CSE 250
Data Structures

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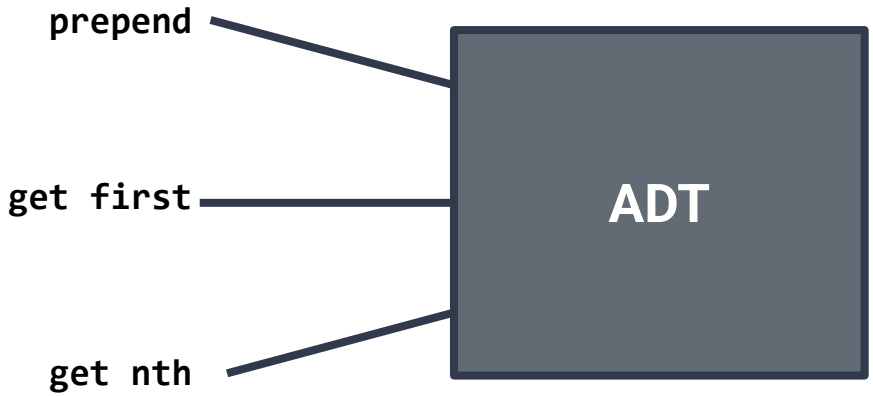
Lec 05: Intro to Complexity

Announcements and Feedback

- AI Quiz, PA0, WA1 due Sunday @ 11:59PM

Thought Experiment

Often, many data structures can satisfy a given ADT…how do you choose?



Thought Experiment

Data Structure 1
- Very fast `prepend`, `get first`
- Very slow `get nth`

Data Structure 2
- Very fast `get nth`, `get first`
- Very slow `prepend`

Data Structure 3
- Very fast `get nth`, `get first`
- Occasionally slow `prepend`

Which is better?

Thought Experiment

Data Structure 1 (LinkedList)

- Very fast **`prepend`**, **`get first`**
- Very slow **`get nth`**

Data Structure 2 (Array)

- Very fast **`get nth`**, **`get first`**
- Very slow **`prepend`**

Data Structure 3 (ArrayList…in reverse)

- Very fast **`get nth`**, **`get first`**
- Occasionally slow **`prepend`**

Which is better?

IT DEPENDS!

Thought Experiment

Data Structure 1 (LinkedList)

- Very **fast** `prepend`, `get first`
- Very **slow** `get nth`

Data Structure 2 (ArrayList)

- Very **fast** `get nth`, `get first`
- Very **slow** `prepend`

Data Structure 3 (ArrayList...in reverse)

- Very **fast** `get nth`, `get first`
- Occasionally **slow** `prepend`

What is "fast"? "slow"?

Attempt #1: Wall-clock time?

- What is fast?
  - 10s? 100ms? 10ns?
  - …it depends on the task
- Algorithm vs Implementation
  - Compare Grace Hopper’s implementation to yours
- What machine are you running on?
  - Your old laptop? A lab machine? The newest, shiniest processor on the market?
- What bottlenecks exist? CPU vs IO vs Memory vs Network…

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Wall-clock time is not terribly useful…

