CSE 250
Data Structures

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Lec 25: Trees

Announcements

- PA2 due last night, submissions close Tuesday
- WA4 due Sunday

(Rooted) Trees

(Even More) Tree Terminology

Rooted, Directed Tree - Has a single root node (node with no parents)

Parent of node X - A node with an out-edge to X (max 1 parent per node)

Child of node X - A node with an in-edge from X

Leaf - A node with no children

Depth of node X - The number of edges in the path from the root to X

Height of node X - The number of edges in the path from X to the deepest leaf

(Even More) Tree Terminology

Level of a node - Depth of the node + 1

Size of a tree (*n*) - The number of nodes in the tree

Height/Depth of a tree (*d*) - Height of the root/depth of the deepest leaf

(Even More) Tree Terminology

Binary Tree - Every vertex has at most 2 children

Complete Binary Tree - All leaves are in the deepest two levels

Full Binary Tree - All leaves are at the deepest level, therefore every vertex has exactly 0 or 2 children, and $d = \log(n)$

Tree Implementation with `Optional`

```
1  public class Tree<T> {
2    Optional<TreeNode<T>> root;
3  }
4
5  public class TreeNode<T> {
6    T value;
7    Optional<TreeNode<T>> leftChild;
8    Optional<TreeNode<T>> rightChild;
9  }
```

Trees are inherently recursive data structures…

So most of the methods we are about to talk about will be very naturally expressed with recursion

Computing Tree Height

The height of a tree is the height of the root

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The children of the root are each roots of the left and right subtrees

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The children of the root are each roots of the left and right subtrees

So we can compute height recursively:

$$h(root) = \begin{cases} \cancel{0}\ \text{-1} & \textbf{if the tree is empty} \\ 1 + max(h(\texttt{root.left}), h(\texttt{root.right})) & \textbf{otherwise} \end{cases}$$

Computing Tree Height

```
1 public int height(Optional<TreeNode<T>> root) {
2   if (!root.isPresent()) return -1;
3   return Math.max(height(root.leftChild), height(root.rightChild)) + 1;
4 }
```

$$h(root) = \begin{cases} \cancel{0}\ -1 & \textbf{if the tree is empty} \\ 1 + max(h(\texttt{root.left}), h(\texttt{root.right})) & \textbf{otherwise} \end{cases}$$

Binary Search Tree

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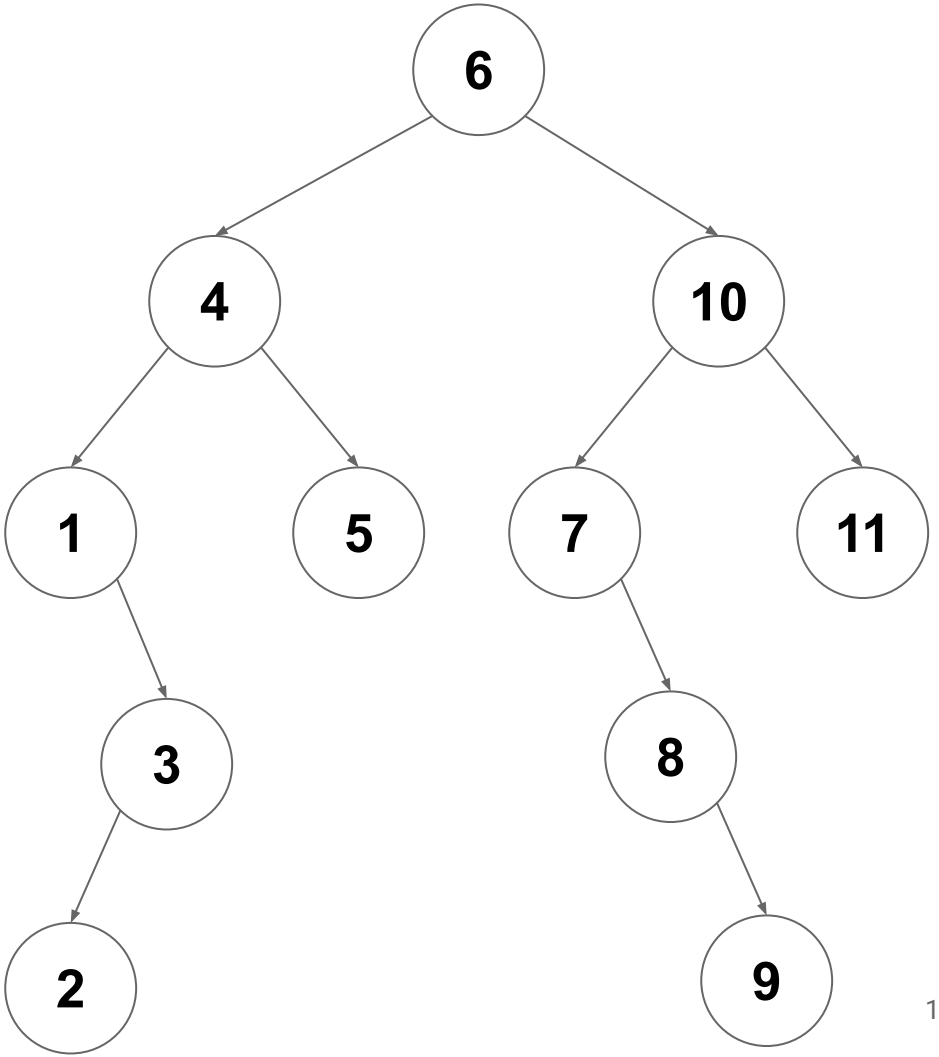
- For every node X_R in the right subtree of node X: X_R**.key** > X**.key**

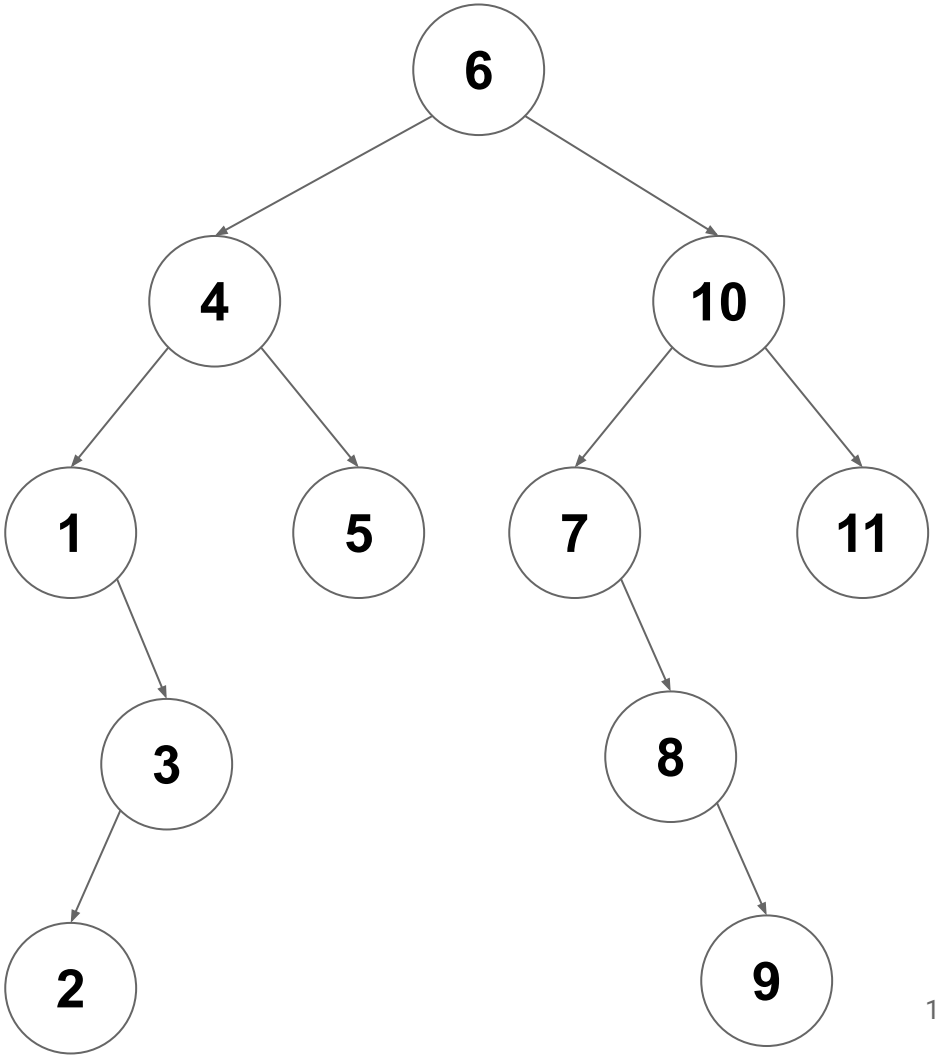
***X* partitions** its children

Is this a valid BST?



Is this a valid BST?

Yes!

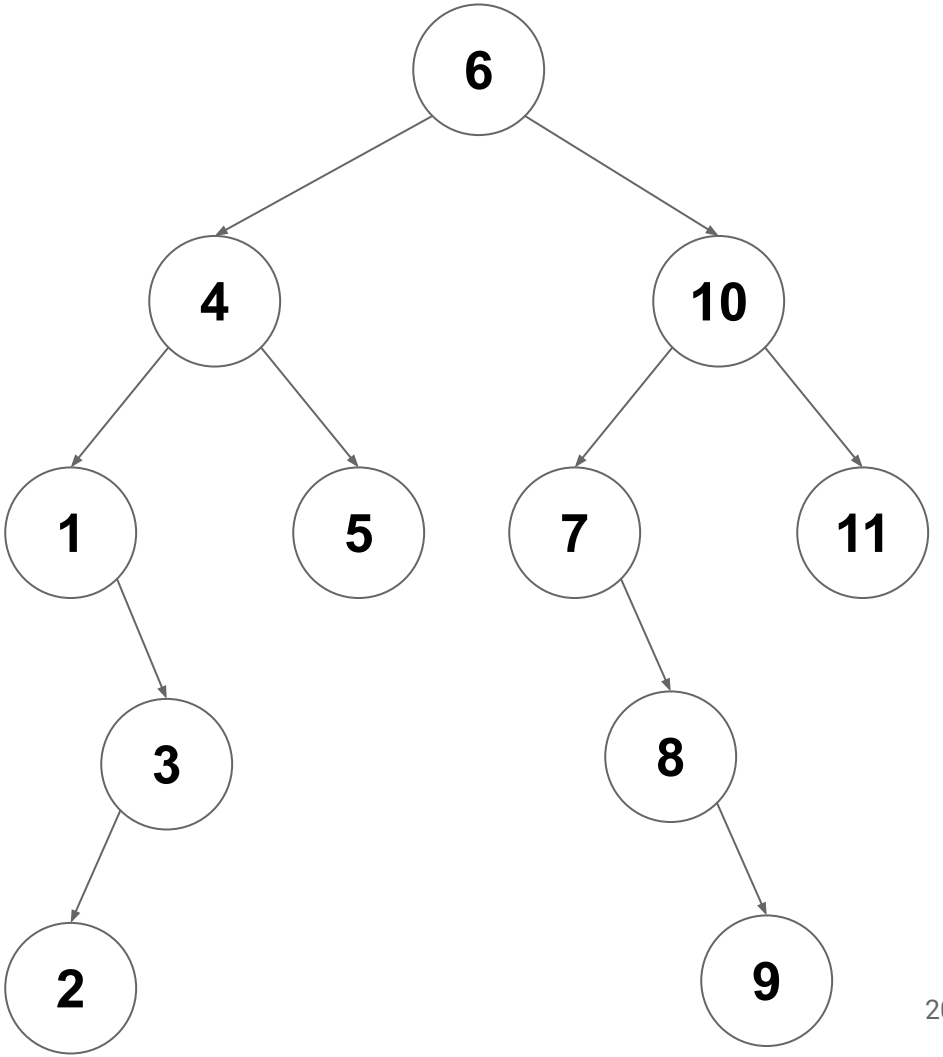


Is this a valid BST?

Yes!

Everything in the left subtree is < 6

Everything in the right subtree is > 6

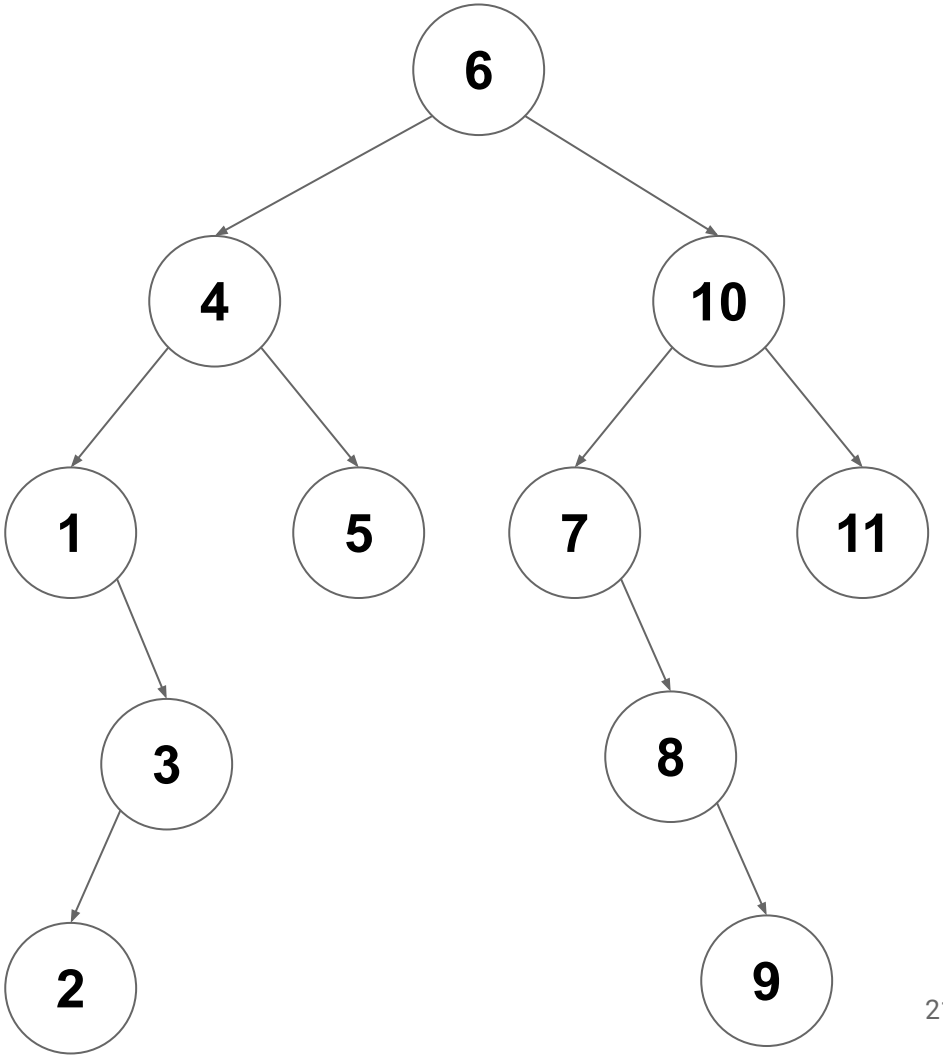


Is this a valid BST?

Yes!

