

# CSE 250

## Data Structures

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**Lec 03: Math Refresher**

# Announcements and Feedback

- Make sure you are on Piazza and AutoLab
- Academic Integrity Quiz due 2/2 @ 11:59PM **(MUST GET 100%)**
- PA0 due 2/2 @ 11:59PM **(MUST GET 100%)**
- WA1 is out now, due 2/2 @ 11:59PM

# Today's Topics

- Summations
- Logarithms

# Summations

$$\sum_{i=j}^k f(i) = f(j) + f(j+1) + \dots + f(k)$$

# Useful Tricks

If  $c$  is a constant (with respect to  $i$ )

$$\sum_{i=j}^k c = \underbrace{(c + c + \dots + c)}_{(k - j + 1) \text{ times}}$$

# Useful Tricks

If  $c$  is a constant (with respect to  $i$ )

$$\begin{aligned}\sum_{i=j}^k c &= (c + c + \dots + c) \\ &= (k - j + 1) \cdot c\end{aligned}$$

# Useful Tricks

If  $c$  is a constant and  $f(i)$  is a function of  $i$ :

$$\sum_{i=j}^k cf(i) = (cf(j) + cf(j+1) + \dots + cf(k))$$

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$$\begin{aligned}\sum_{i=j}^k cf(i) &= (cf(j) + cf(j+1) + \dots + cf(k)) \\ &= c(f(j) + f(j+1) + \dots + f(k))\end{aligned}$$



# Useful Tricks

If  $c$  is a constant and  $f(i)$  is a function of  $i$ :

$$\begin{aligned}\sum_{i=j}^k cf(i) &= (cf(j) + cf(j+1) + \dots + cf(k)) \\ &= c(f(j) + f(j+1) + \dots + f(k)) \\ &= c \sum_{i=j}^k f(i)\end{aligned}$$

# Useful Tricks

If  $f(i)$  and  $g(i)$  are functions of  $i$ :

$$\sum_{i=j}^k f(i) + g(i) = (f(j) + g(j)) + (f(j+1) + g(j+1)) + \dots + (f(k) + g(k))$$

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If  $f(i)$  and  $g(i)$  are functions of  $i$ :

$$\begin{aligned}\sum_{i=j}^k f(i) + g(i) &= (f(j) + g(j)) + (f(j+1) + g(j+1)) + \dots + (f(k) + g(k)) \\ &= (f(j) + f(j+1) + \dots + f(k)) + (g(j) + g(j+1) + \dots + g(k)) \\ &= \left( \sum_{i=j}^k f(i) \right) + \left( \sum_{i=j}^k g(i) \right)\end{aligned}$$

# Useful Tricks

If  $j < l < k$ :

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If  $j < l < k$ :

$$\begin{aligned}\sum_{i=j}^k f(i) &= f(j) + \dots + f(k) \\ &= f(j) + \dots + f(l-1) + f(l) + \dots + f(k)\end{aligned}$$

# Useful Tricks

If  $j < l < k$ :

$$\begin{aligned}\sum_{i=j}^k f(i) &= f(j) + \dots + f(k) \\ &= f(j) + \dots + f(l-1) + f(l) + \dots + f(k) \\ &= \left( \sum_{i=j}^{l-1} f(i) \right) + \left( \sum_{i=l}^k f(i) \right)\end{aligned}$$

# Useful Tricks

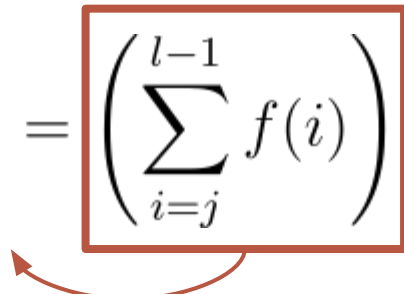
If  $j < l < k$ :

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If  $j < l < k$ :

$$\left( \sum_{i=j}^k f(i) \right) = \left( \sum_{i=j}^{l-1} f(i) \right) + \left( \sum_{i=l}^k f(i) \right)$$