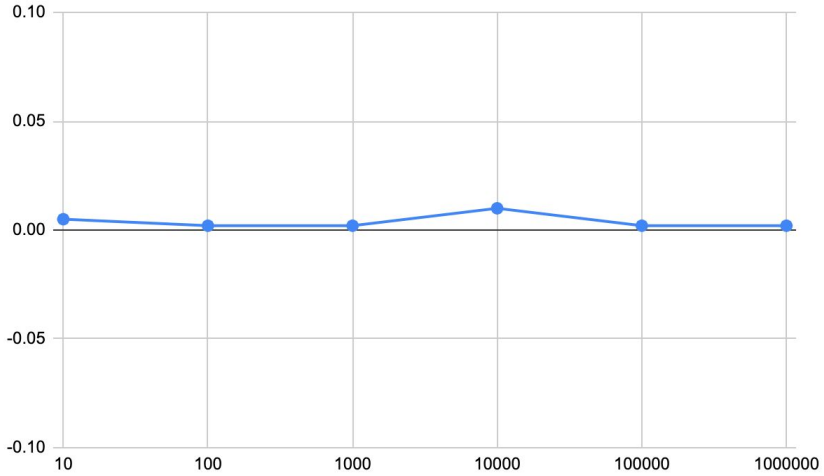
Analysis Checklist

1. Don’t think in terms of wall-time, think in terms of “number of steps”

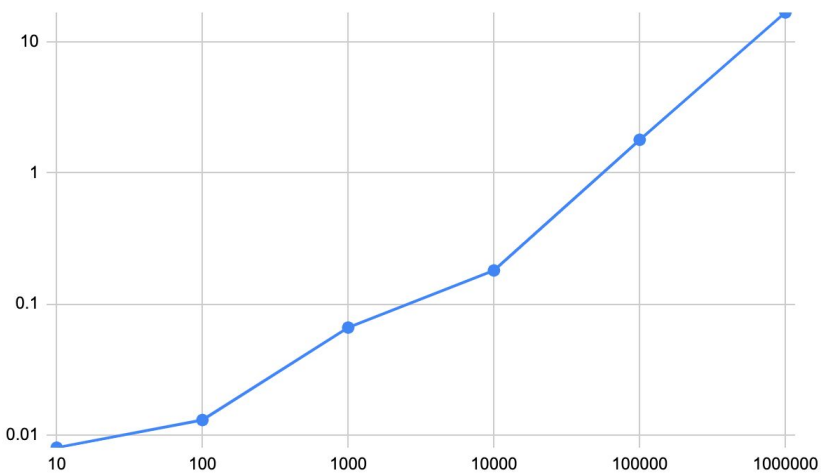
Let’s do a quick demo…

Comparing Random Access for Array vs List

Array



List



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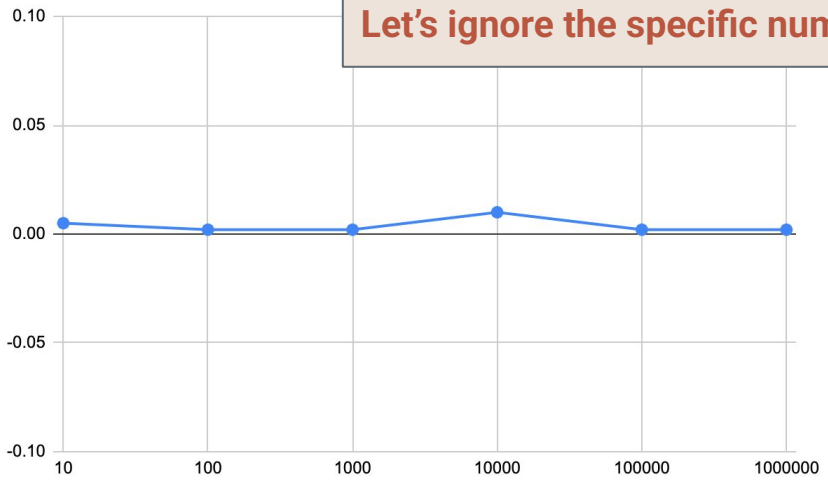

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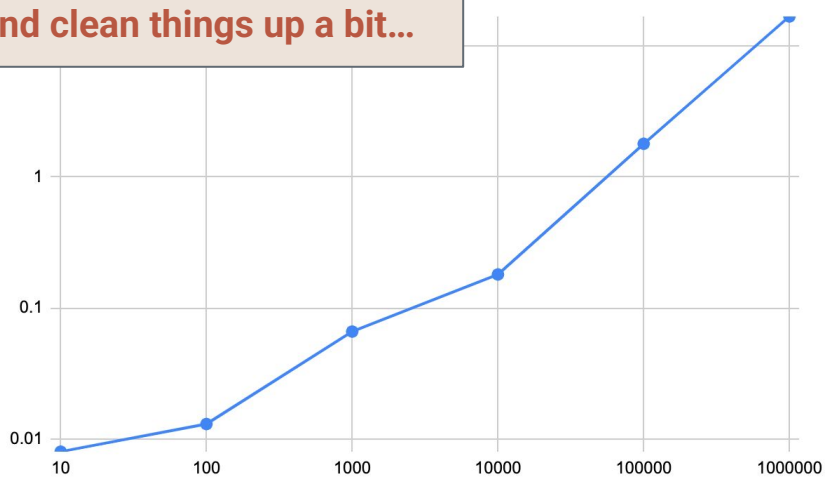
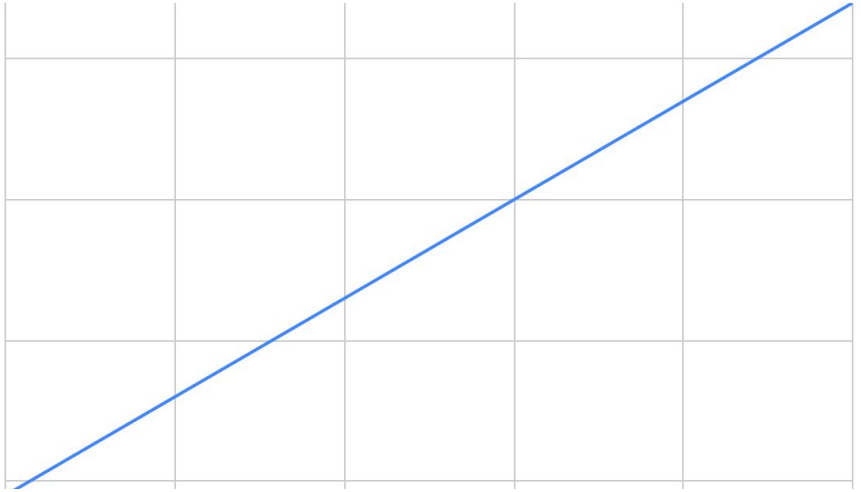


Let's ignore the specific numbers and clean things up a bit...