AND:

The left subtree is a BST
The right subtree is a BST



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4. Is ***k*** greater than **`root.value`**'s key? (If yes, search the right subtree)

find

```
1  public Optional<TreeNode<T>> find(T target, Optional<TreeNode<T>> root) {
2    if (!root.isPresent()) return Optional.empty();
3    if (target < root.value) {
4      return find(target, root.leftChild);
5    } else if (target > root.value()) {
6      return find(target, root.rightChild);
7    } else {
8      return root;
9    }
10 }
```

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```

If the current node we are looking at is empty, then the value we are looking for is not in the tree

find

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```

Otherwise, search the BST whose root is the left child if the target is less than our current value

find

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7    } else {
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9    }
10 }
```

… or search the BST whose root is the right child if the target is greater than our current value…

find

```
1  public Optional<TreeNode<T>> find(T target, Optional<TreeNode<T>> root) {
2    if (!root.isPresent()) return Optional.empty();
3    if (target < root.value) {
4      return find(target, root.leftChild);
5    } else if (target > root.value()) {
6      return find(target, root.rightChild);
7    } else {
8      return root;
9    }
10 }
```

… or the target is equal to our current value so return our current node

find

```
1  public Optional<TreeNode<T>> find(T target, Optional<TreeNode<T>> root) {
2    if (!root.isPresent()) return Optional.empty();
3    if (target < root.value) {
4      return find(target, root.leftChild);
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```

What's the complexity?

find

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```

*What's the complexity? (how many times do we call **find**)?*

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1  public Optional<TreeNode<T>> find(T target, Optional<TreeNode<T>> root) {
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9    }
10 }
```

What's the complexity? (how many times do we call **`find`***)?* ***O(d)***

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3. Is ***k*** less than **`root.value`**'s key? (call insert on left subtree)
4. Is ***k*** greater than **`root.value`**'s key? (call insert on right subtree)

```java
public TreeNode<T> insert(T target, TreeNode<T> root) {
  if (target < root.value) {
    if (root.leftChild.isPresent()) {
      return insert(target, root.leftChild.get());
    } else {
      root.leftChild = Optional.of(new TreeNode<>(target));
      return root.leftChild.get();
    }
  } else if (target > root.value) {
    if (root.rightChild.isPresent()) {
      return insert(target, root.rightChild.get());
    } else {
      root.rightChild = Optional.of(new TreeNode<>(target));
      return root.rightChild.get();
    }
  } else {
    return root; // The value is already in the tree
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8      }
9    } else if (target > root.value) {
10     if (root.rightChild.isPresent()) {
11       return insert(target, root.rightChild.get());
12     } else {
13       root.rightChild = Optional.of(new TreeNode<>(target));
14       return root.rightChild.get();
15     }
16   } else {
17     return root; // The value is already in the tree
18 }}
```

If the target is smaller than our current root and the left subtree exists, insert into the left subtree

```
1  public TreeNode<T> insert(T target, TreeNode<T> root) {
2    if (target < root.value) {
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```

If the target is smaller than our current root and the left subtree does not exist, insert the new node as the left subtree

```
1  public TreeNode<T> insert(T target, TreeNode<T> root) {
2    if (target < root.value) {
3      if (root.leftChild.isPresent()) {
4        return insert(target, root.leftChild.get());
5      } else {
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7        return root.leftChild.get();
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```

Repeat the same logic for the right tree

```
1  public TreeNode<T> insert(T target, TreeNode<T> root) {
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```

Otherwise the value already exists so don't insert

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Goal: Insert a new item with key ***k*** in a BST rooted at **`root`**

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3. Is ***k*** less than **`root.value`**'s key? (call insert on left subtree)
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What is the complexity?

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4. Is ***k*** greater than **`root.value`**'s key? (call insert on right subtree)

What is the complexity? ***O*(*d*)**

Remove

Goal: Remove the item with key ***k*** from a BST rooted at **`root`**

1. **`find`** the item
2. Replace the found node with the right subtree
3. Insert the left subtree under the right

We'll look at this in more detail later, but for now…

What's the complexity?

Remove

Goal: Remove the item with key ***k*** from a BST rooted at **`root`**

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What's the complexity? ***O*(*d*)**

BST Operations

Operation	Runtime
`find`	$O(d)$
`insert`	$O(d)$
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Can we do better?

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What is the runtime in terms of n*?* $O(n)$

What about the lower bound? $\Omega(\log(n))$

Can we do better? **TBD...**

The Set ADT

`void add(T element)`

Store one copy of **`element`** if not already present

`boolean contains(T element)`

Return true if **`element`** is present in the set

`boolean remove(T element)`

Remove **`element`** if present, or return false if not

The Set ADT

`void add(T element)` → **`BST.insert(...) in O(n)`**

Store one copy of **`element`** if not already present

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Return true if **`element`** is present in the set

`boolean remove(T element)` → **`BST.remove(...) in O(n)`**

Remove **`element`** if present, or return false if not

Sets and Bags

So we could use this specification of a BST to implement a Set

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What about bags? How could we change our BST to implement a Bag?

Idea 1: Allow multiple copies ($X_L \leq X$ instead of $<$)

Idea 2: Only store one copy of each element, but also store a count

Collection ADTs

Property	Sequence	List	Set	Bag
Explicit Order	✓	✓		
Enforced Uniqueness			✓	
Fixed Size	✓			
Iterable	✓	✓	✓	✓

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Fixed Size	✓			
Iterable	✓	✓	✓	✓

Tree Traversals

Goal: Visit every element of a tree (in linear time?)

Pre-Order (top-down)

Visit the `root`, then the `left` subtree, then the `right` subtree

In-Order

Visit the `left` subtree, then the `root`, then the `right` subtree

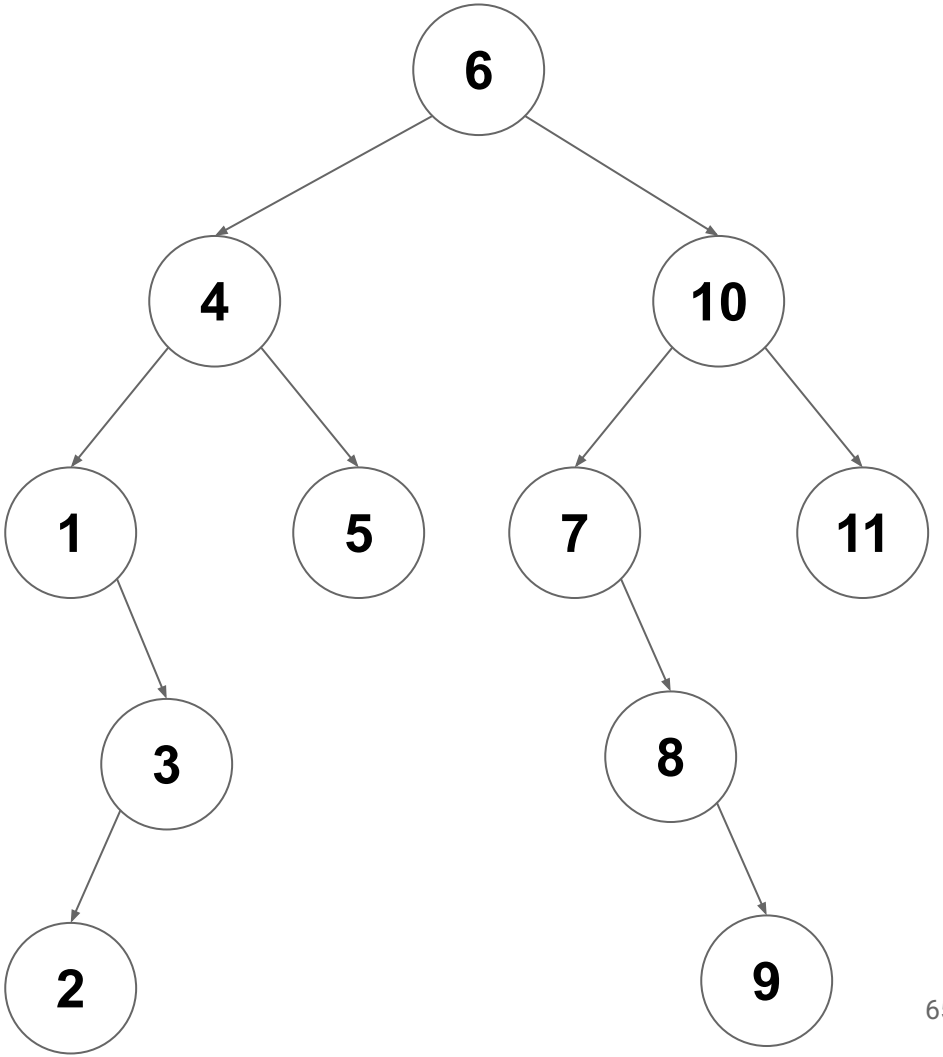
Post-Order (bottom-up)

Visit the `left` subtree, then the `right` subtree, then the `root`

Tree Traversal: In-Order

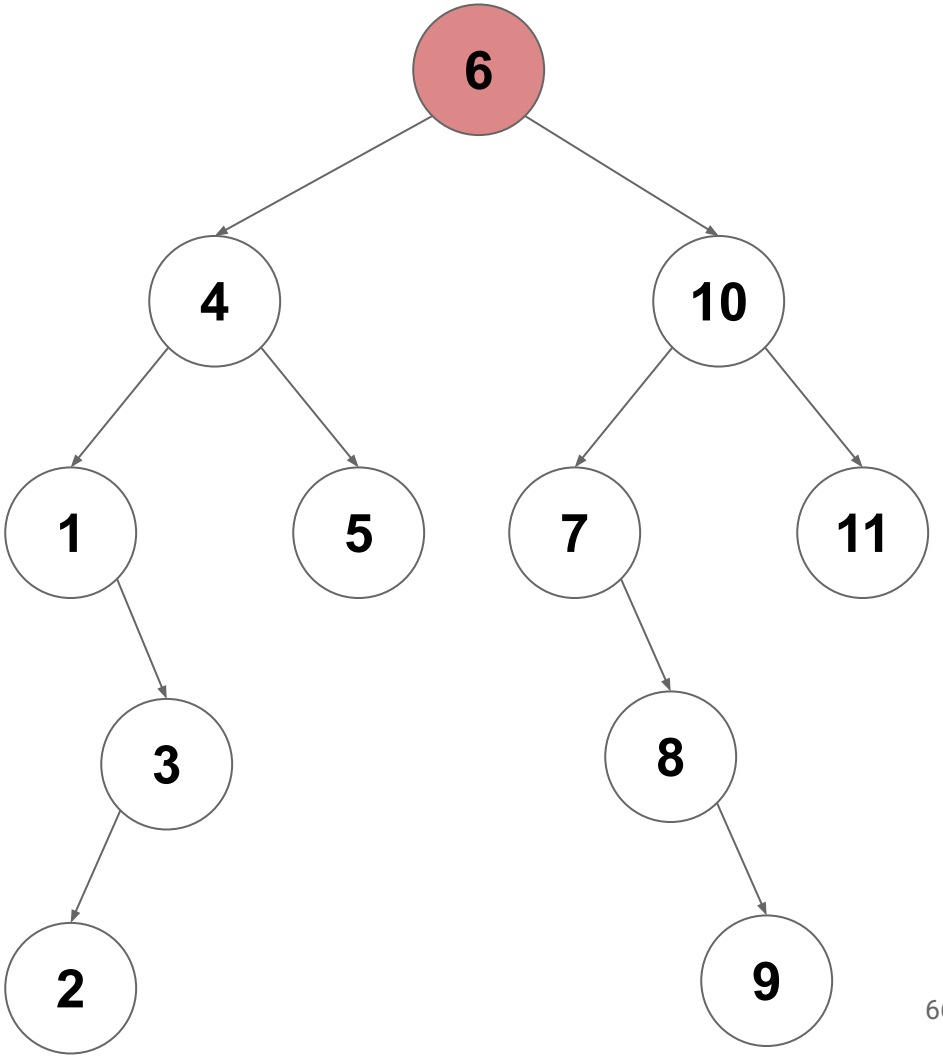
```
1  void inOrderVisit(Optional<TreeNode<T>> root, Visitor v) {
2    if (root.isPresent()) {
3      inOrderVisit(root.get().leftChild, v);
4      v.visit(root.get().value);
5      inOrderVisit(root.get().rightChild, v);
6    }
7  }
```

In-Order Traversal on a BST



In-Order Traversal on a BST

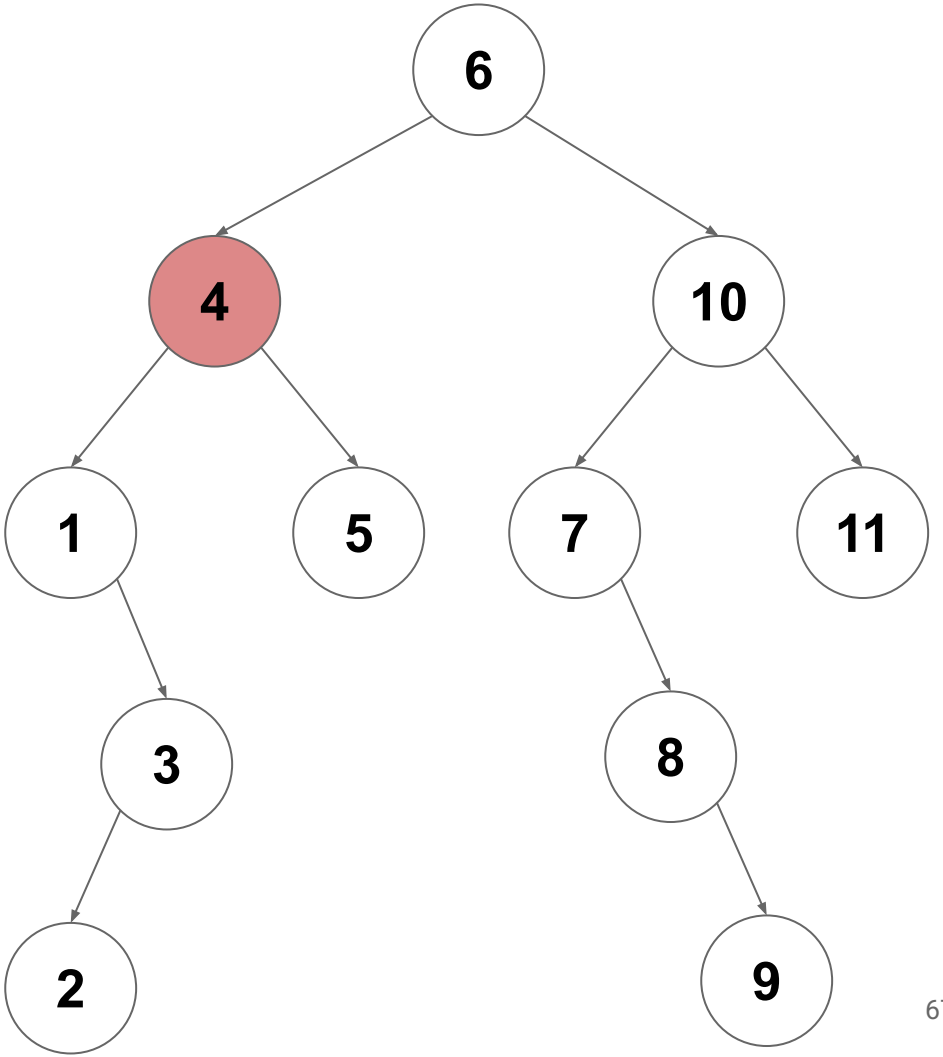
`inorderVisit(6)`



In-Order Traversal on a BST

```
inorderVisit(6)

inorderVisit(4)
```



In-Order Traversal on a BST

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inorderVisit(1)
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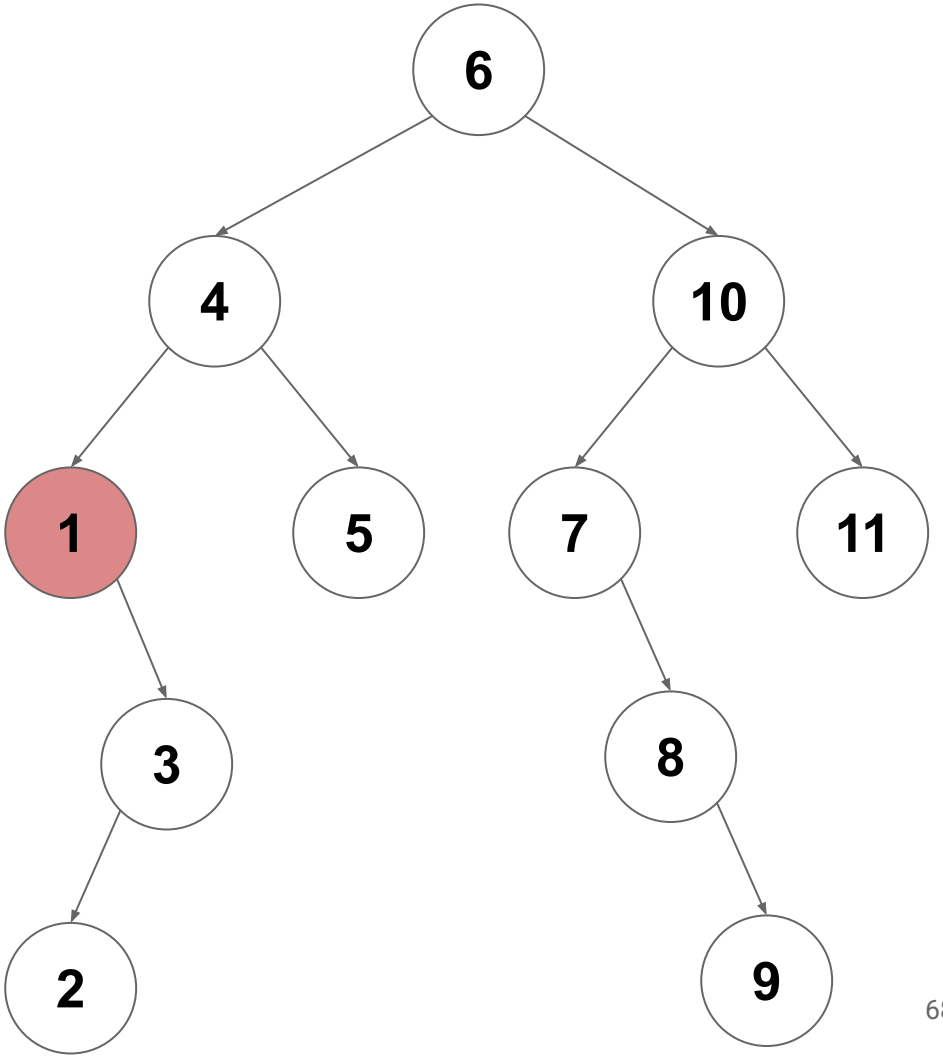


