

$$f_7(n) = \sum_{i=1}^{n} \sum_{j=1}^{n} i \tag{7}$$

$$f_8(n) = \sum_{i=1}^{2^n} \sum_{j=0}^{n-1} 2^j \tag{8}$$

$$f_9(n) = \sum_{i=6}^{6n} \sum_{j=0}^{\log(i)-1} 2^j \tag{9}$$

$$f_{10}(n) = \sum_{i=-3}^{n} i \tag{10}$$

Summation Rules

S1. $\sum_{i=j}^{k} c = (k-j+1)c$

S2. $\sum_{i=j}^{k}(cf(i)) = c\sum_{i=j}^{k} f(i)$

S3. $\sum_{i=j}^{k}(f(i)+g(i)) = \left(\sum_{i=j}^{k} f(i)\right) + \left(\sum_{i=j}^{k} g(i)\right)$

S4. $\sum_{i=j}^{k}(f(i)) = \left(\sum_{i=\ell}^{k}(f(i))\right) - \left(\sum_{i=\ell}^{j-1}(f(i))\right)$ **(for any $\ell < j$)**

S5. $\sum_{i=j}^{k} f(i) = f(j) + f(j+1) + \ldots + f(k-1) + f(k)$

S6. $\sum_{i=j}^{k} f(i) = f(j) + \ldots + f(\ell-1) + \left(\sum_{i=\ell}^{k} f(i)\right)$ **(for any $j < \ell \leq k$)**

S7. $\sum_{i=j}^{k} f(i) = \left(\sum_{i=j}^{\ell} f(i)\right) + f(\ell+1) + \ldots + f(k)$ **(for any $j \leq \ell < k$)**

S8. $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$

S9. $\sum_{i=0}^{k} 2^i = 2^{k+1} - 1$

Log Rules

L1. $\log(n^a) = a\log(n)$

L2. $\log(an) = \log(a) + \log(n)$

L3. $\log(\frac{n}{a}) = \log(n) - \log(a)$

L4. $\log_b(n) = \frac{\log_c(n)}{\log_c(b)}$

L5. $\log(2^n) = 2^{\log(n)} = n$

Inequality Rules

I1. $f(n) \leq g(n)$ is true if you can find some $h(n)$ where $f(n) \leq h(n)$ and $h(n) \leq g(n)$

I2. $f(n) \leq g(n)$ is true if you can find some $h(n)$ where $f(n) - h(n) \leq g(n) - h(n)$

I3. $f(n) \leq g(n)$ is true if you can find some $h(n) \geq 0$ (for all n) such that $f(n) \cdot h(n) \leq g(n) \cdot h(n)$

I4. Take it as a given that: $\theta(1) \leq \theta(\log(n)) \leq \theta(n) \leq \theta(n^2) \leq \theta(n^k)$ **(for $k > 2$)** $\leq \theta(2^n)$