Comparing Random Access for Array vs List

Array

List



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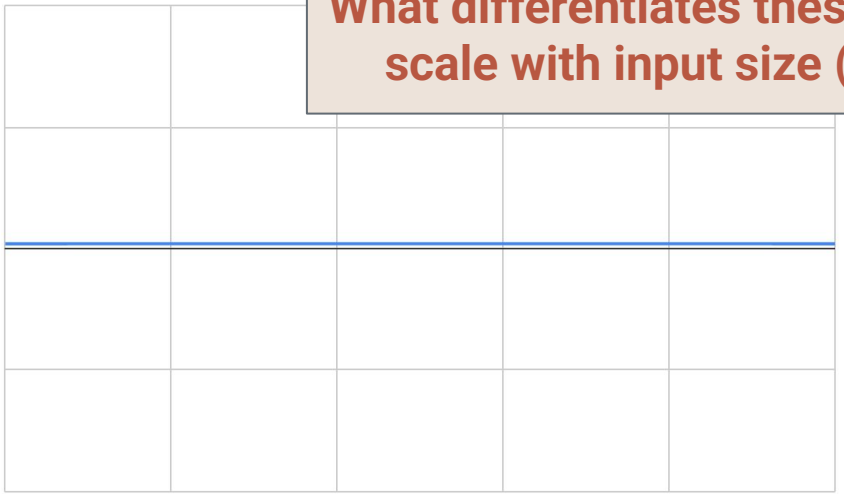

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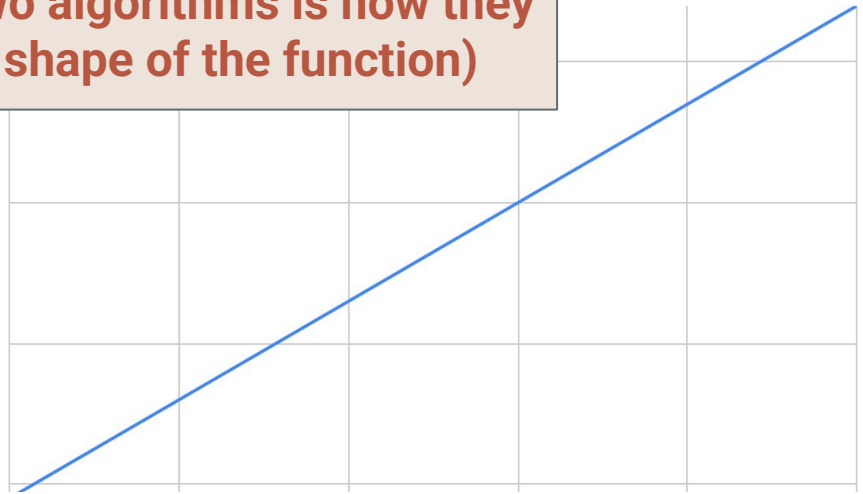

What differentiates these two algorithms is how they scale with input size (the shape of the function)

Analysis Checklist

1. Don’t think in terms of wall-time, think in terms of “number of steps”
2. **To give a useful solution, we should take “scale” into account**
    - **How does the runtime change as we change the size of the input?**

Counting Steps

```
1 public void userFullName(User[] users, int id) {
2   User user = users[id];
3   String fullName = user.firstName + user.lastName;
4   return fullName;
5 }
```

Counting Steps

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1  public void userFullName(User[] users, int id) {
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How many steps does this function take?

Counting Steps

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1  public void userFullName(User[] users, int id) {
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4    return fullName;
5  }
```

7 steps…ish? Maybe? What the heck is a step?

Counting Steps