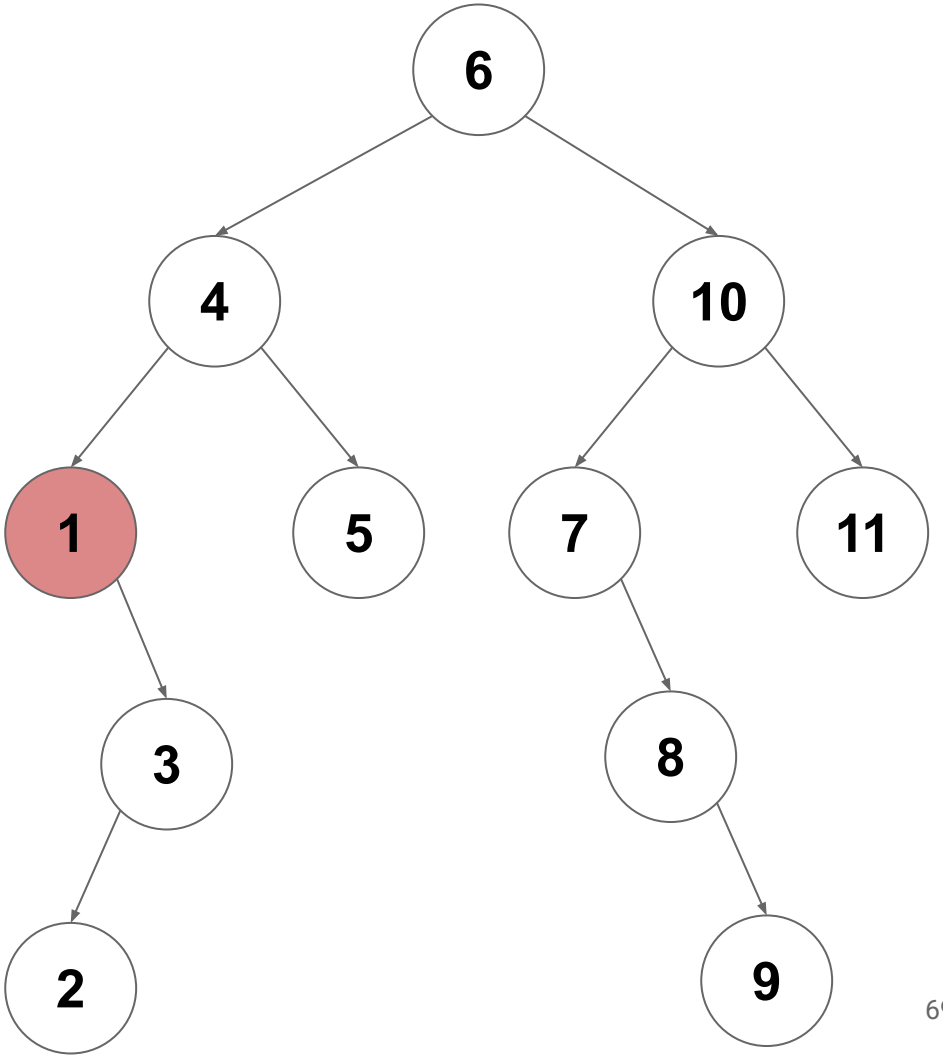
In-Order Traversal on a BST

```
inorderVisit(6)

inorderVisit(4)

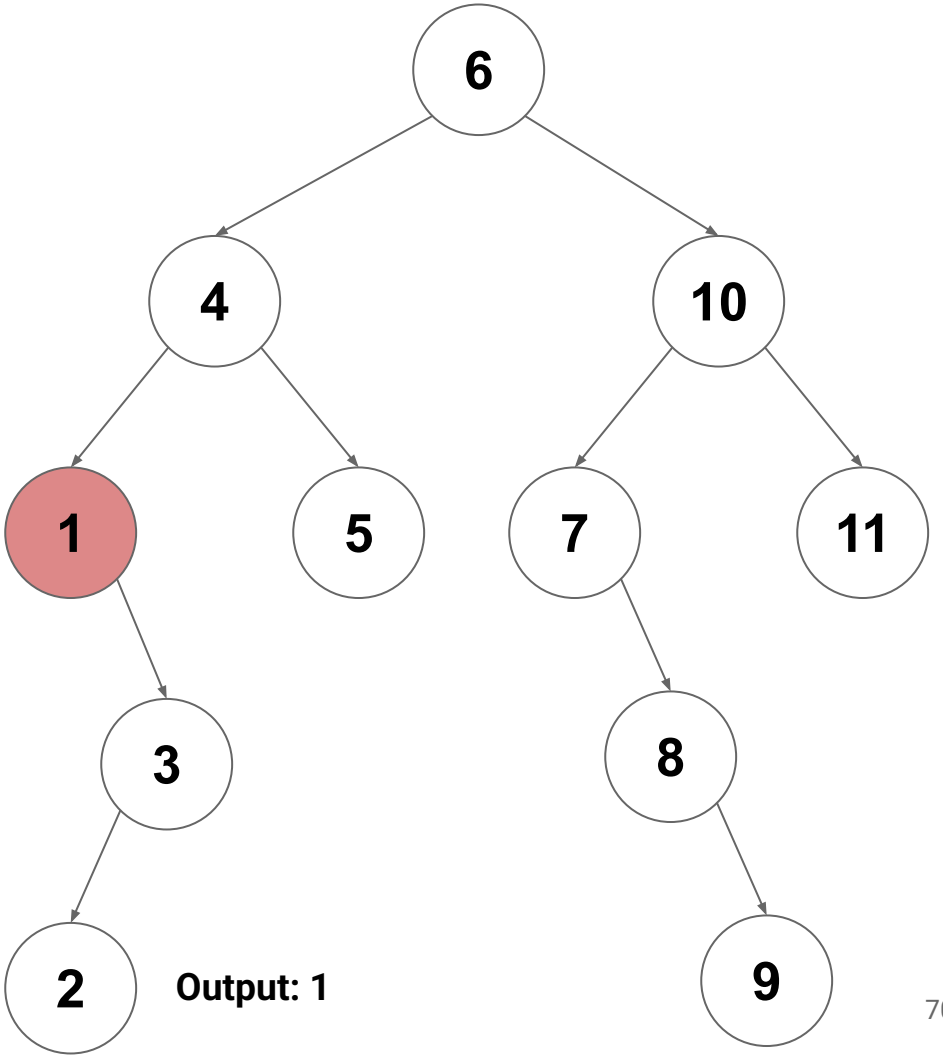
inorderVisit(1)

inorderVisit(empty)
```



In-Order Traversal on a BST

```
inorderVisit(6)
inorderVisit(4)
inorderVisit(1)
visit(1)
```



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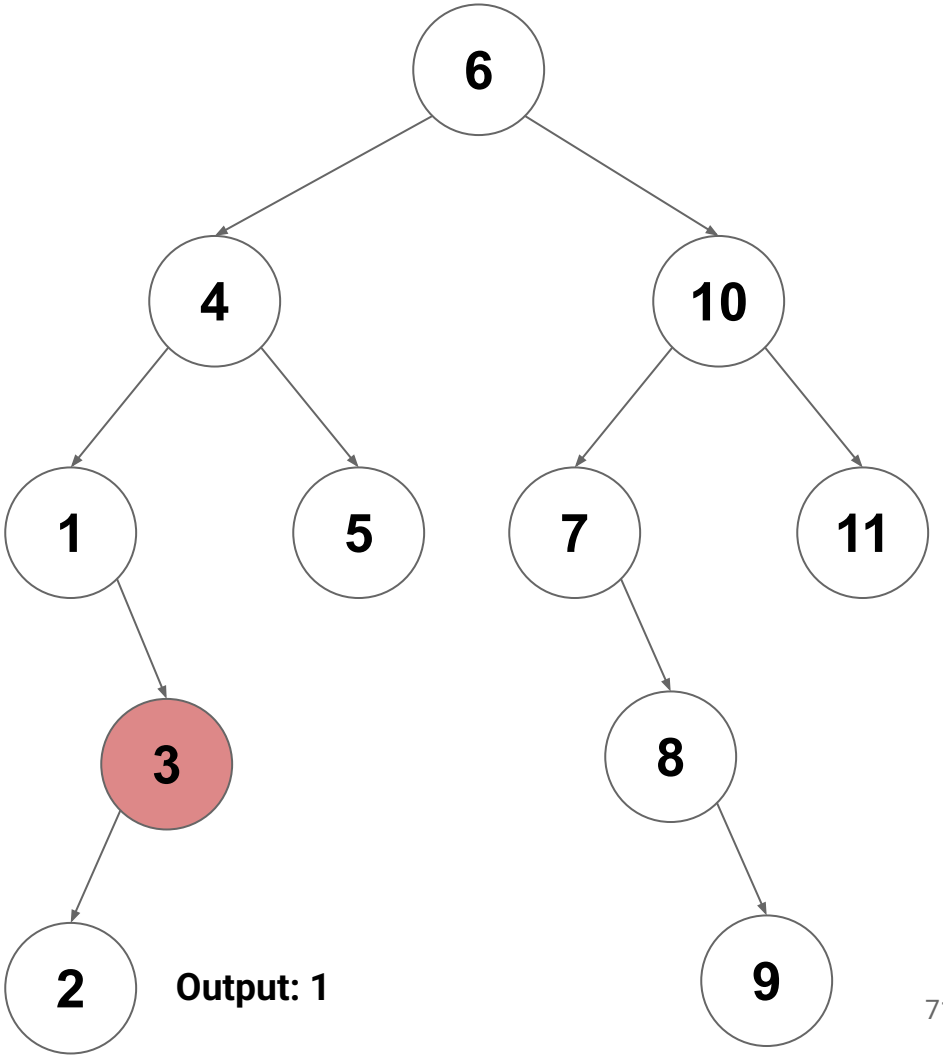

In-Order Traversal on a BST

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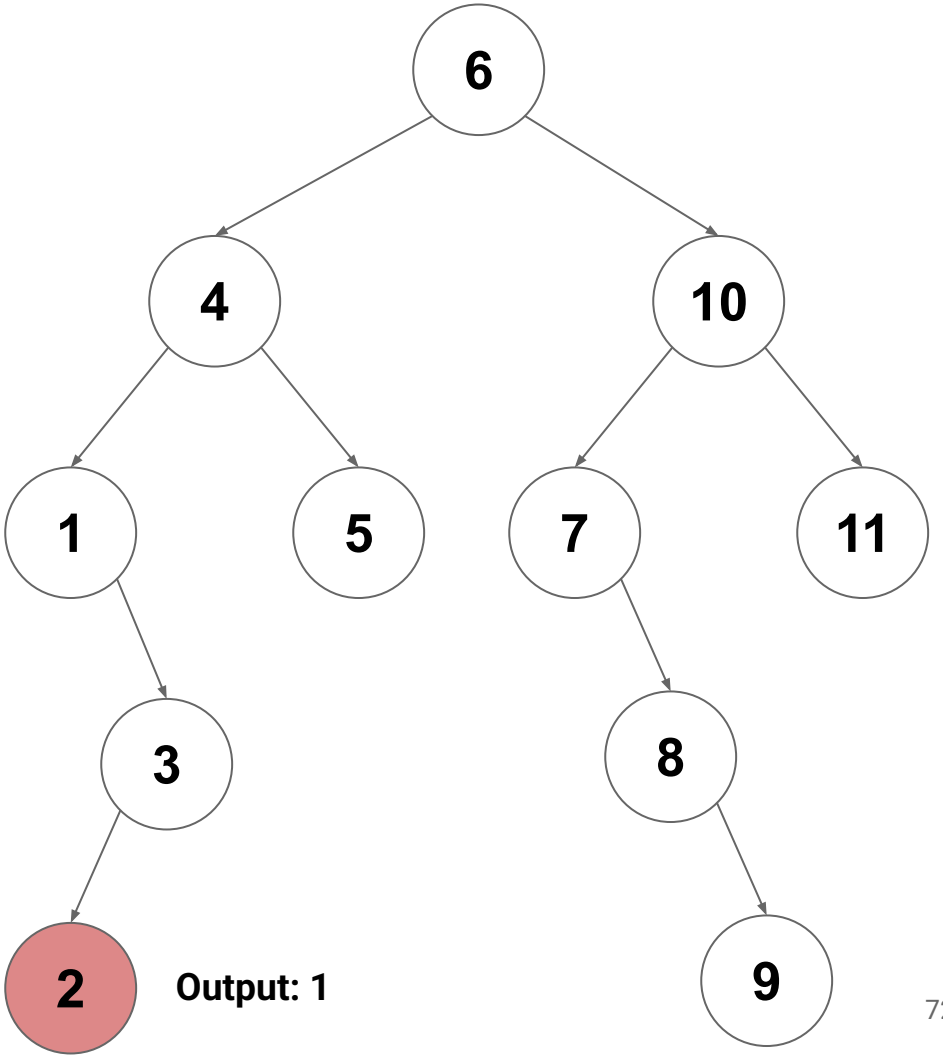
inorderVisit(6)
inorderVisit(4)
inorderVisit(1)
inorderVisit(3)
inorderVisit(2)
```



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visit(2)
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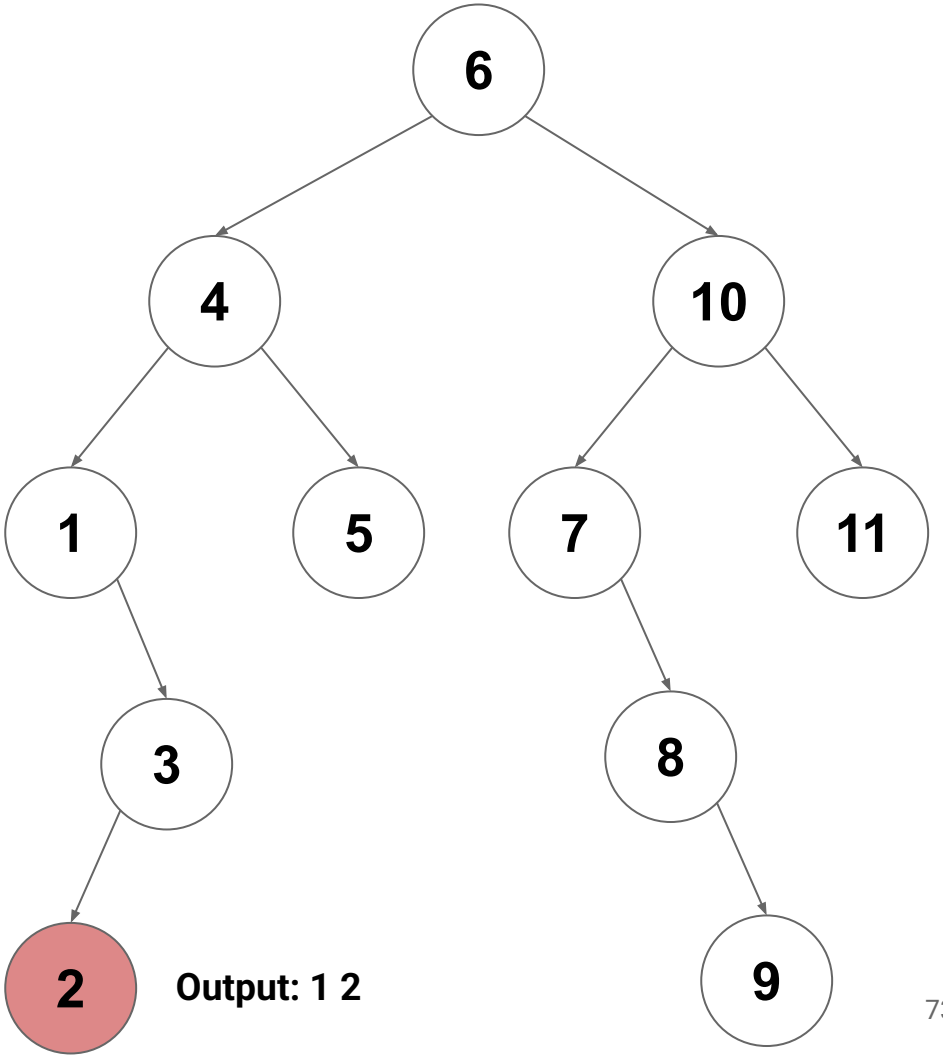
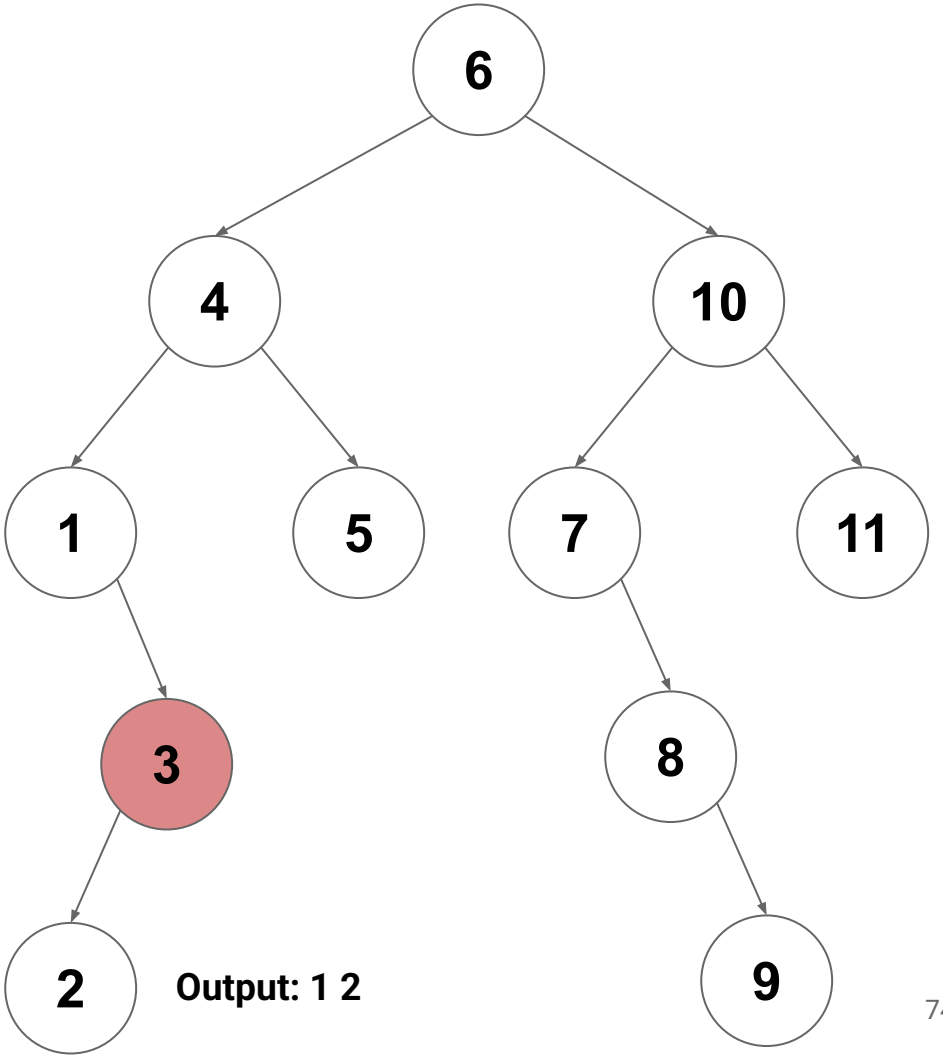


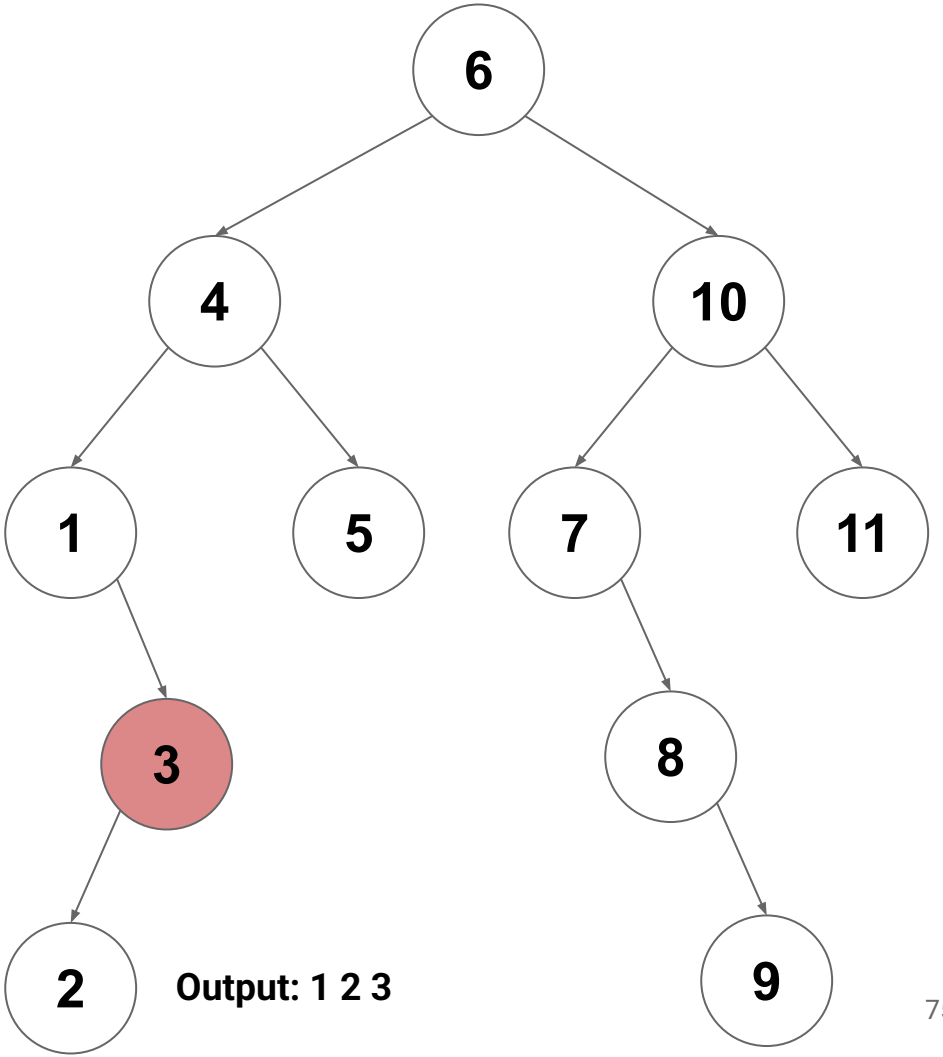
In-Order Traversal on a BST