Subtract to other side

# Useful Tricks

If  $j < l < k$ :

$$\left( \sum_{i=j}^k f(i) \right) = \left( \sum_{i=j}^{l-1} f(i) \right) + \left( \sum_{i=l}^k f(i) \right)$$

$$\left( \sum_{i=j}^k f(i) \right) - \left( \sum_{i=j}^{l-1} f(i) \right) = \left( \sum_{i=l}^k f(i) \right)$$

# Series

Some common closed form solutions:

$$\sum_{i=1}^k i = \frac{k(k+1)}{2}$$

$$\sum_{i=0}^k 2^i = 2^{k+1} - 1$$

# Summary

- The previous rules will always be provided on WAs and exams
- Usually the goal will be to reduce some complicated summation to a simpler form without a summation
  - Some of the rules get rid of summations
  - Some allow you to manipulate summations/bounds so that you can apply rules that get rid of summations
- Be cognizant of what variables are constant ***with respect to the summation variable*** and which one aren't

# Logarithms

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is division

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# Logarithms

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$$a^n = \underbrace{a \cdot a \cdot \dots \cdot a}_{\mathbf{a} \text{ multiplied together } \mathbf{n} \text{ times}}$$

The inverse operation  
is  $\log_a$



# Logarithms

$\log_a(b)$  = the number of times you multiply  $a$  together to get  $b$

$$\log_2(32) = 5$$

$$\log_3(27) = 3$$

$$\log_2\left(\frac{1}{8}\right) = -3$$

$$\log_2(2^{10}) = 10$$

# Logarithms

Logarithm is the inverse exponent

$$b^{\log_b(n)} = n = \log_b(b^n)$$

# Product Rule

Let's say  $n = a \cdot b$

How are  $\log_2(n)$ ,  $\log_2(a)$ , and  $\log_2(b)$  related?

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$$n = \underbrace{2 \cdot \dots \cdot 2}_{\log_2(n) \text{ times}} = \underbrace{2 \cdot \dots \cdot 2}_{\log_2(a) \text{ times}} \cdot \underbrace{2 \cdot \dots \cdot 2}_{\log_2(b) \text{ times}}$$

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$$\log_2(n) = \log_2(ab) = \log_2(a) + \log_2(b)$$

# Exponent Rule

$$\log_2(a^n) = \log_2(a \cdot \dots \cdot a)$$



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$$\begin{aligned}\log_2(a^n) &= \log_2(\overbrace{a \cdot \dots \cdot a}^{n \text{ times}}) \\ &= \log_2(a) + \log_2(\overbrace{a \cdot \dots \cdot a}^{n-1 \text{ times}})\end{aligned}$$

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# Exponent Rule

$$\begin{aligned}\log_2(a^n) &= \log_2(a \cdot \dots \cdot a) \\ &= \log_2(a) + \log_2(a \cdot \dots \cdot a) \\ &= \log_2(a) + \log_2(a) + \log_2(a \cdot \dots \cdot a) \\ &= \log_2(a) + \dots + \log_2(a) \\ &= n \cdot \log_2(a)\end{aligned}$$

# Division Rule

$$\log_2\left(\frac{a}{b}\right) = \log_2\left(a \cdot \frac{1}{b}\right)$$

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# Change of Base

$$b^m = n$$

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$$\log_b(n) = \frac{\log_c(n)}{\log_c(b)}$$

# Summary

|                       |  |
|-----------------------|--|
| <b>Exponent Rule</b>  | $\log(n^a) = a \log(n)$                            |
| <b>Product Rule</b>   | $\log(ab) = \log(a) + \log(b)$                     |
| <b>Division Rule</b>  | $\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$ |
| <b>Change of Base</b> | $\log_b(n) = \frac{\log_c(n)}{\log_c(b)}$          |
| <b>Inverse</b>        | $b^{\log_b(n)} = \log_b(b^n) = n$                  |

*\* for this class, always assume base 2 unless otherwise stated \**