```
1  public void updateUsers(User[] users) {
2    x = 1;
3    for(User user : users) {
4      user.id = x;
5      x = x + 1;
6    }
7  }
```

Counting Steps

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$$1 + \sum_{user \in users}$$

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$$1 + \sum_{user \in users} 4$$

Counting Steps

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$$1 + \sum_{user \in users} 4 = 1 + 4 \cdot |\texttt{users}|$$

Steps to "Functions"

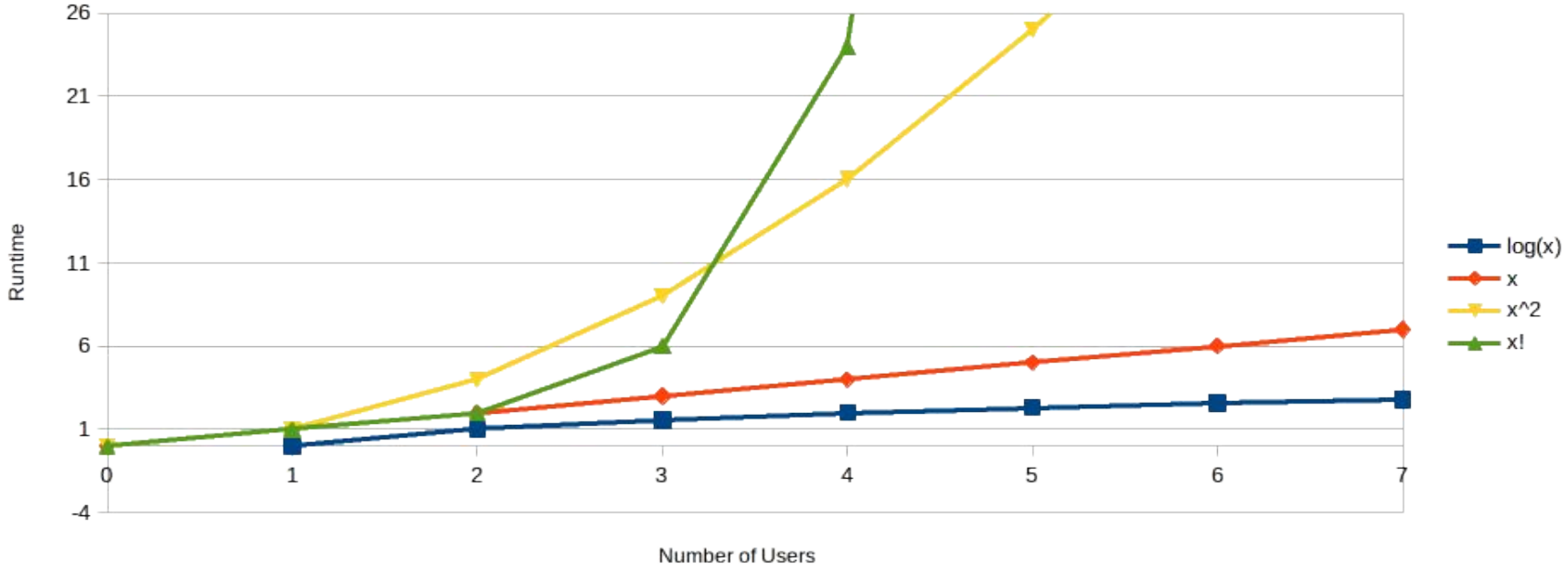
Now that we have number of steps* in terms of summations…

…which we can simplify (like in WA1) into mathematical functions…

We can start analyzing runtime as a function

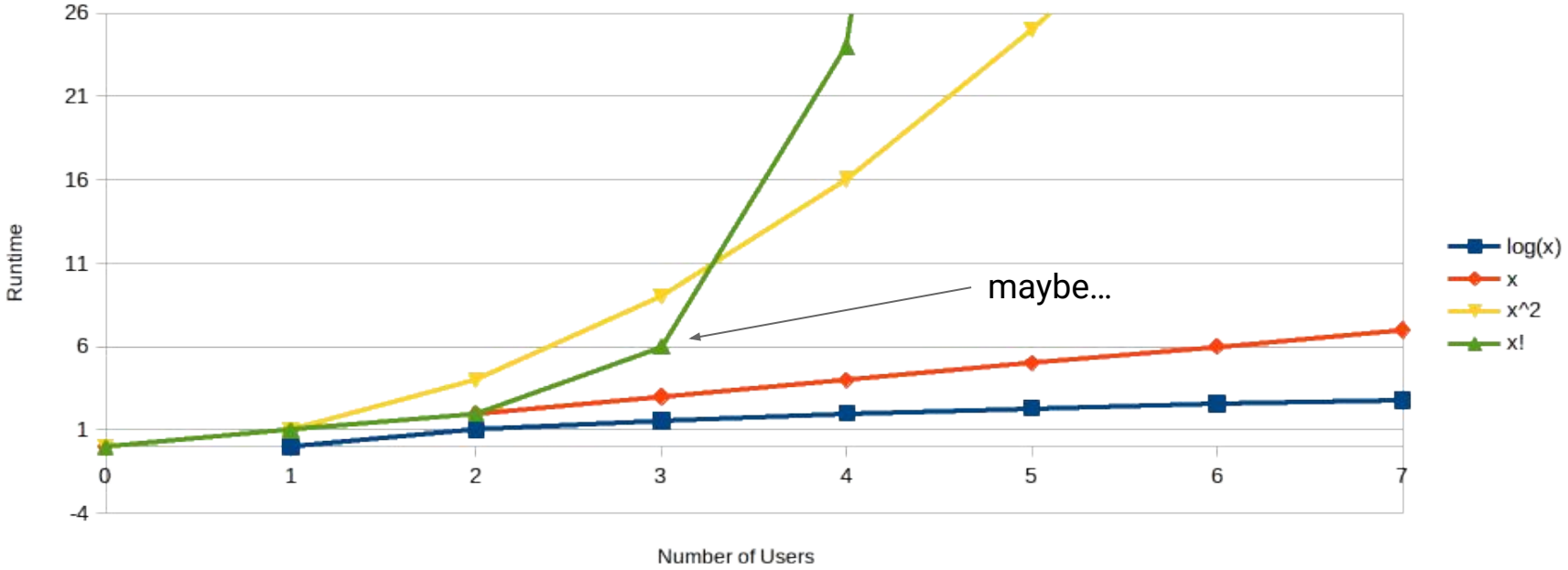
** we'll give a better definition of what a "step" is later*

Runtime as a Function