```
inorderVisit(6)
inorderVisit(4)
inorderVisit(1)
inorderVisit(3)
```



In-Order Traversal on a BST

```
inorderVisit(6)
inorderVisit(4)
inorderVisit(1)
visit(3)
```

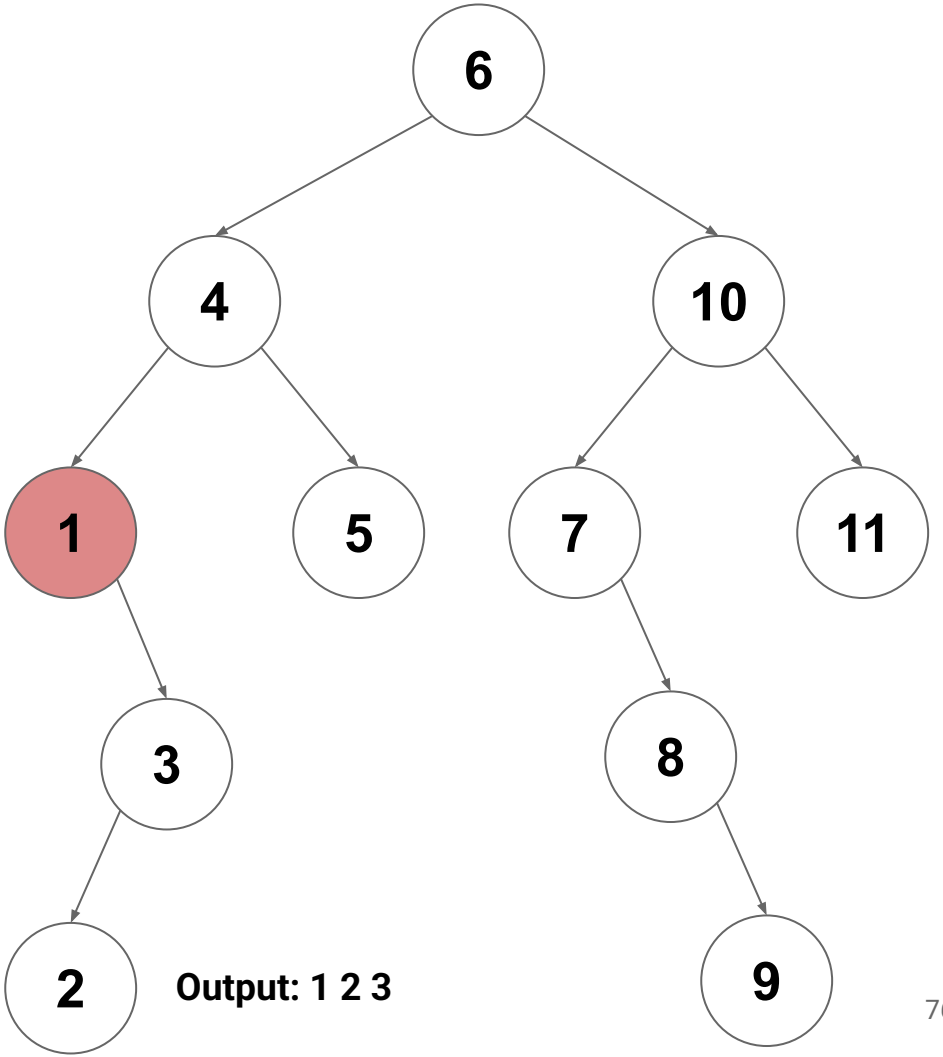


In-Order Traversal on a BST

```
inorderVisit(6)

inorderVisit(4)

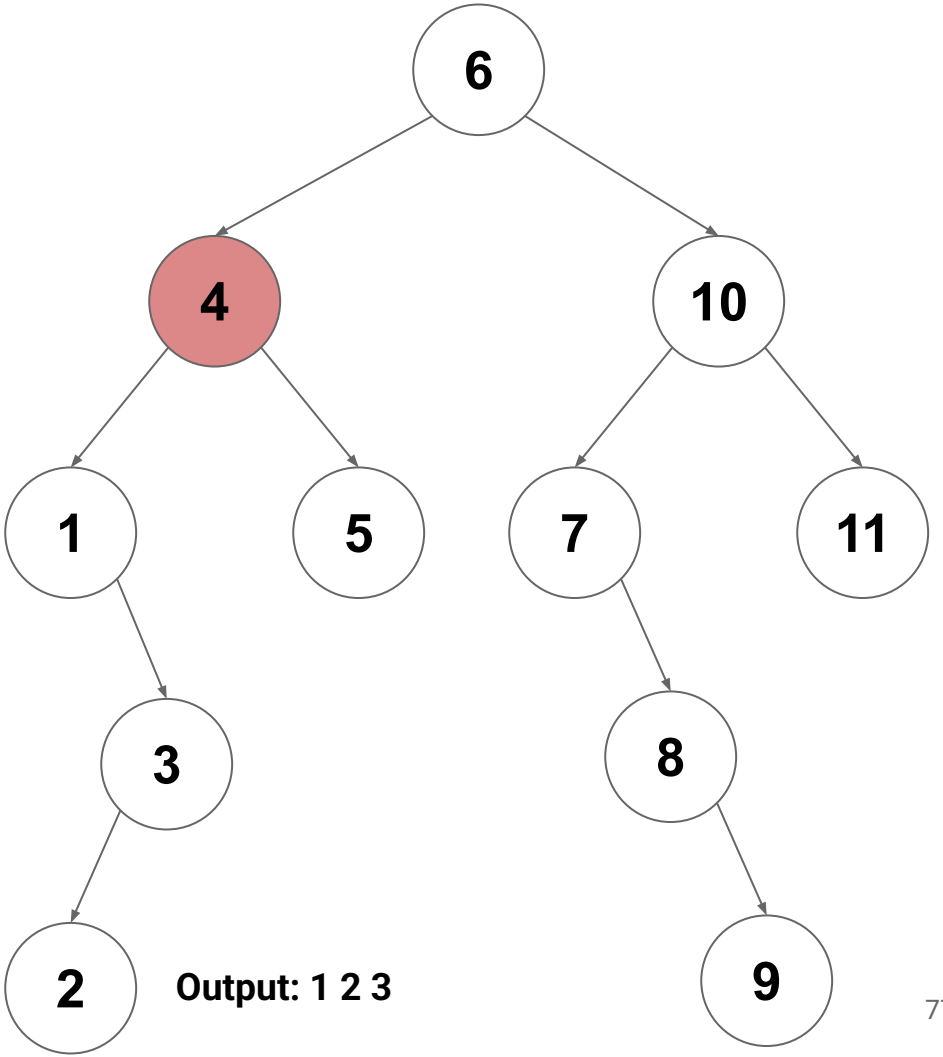
inorderVisit(1)
```



In-Order Traversal on a BST

```
inorderVisit(6)

inorderVisit(4)
```

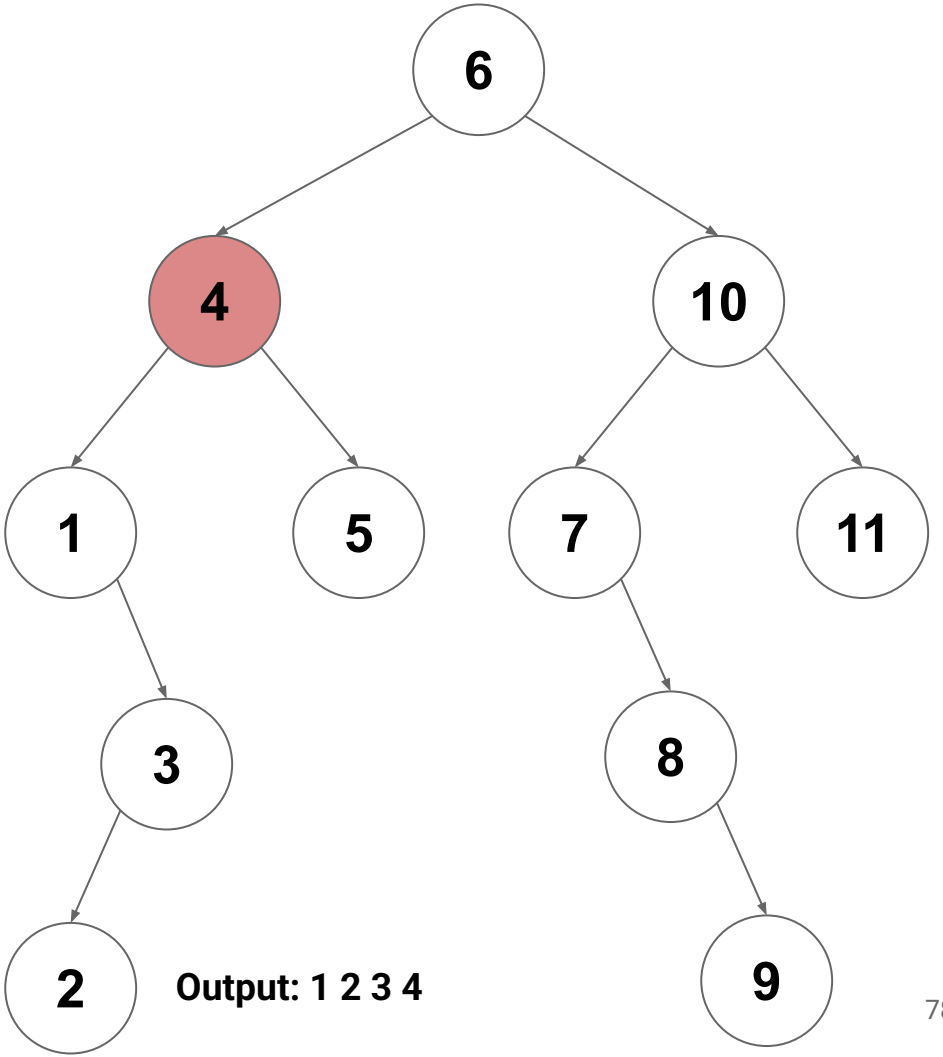


In-Order Traversal on a BST

```
inorderVisit(6)

inorderVisit(4)

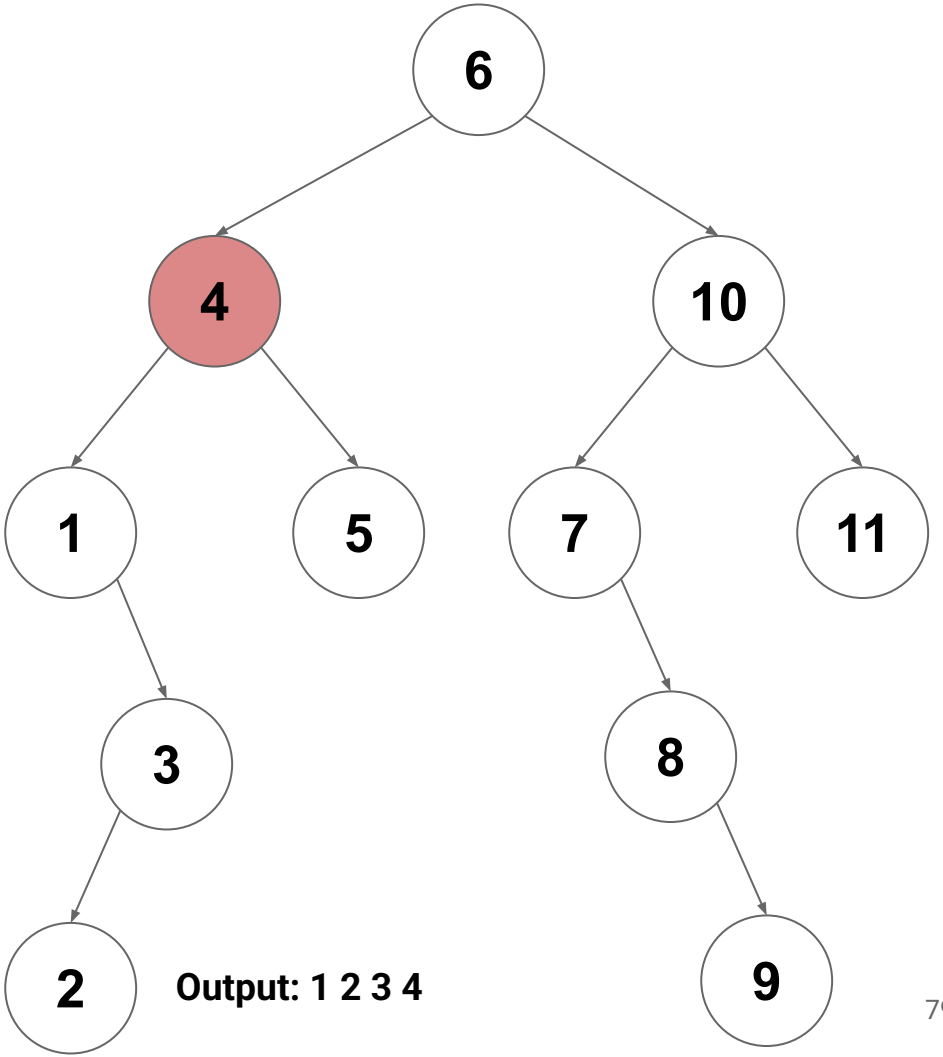
visit(4)
```



In-Order Traversal on a BST

```
inorderVisit(6)

inorderVisit(4)
```

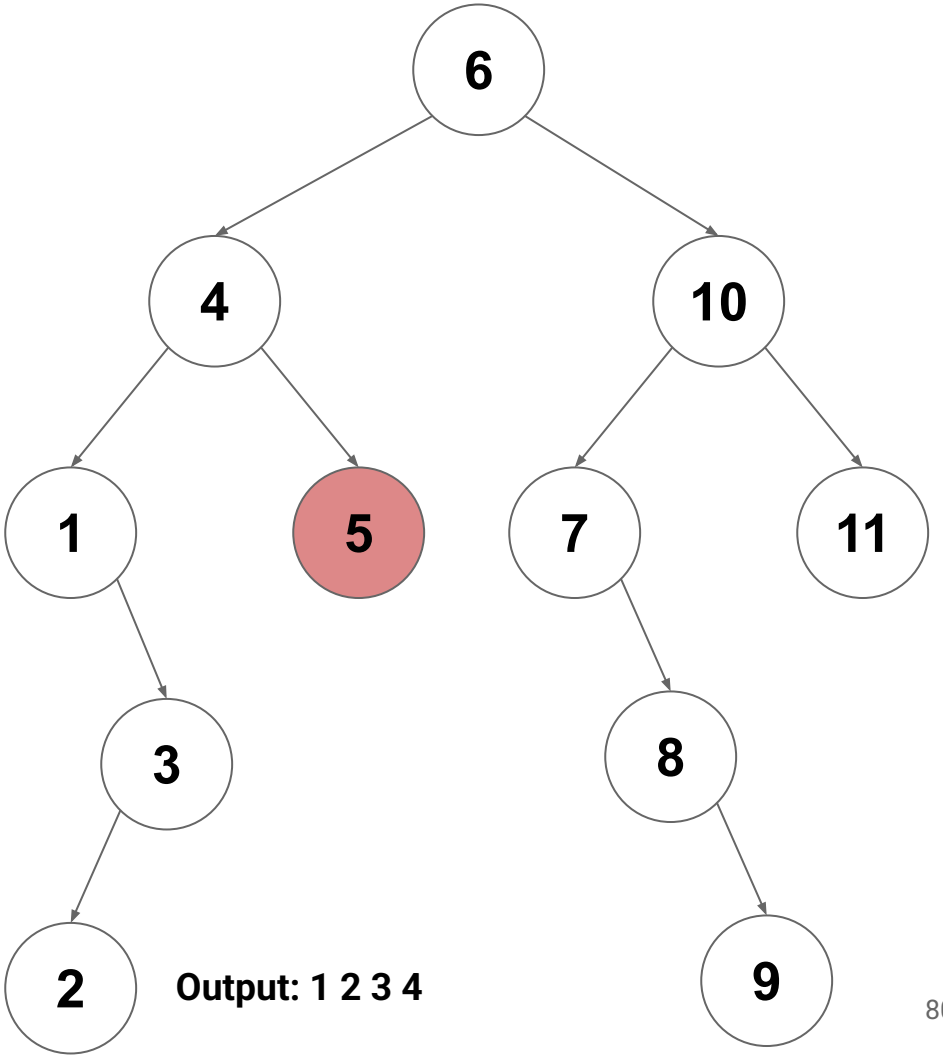


In-Order Traversal on a BST

```
inorderVisit(6)

inorderVisit(4)

inorderVisit(5)
```

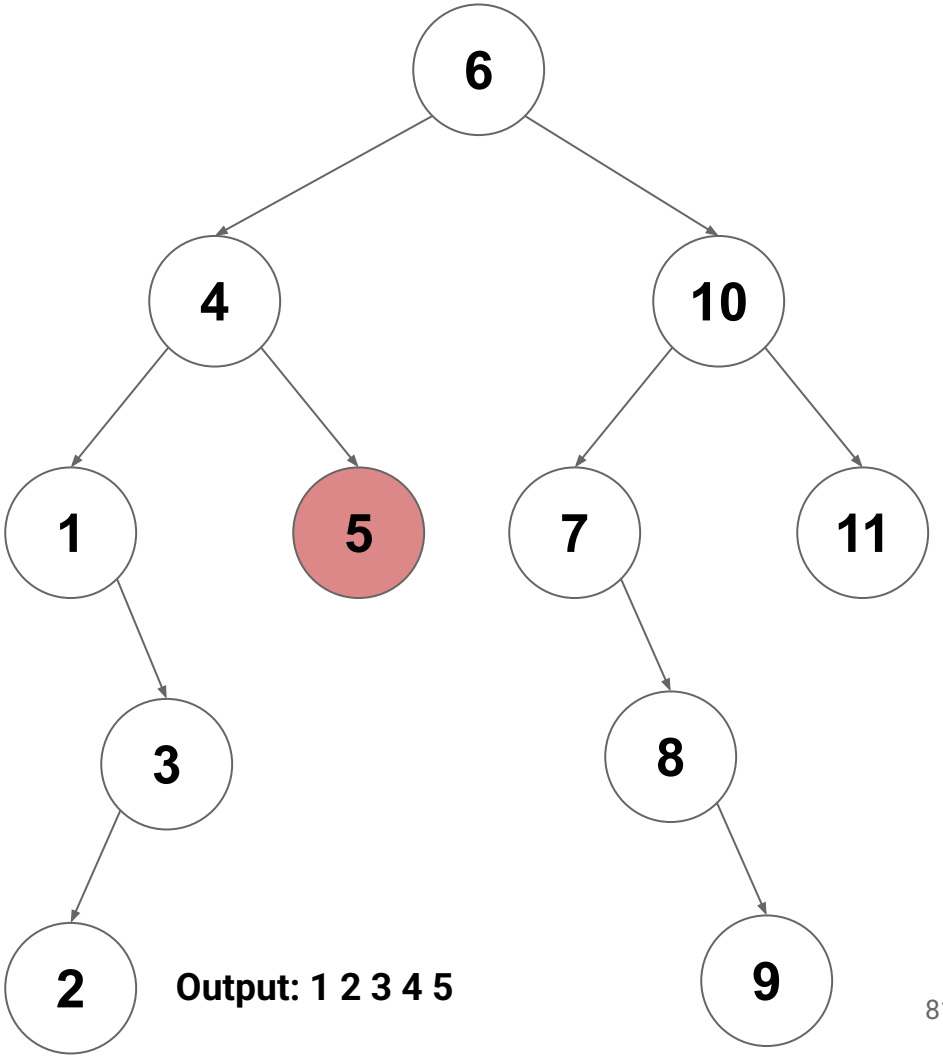


In-Order Traversal on a BST

```
inorderVisit(6)

inorderVisit(4)

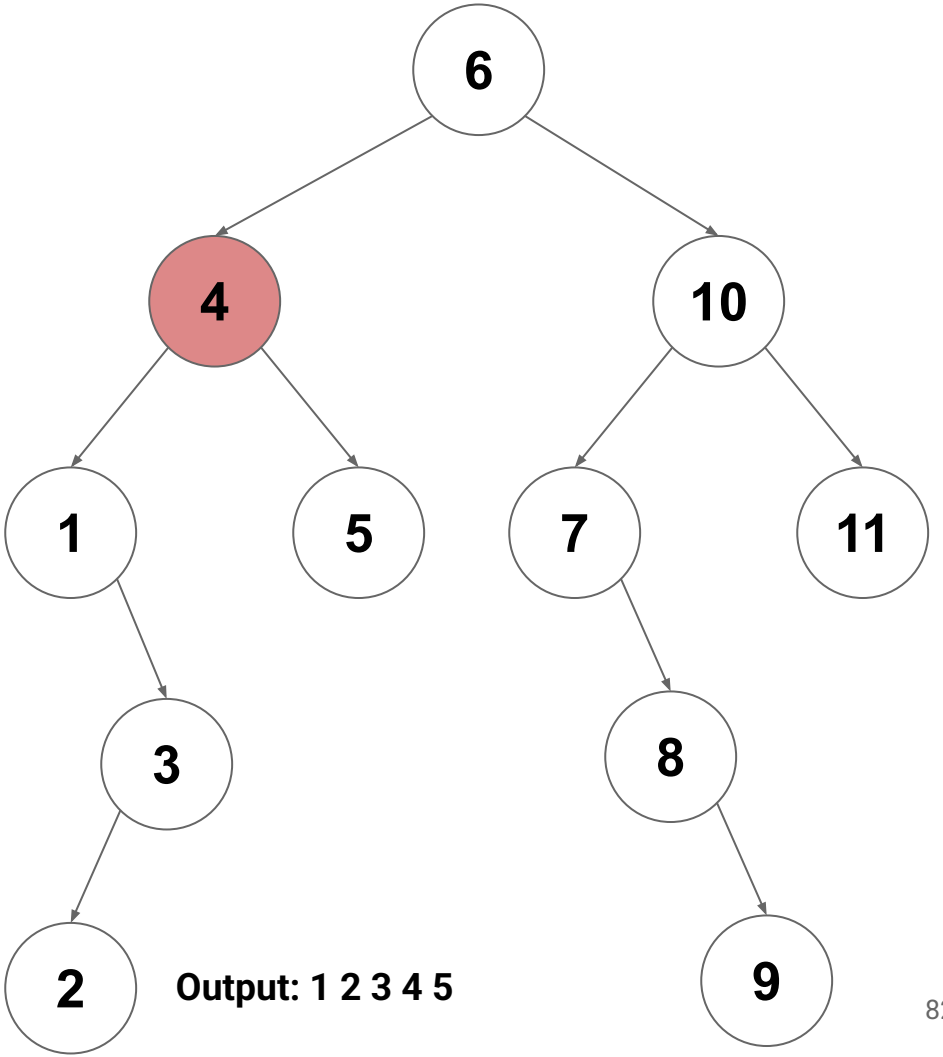
visit(5)
```



In-Order Traversal on a BST

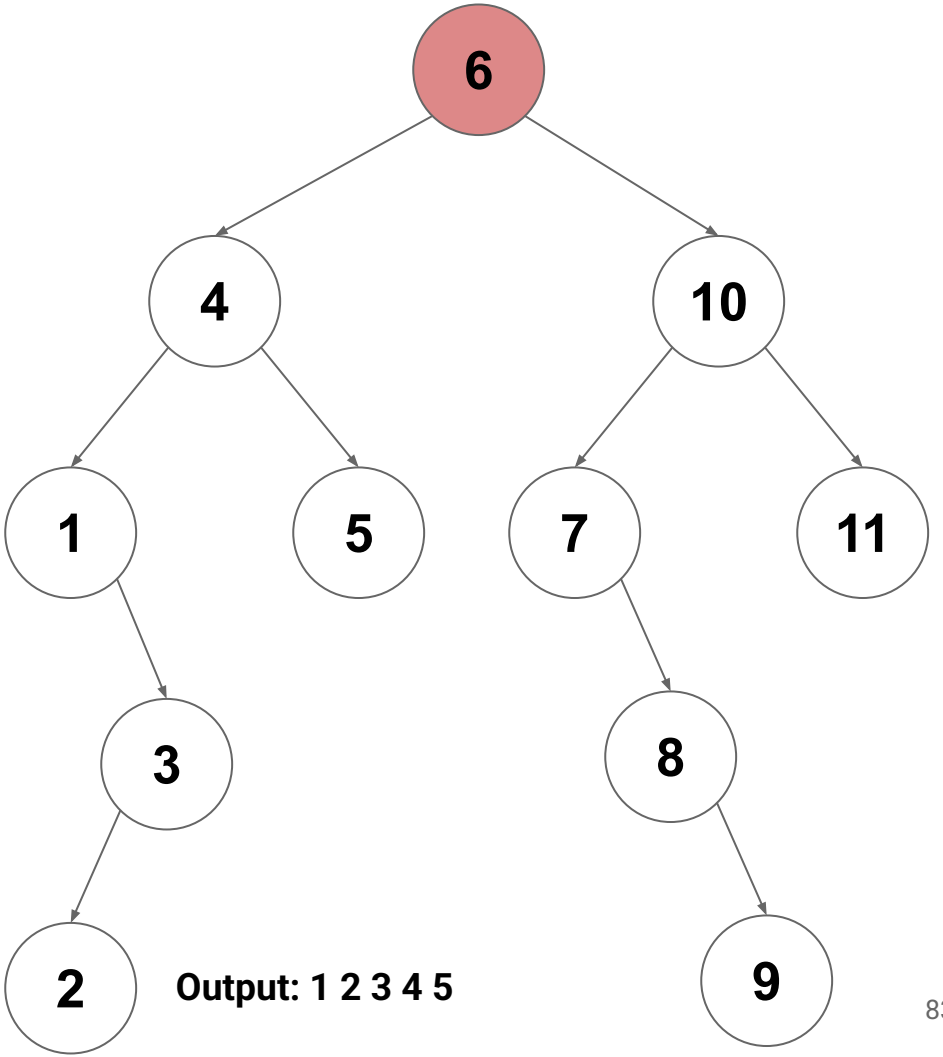
```
inorderVisit(6)

inorderVisit(4)
```



In-Order Traversal on a BST

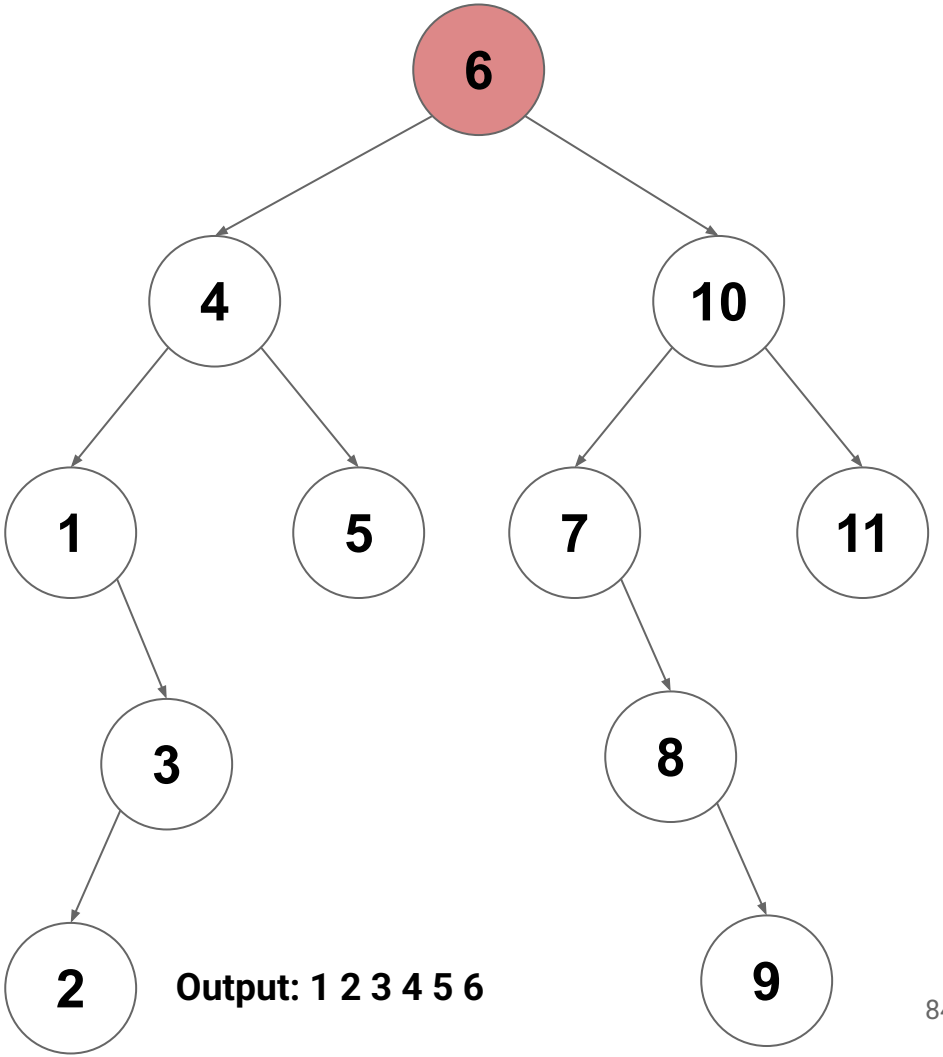
`inorderVisit(6)`



In-Order Traversal on a BST