Would you consider an algorithm that takes |Users|! number of steps?

Runtime as a Function



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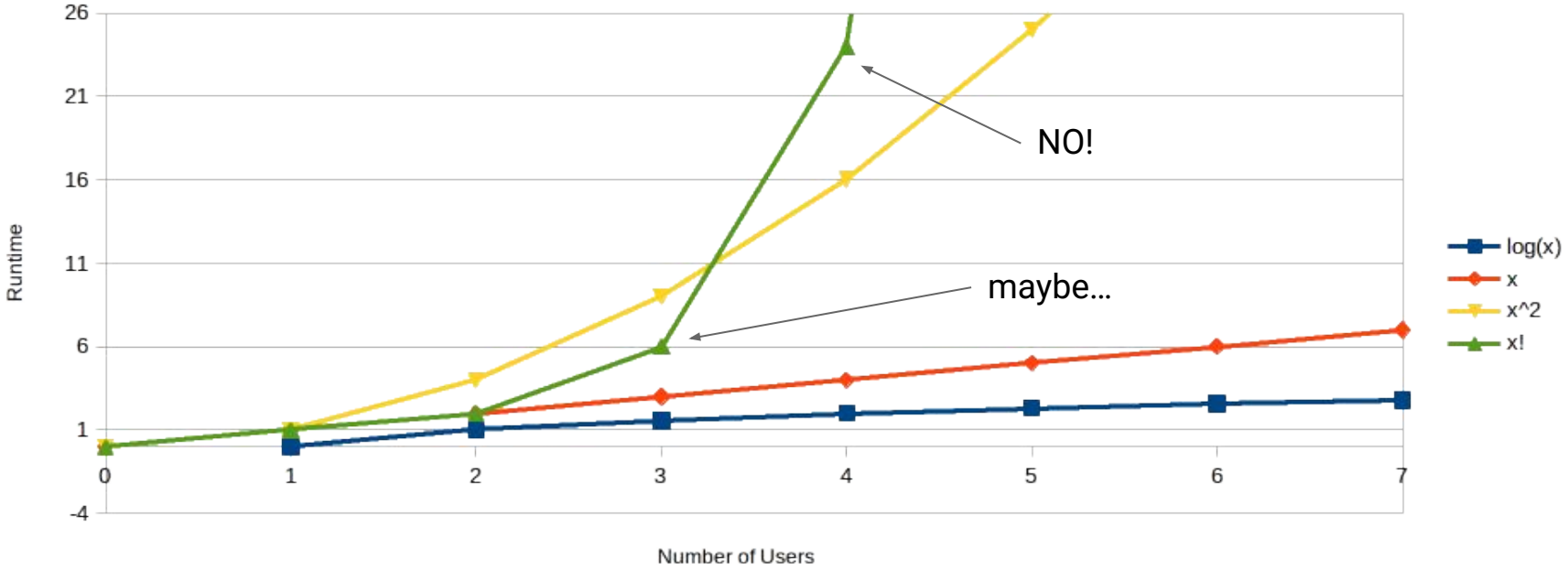

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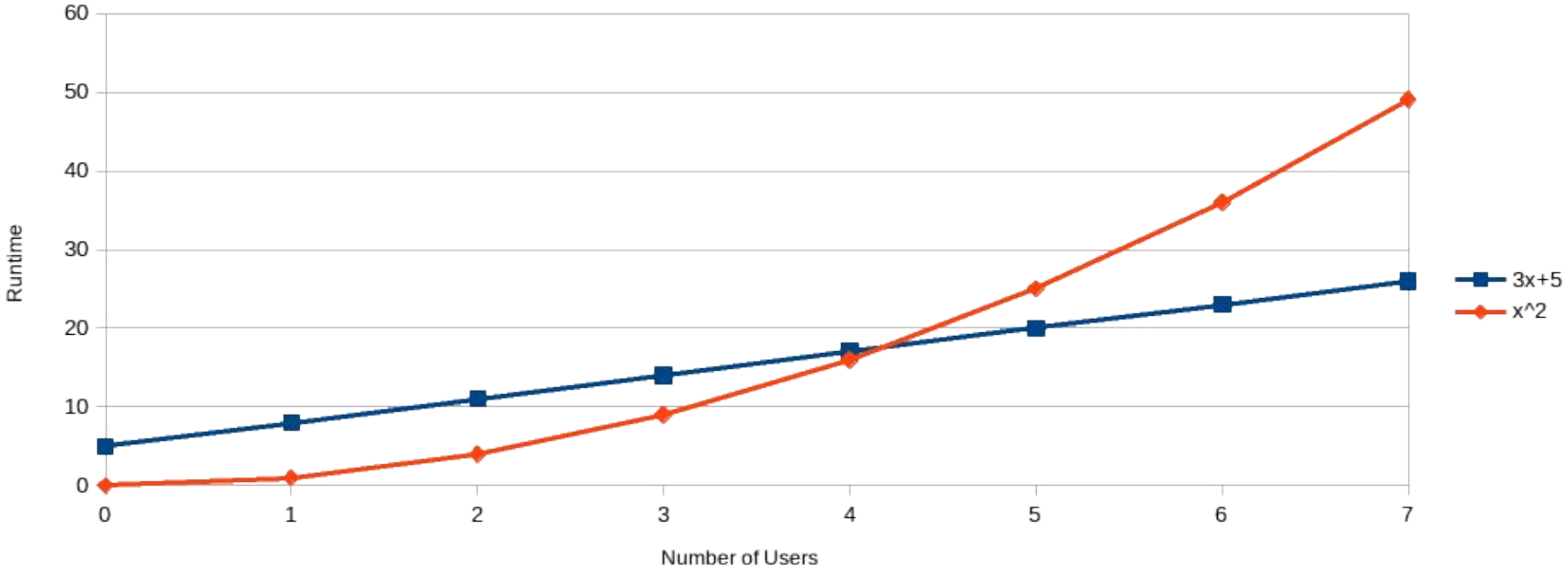

Which is better? 3x|Users|+5 or $|Users|^2$

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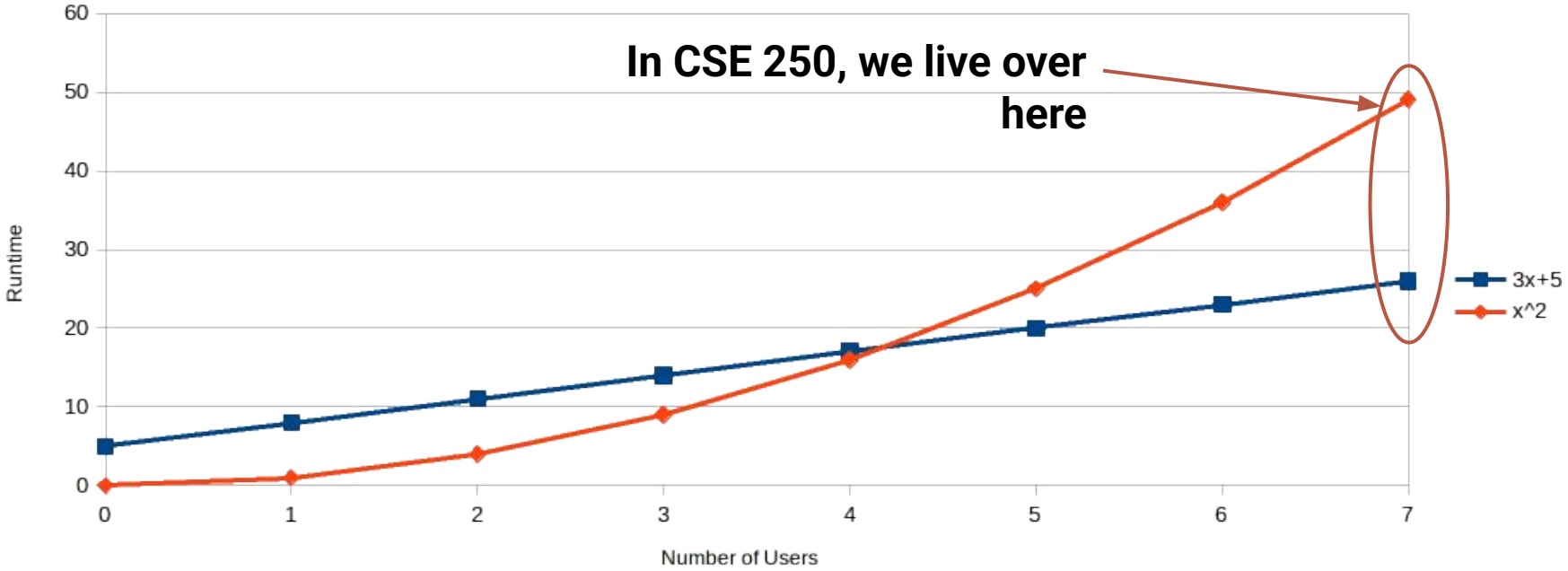
3. **Focus on “large” inputs**
    - **Rank functions based on how they behave at large scales**

Runtime as a Function



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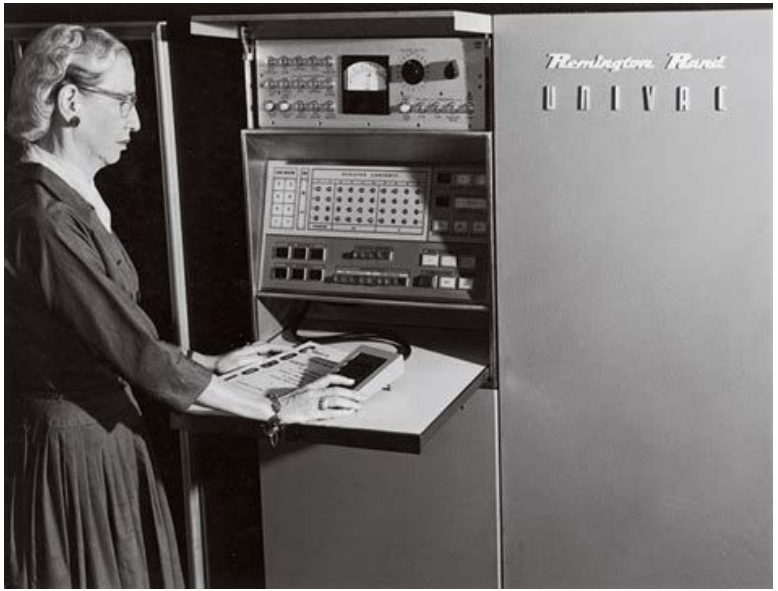
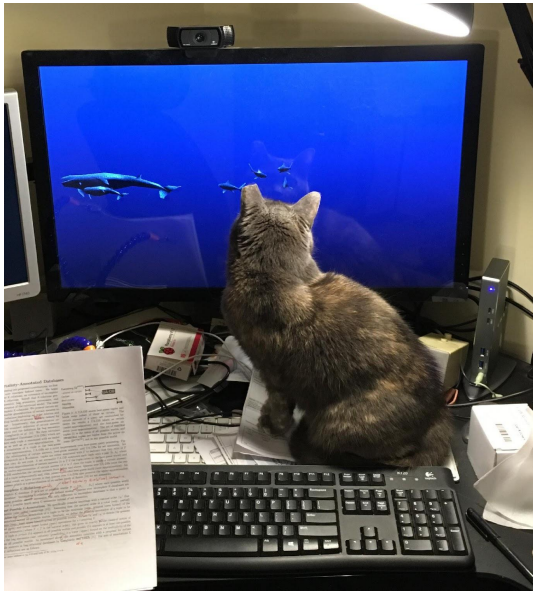
Goal: Ignore implementation details



vs

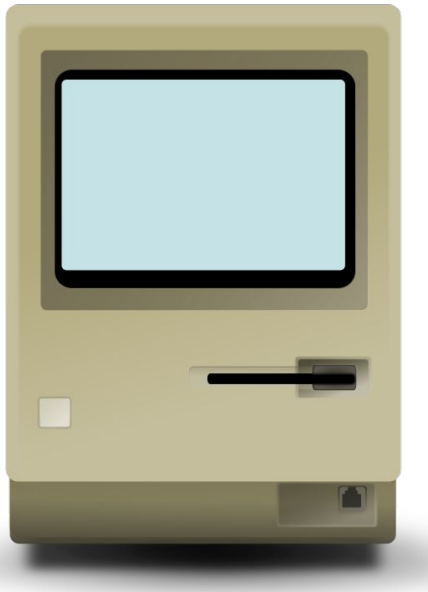


Seasoned Pro Implementation

Error 23: Cat on Keyboard