```
inorderVisit(6)

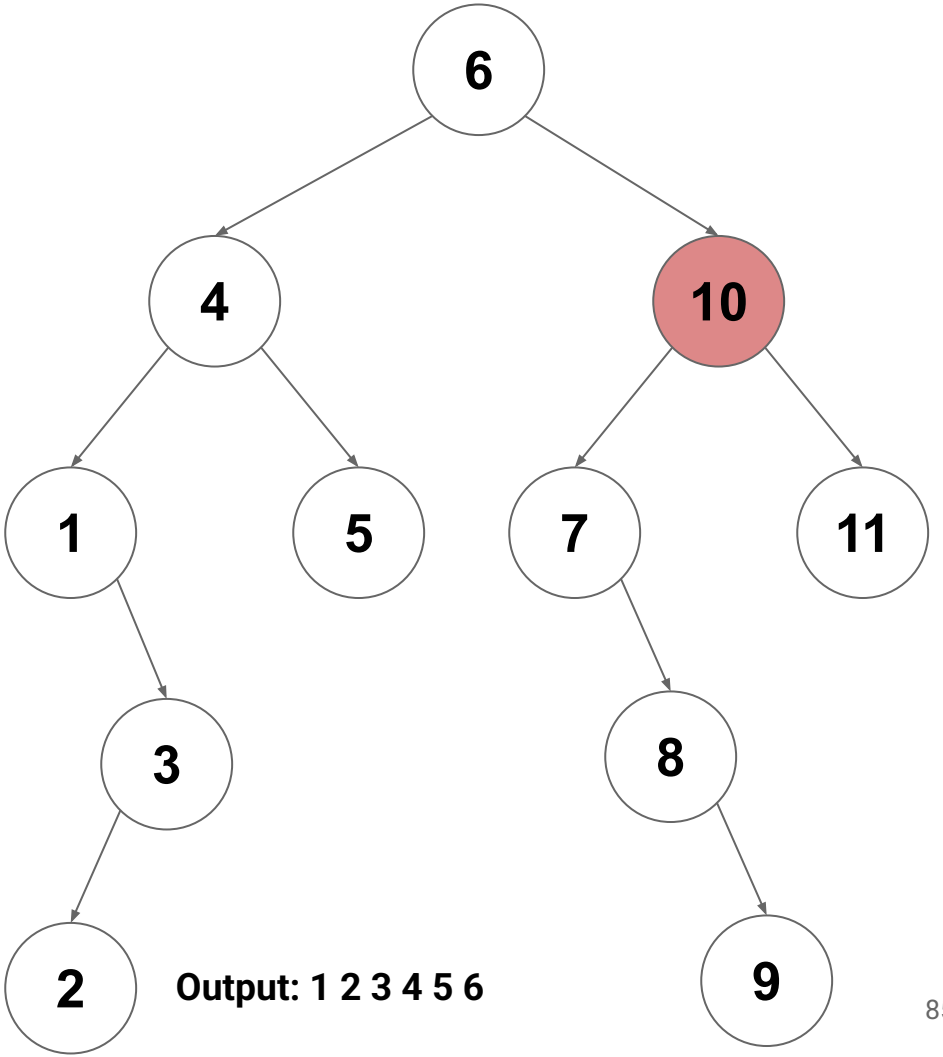
visit(6)
```



In-Order Traversal on a BST

```
inorderVisit(6)

inorderVisit(10)
```

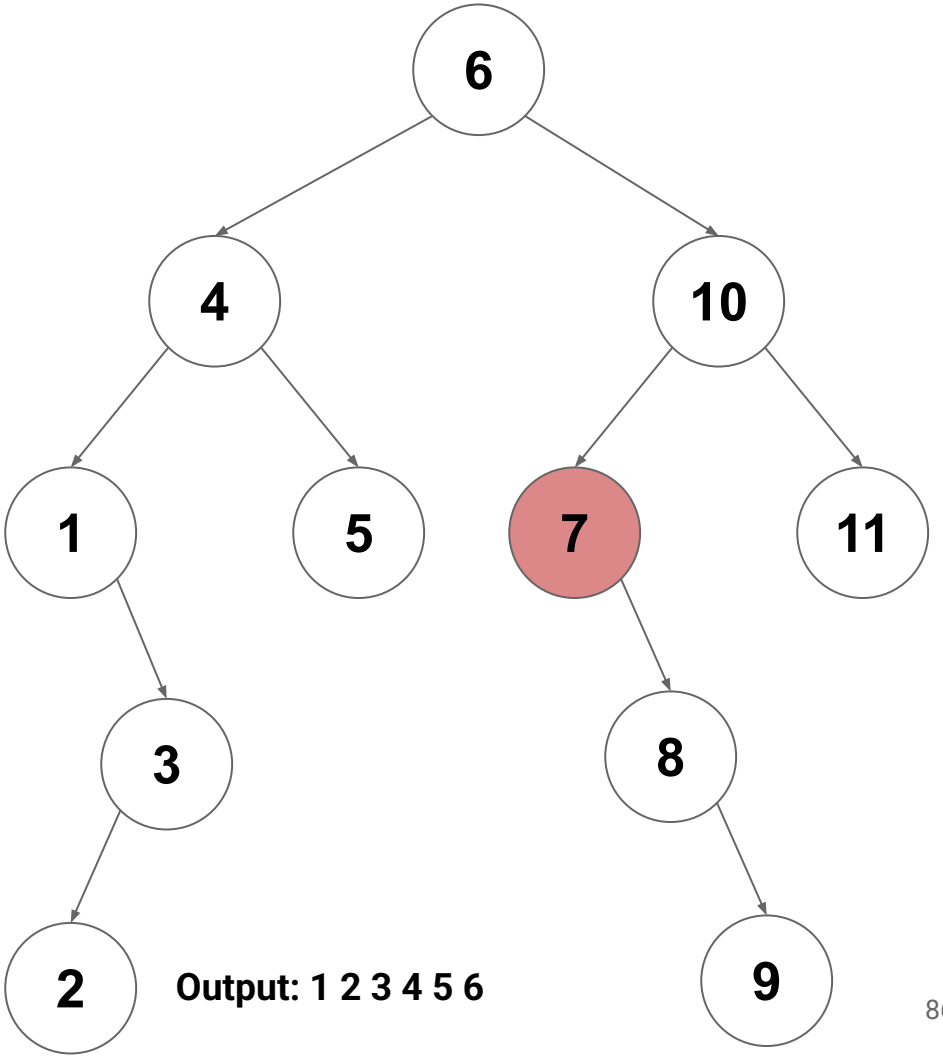


In-Order Traversal on a BST

```
inorderVisit(6)

inorderVisit(10)

inorderVisit(7)
```



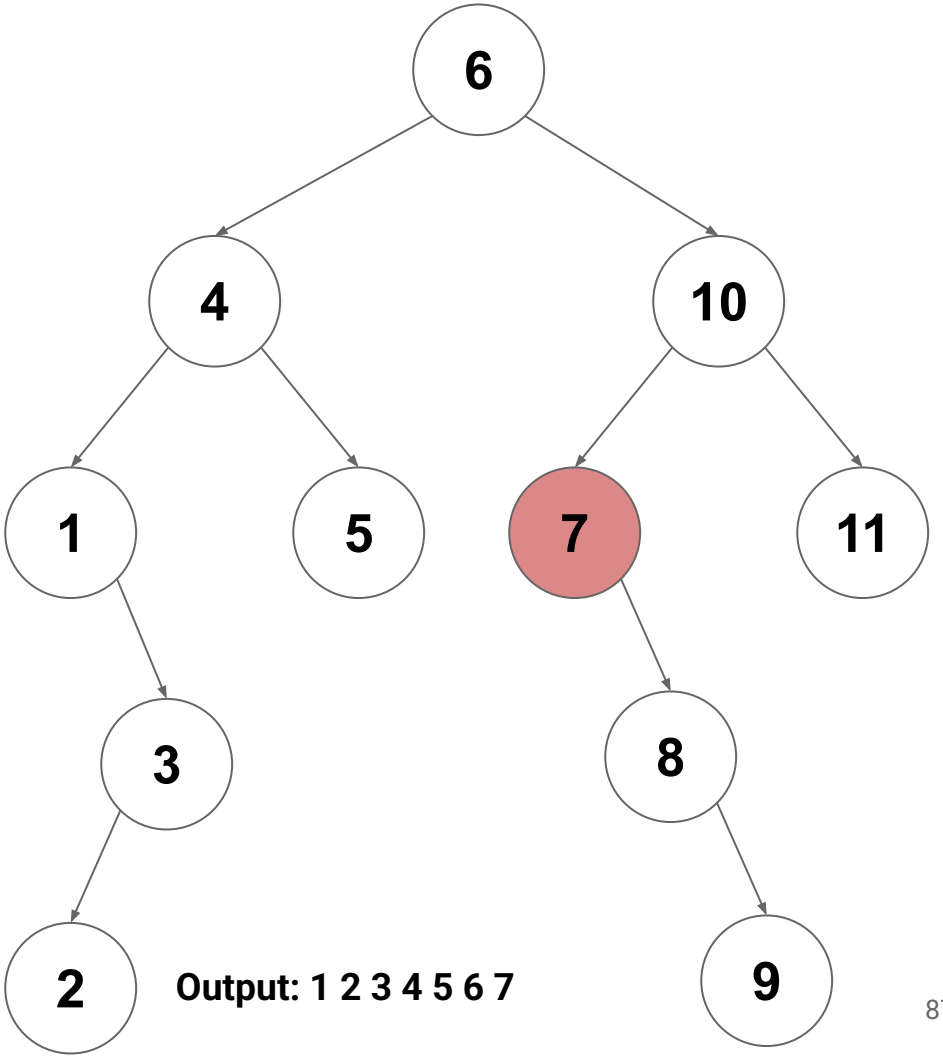
In-Order Traversal on a BST

```
inorderVisit(6)

inorderVisit(10)

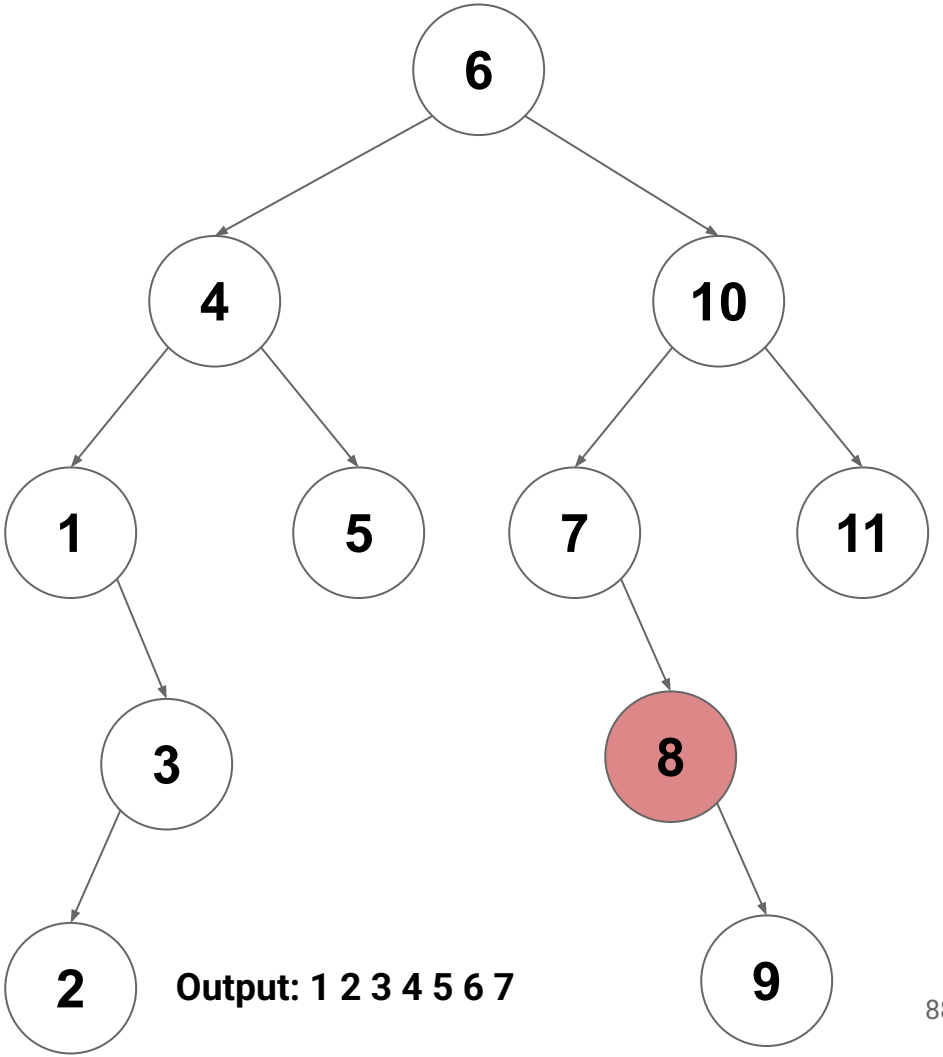
inorderVisit(7)

visit(7)
```



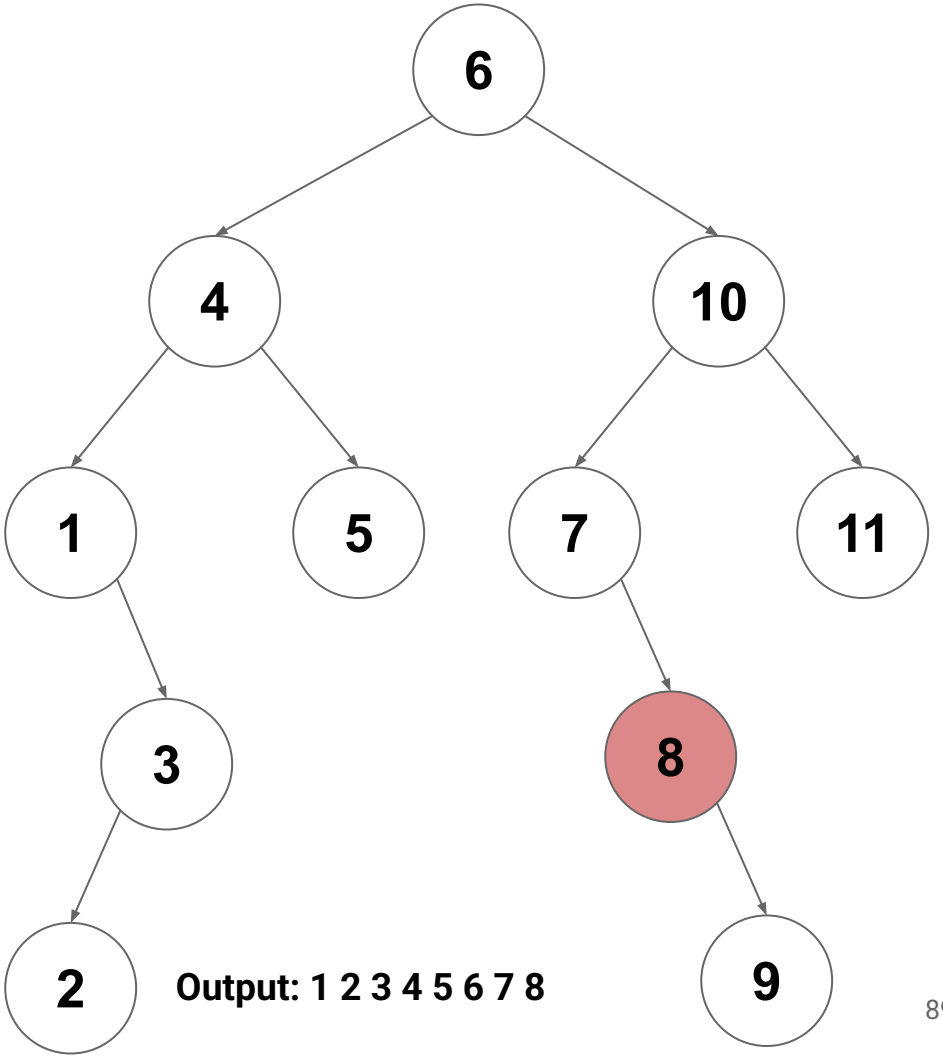
In-Order Traversal on a BST