Goal: Ignore execution environment

vs



Intel i9

Motorola 68000

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Goal: Judge the Algorithm Itself

- How fast is a step? Don’t care
  - Only count number of steps
- Can this be done in two steps instead of one?
  - “3 steps per user” vs “some number of steps per user”
  - Sometimes we don’t care…sometimes we do

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4. **Decouple algorithm from infrastructure/implementation**
    - **Asymptotic notation…?**

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Attempt #2: Growth Functions

Not a function in code…but a mathematical function:

$$T(n)$$

n: The “size” of the input

ie: number of users,rows, pixels, etc

***T*(n): The number of “steps” taken for input of size n**

ie: 20 steps per user, where n = |Users|, is 20 x n

Some Basic Assumptions:

Problem sizes are non-negative integers

$$n \in \{0, 1, 2, 3, \ldots\} = \{0\} \cup \mathbb{Z}^+$$

We can't reverse time…(obviously)

$$T(n) > 0$$

Smaller problems aren't harder than bigger problems

$$n_1 < n_2 \Rightarrow T(n_1) \leq T(n_2)$$

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$T: \{0\} \cup \mathbb{Z}^+ \rightarrow \mathbb{R}^+$

T is non-decreasing

First Problem...

We are still implementation dependent...

$$T_1(n) = 19n$$

$$T_2(n) = 20n$$

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Does 1 extra step per element really matter...?

Is this just an implementation detail?

First Problem…

We are still implementation dependent…

$$T_1(n) = 19n$$

$$T_2(n) = 20n$$

$$T_3(n) = 2n^2$$