```
inorderVisit(6)
inorderVisit(10)
inorderVisit(7)
inorderVisit(8)
```



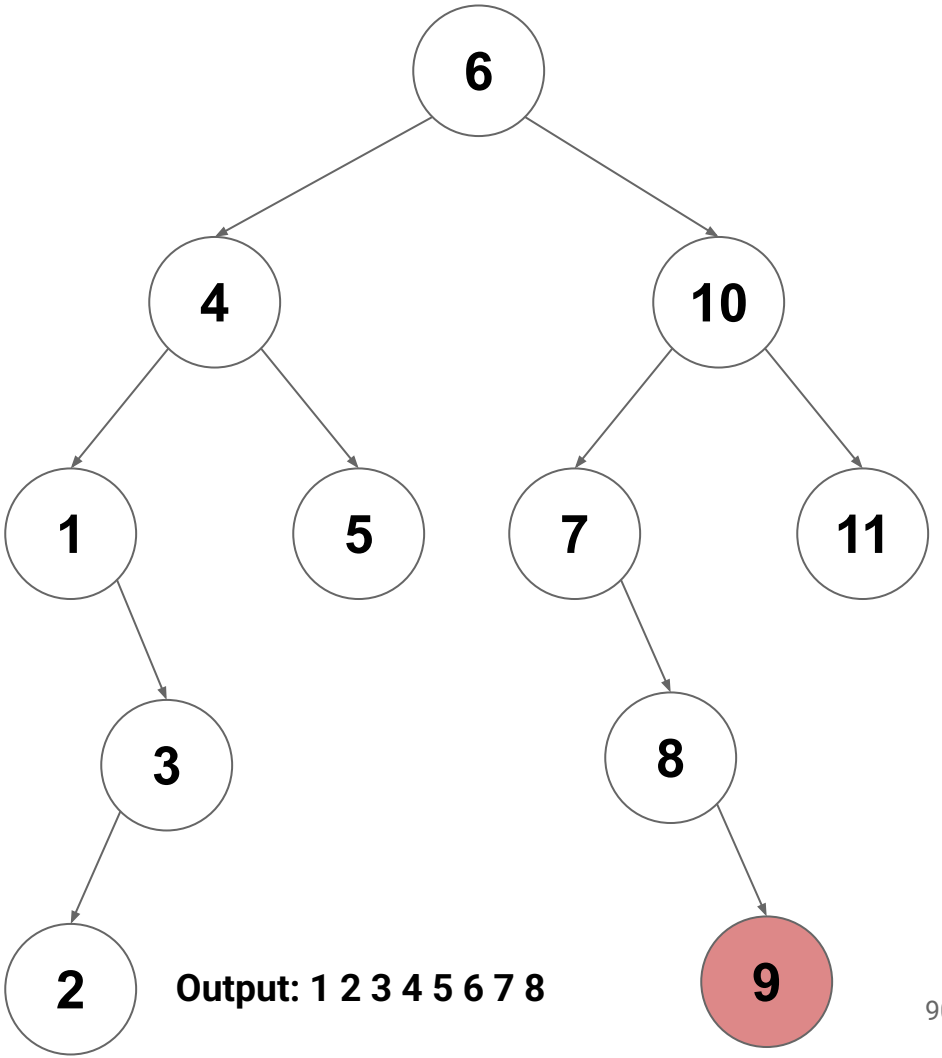
In-Order Traversal on a BST

```
inorderVisit(6)
inorderVisit(10)
inorderVisit(7)
inorderVisit(8)
visit(8)
```



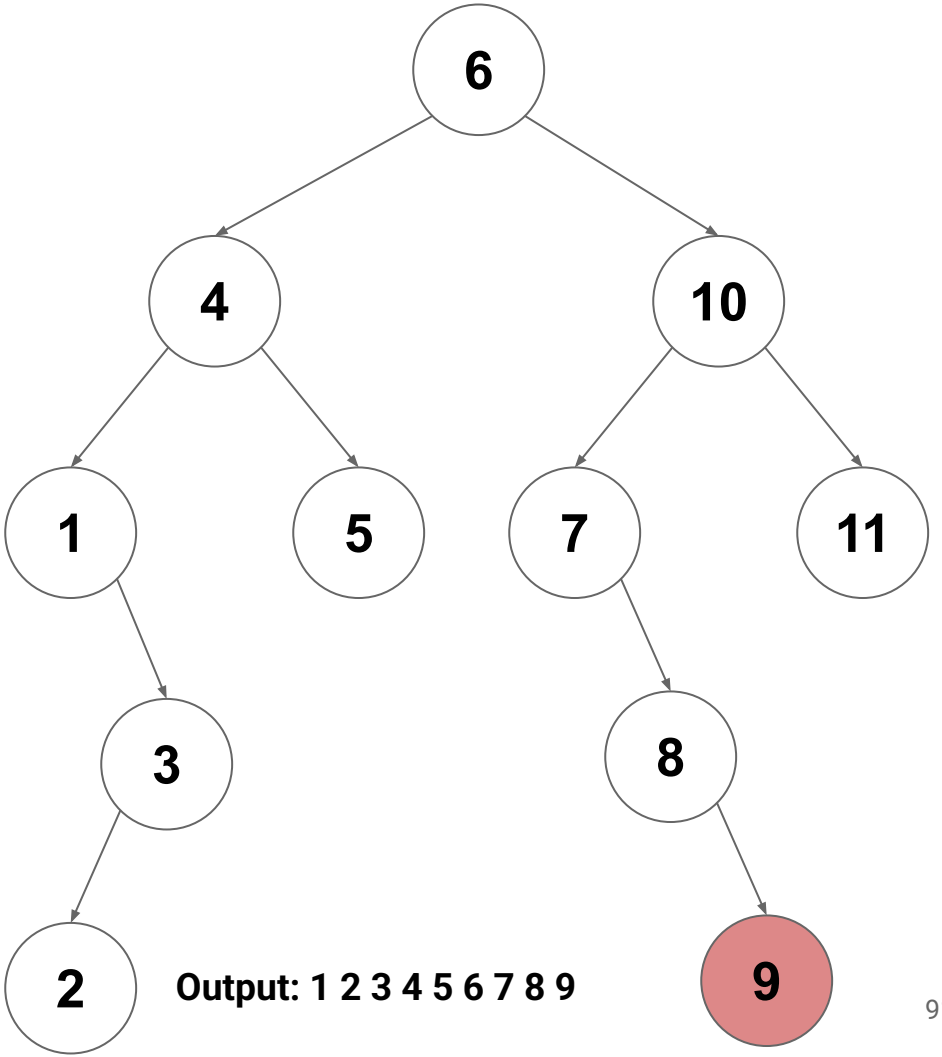
In-Order Traversal on a BST

```
inorderVisit(6)
inorderVisit(10)
inorderVisit(7)
inorderVisit(8)
inorderVisit(9)
```



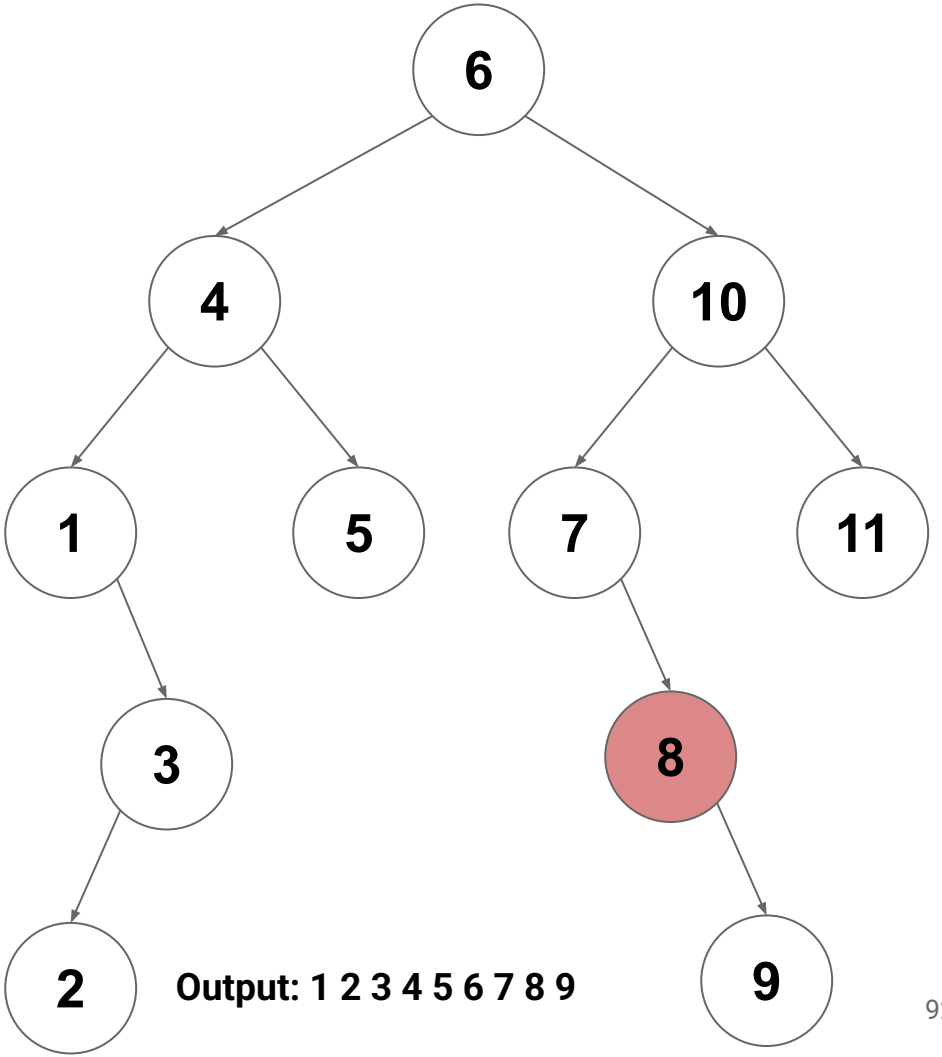
In-Order Traversal on a BST

```
inorderVisit(6)
inorderVisit(10)
inorderVisit(7)
inorderVisit(8)
visit(9)
```



In-Order Traversal on a BST

```
inorderVisit(6)
inorderVisit(10)
inorderVisit(7)
inorderVisit(8)
```

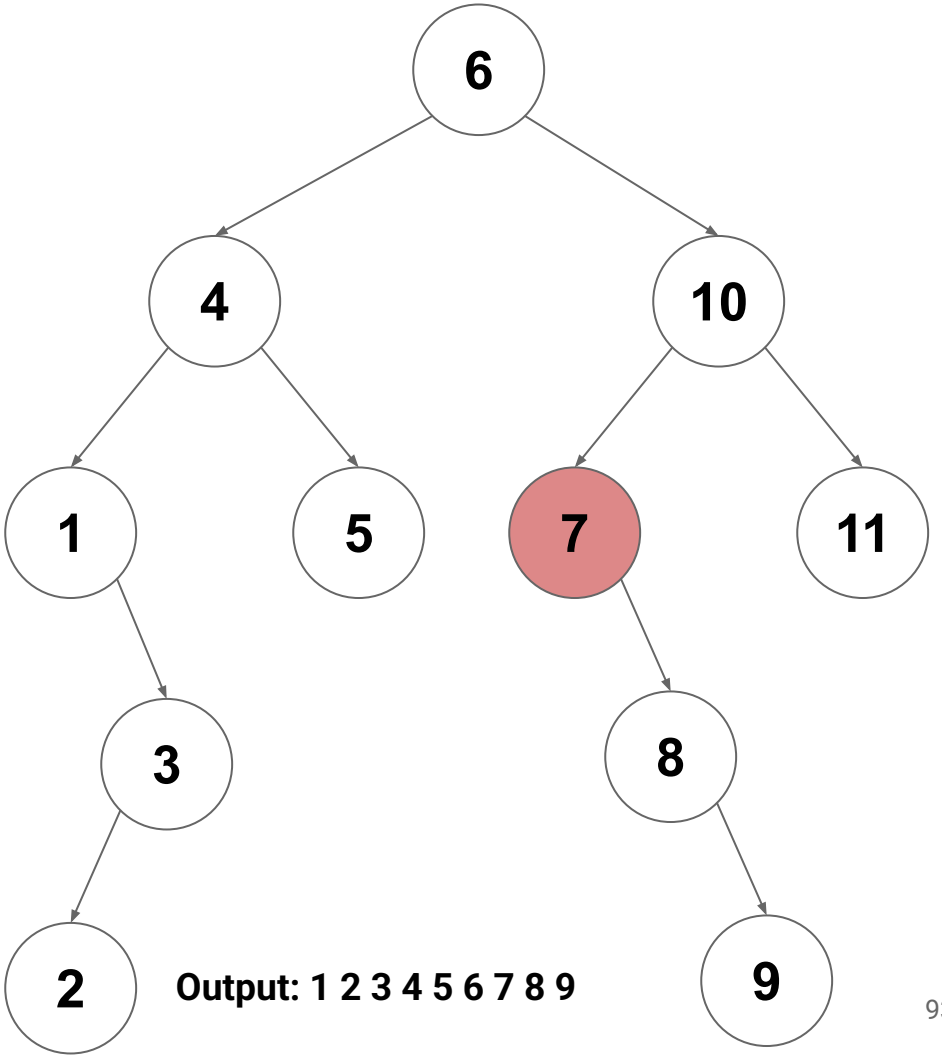


In-Order Traversal on a BST

```
inorderVisit(6)

inorderVisit(10)

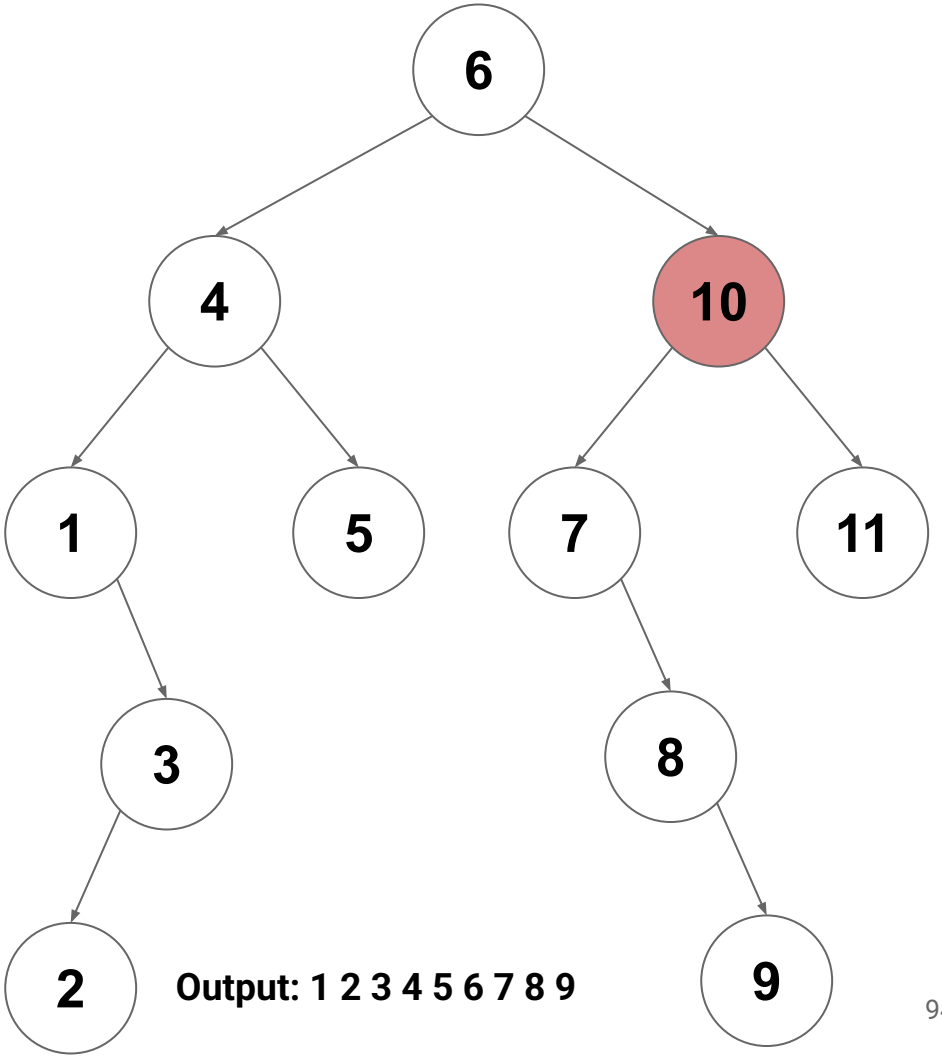
inorderVisit(7)
```



In-Order Traversal on a BST