T_1 and T_2 are much more “similar” to each other than they are to T_3

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$$T_1(n) = 19n$$

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$$T_3(n) = 2n^2$$

T_1 and T_2 are much more “similar” to each other than they are to T_3

How do we capture this idea formally?

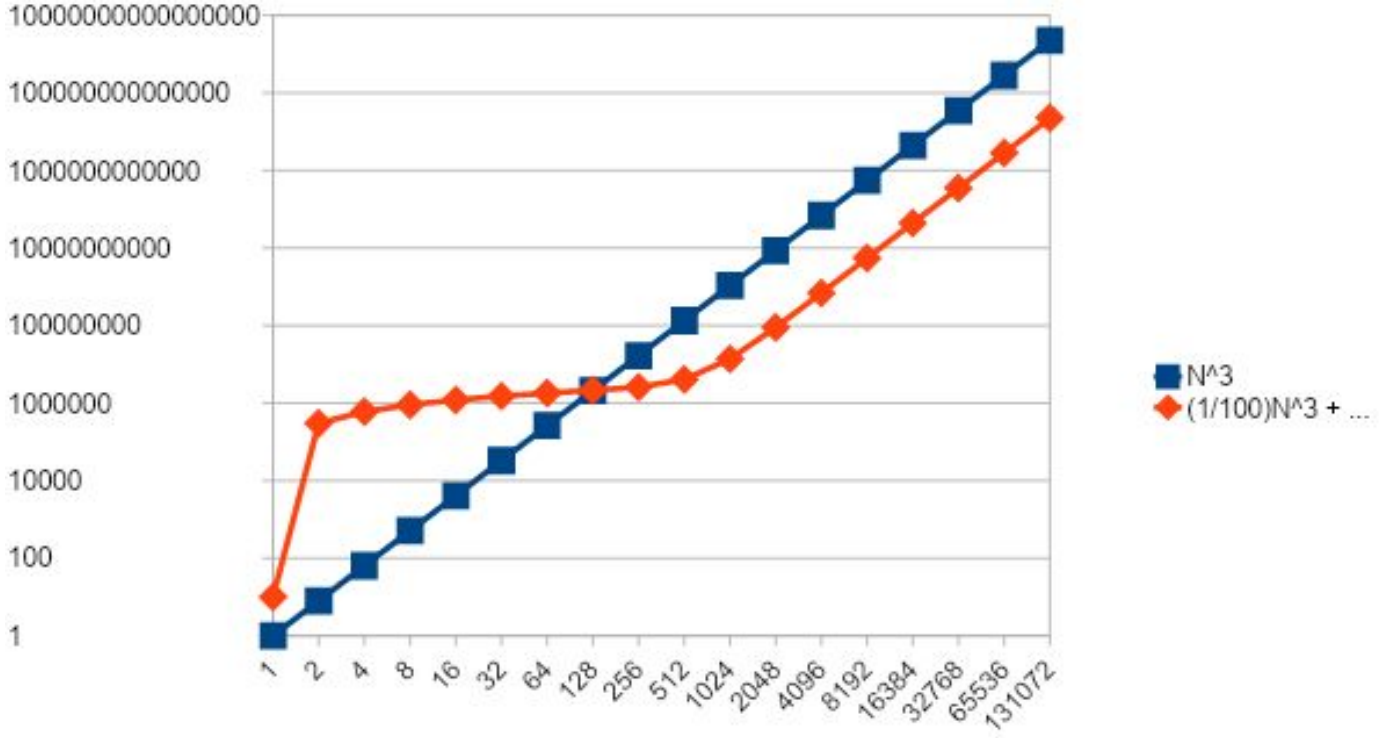
How Do We Capture Behavior at Scale?

Consider the following two functions:

$$\frac{1}{100}n^3 + 10n + 1000000\log(n)$$

$$n^3$$

How Do We Capture Behavior at Scale?



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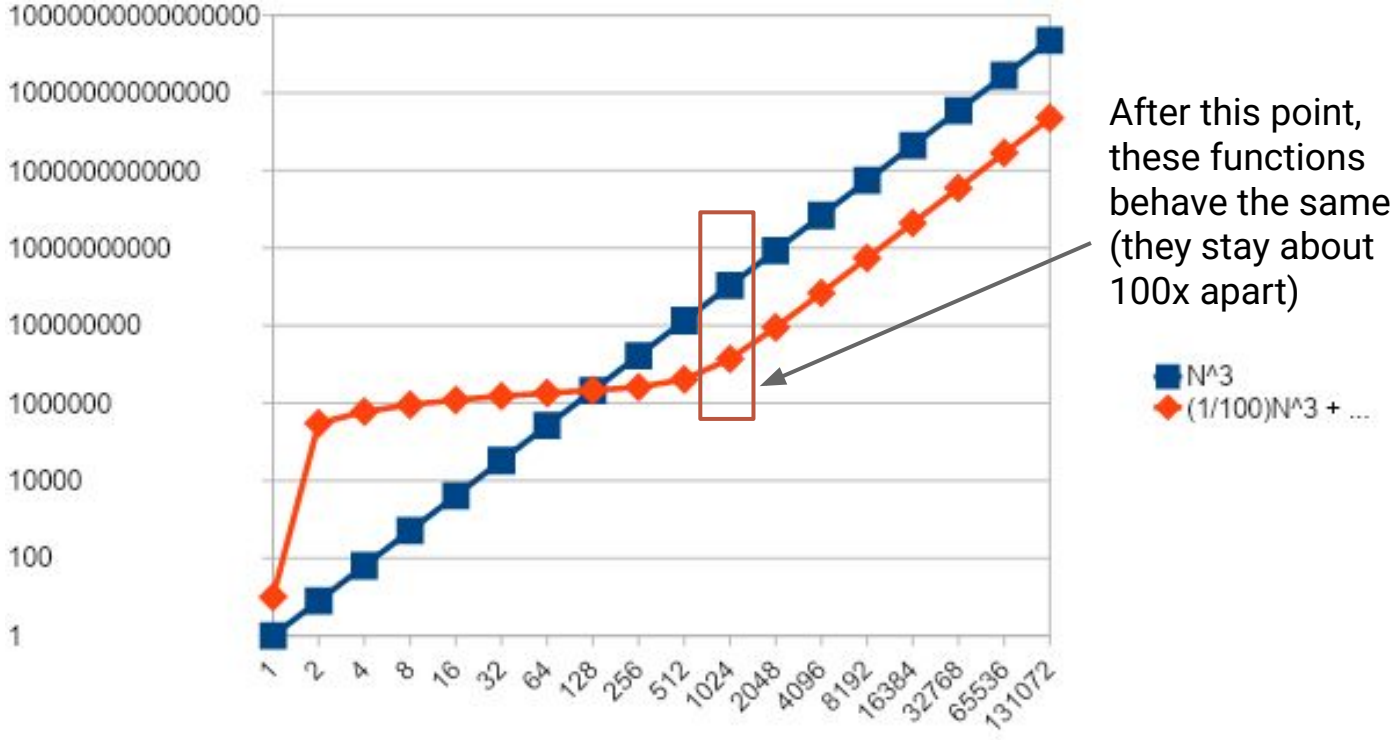

Attempt #3: Asymptotic Analysis

We want to organize runtimes (growth functions) into different ***Complexity Classes***

Within the same complexity class, runtimes “behave the same”/"have the same shape" (at scale)