```
inorderVisit(6)

inorderVisit(10)
```

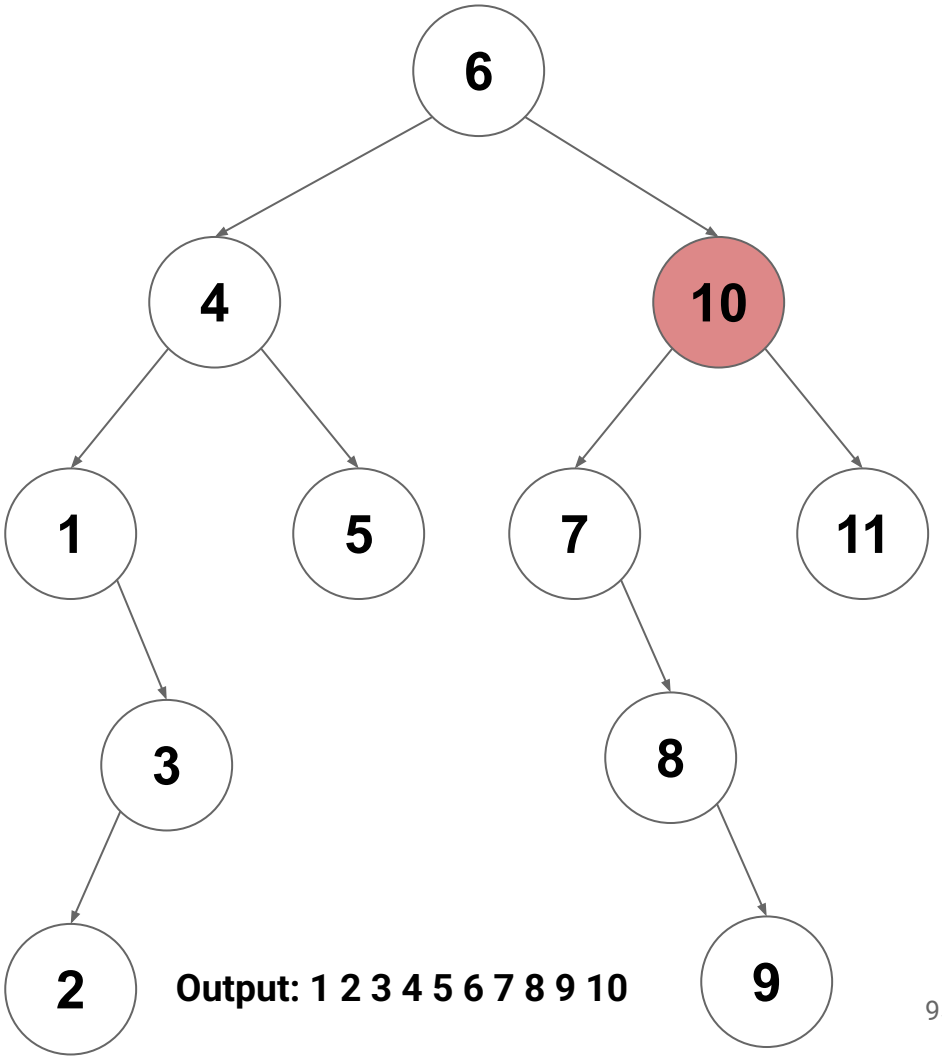


In-Order Traversal on a BST

```
inorderVisit(6)

inorderVisit(10)

visit(10)
```

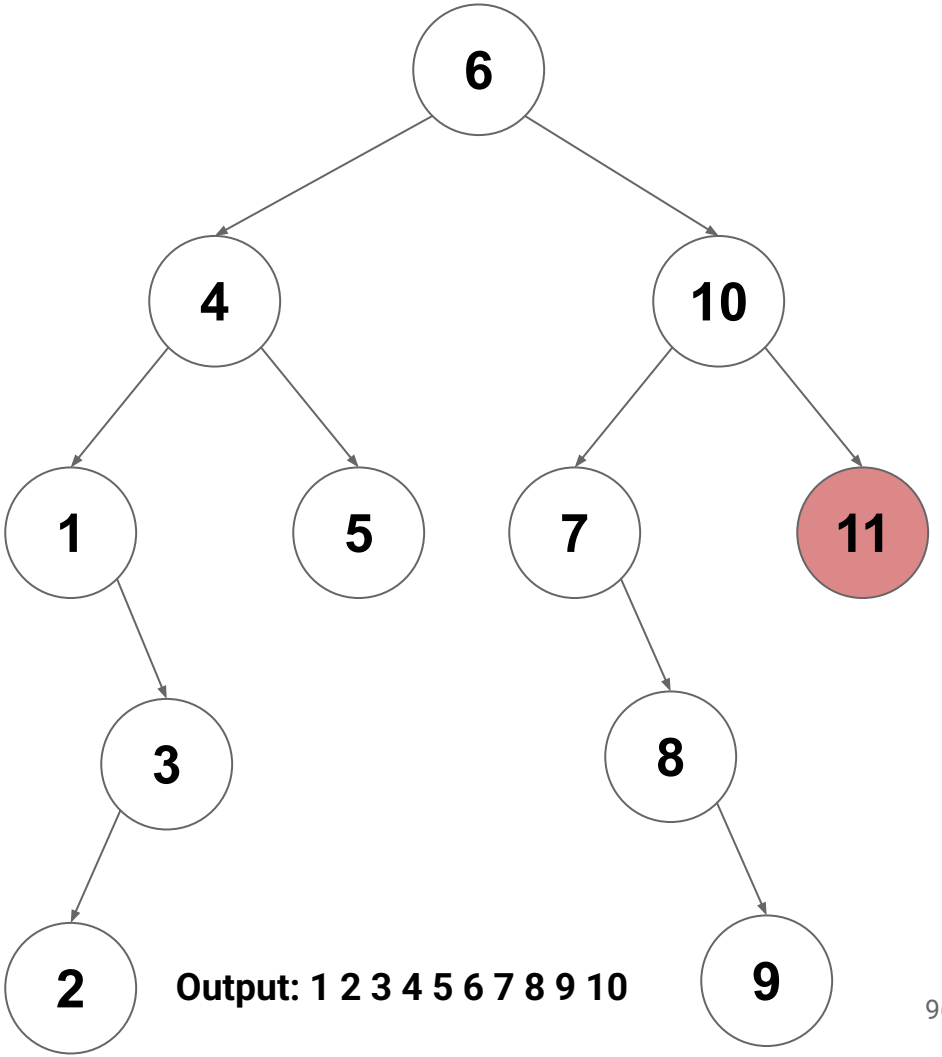


In-Order Traversal on a BST

```
inorderVisit(6)

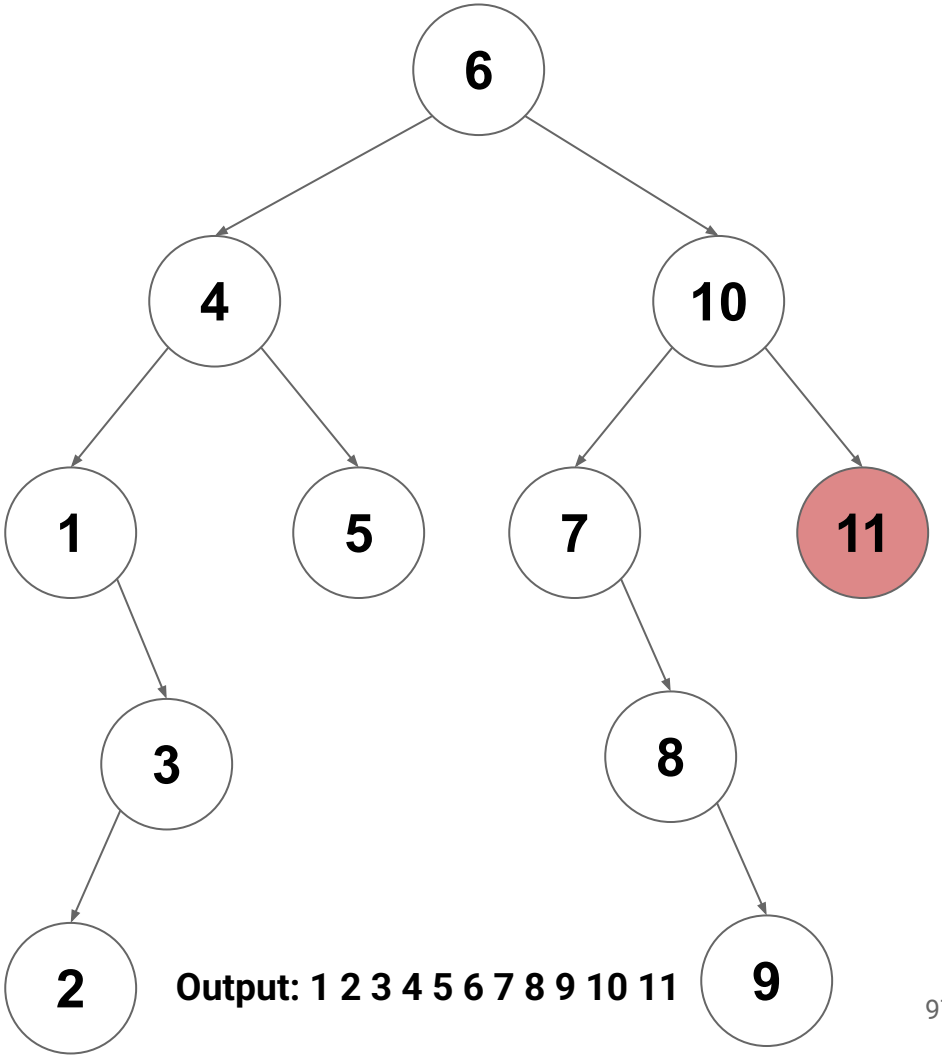
inorderVisit(10)

inorderVisit(11)
```



In-Order Traversal on a BST

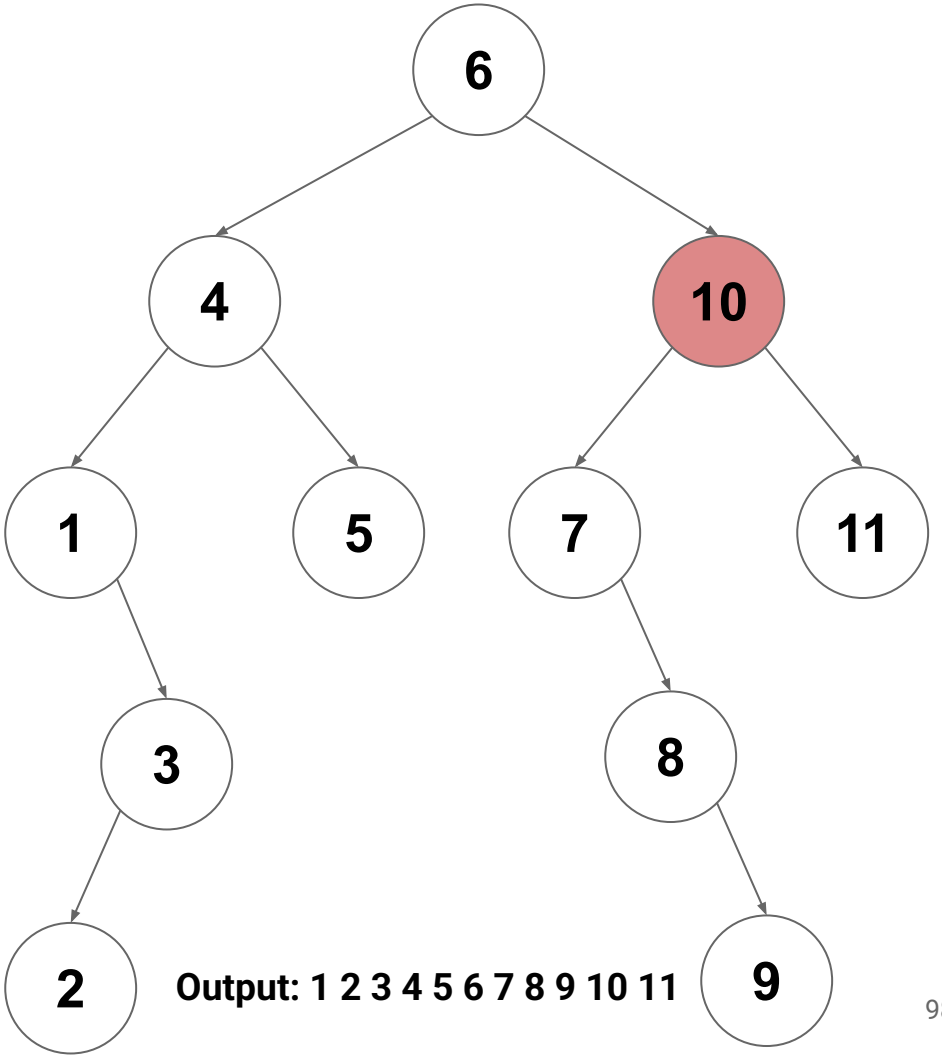
```
inorderVisit(6)
inorderVisit(10)
inorderVisit(11)
visit(11)
```



In-Order Traversal on a BST

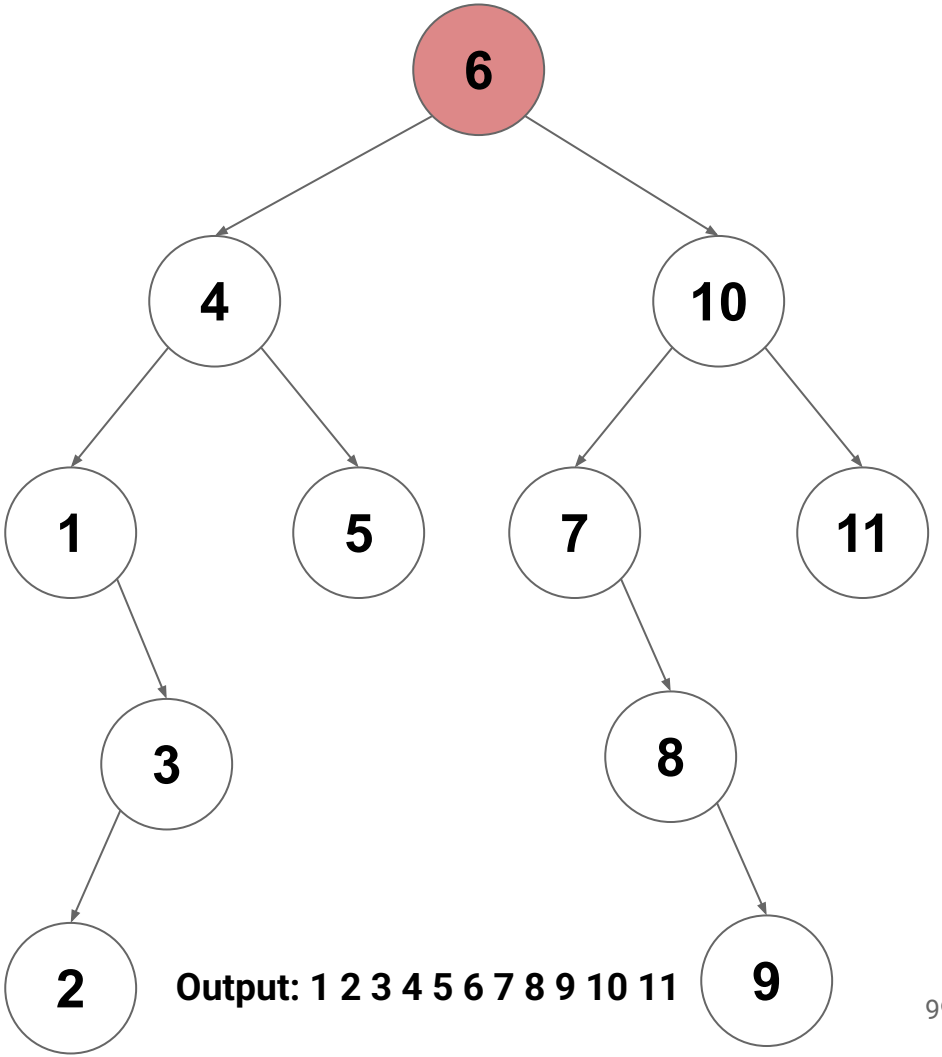
```
inorderVisit(6)

inorderVisit(10)
```

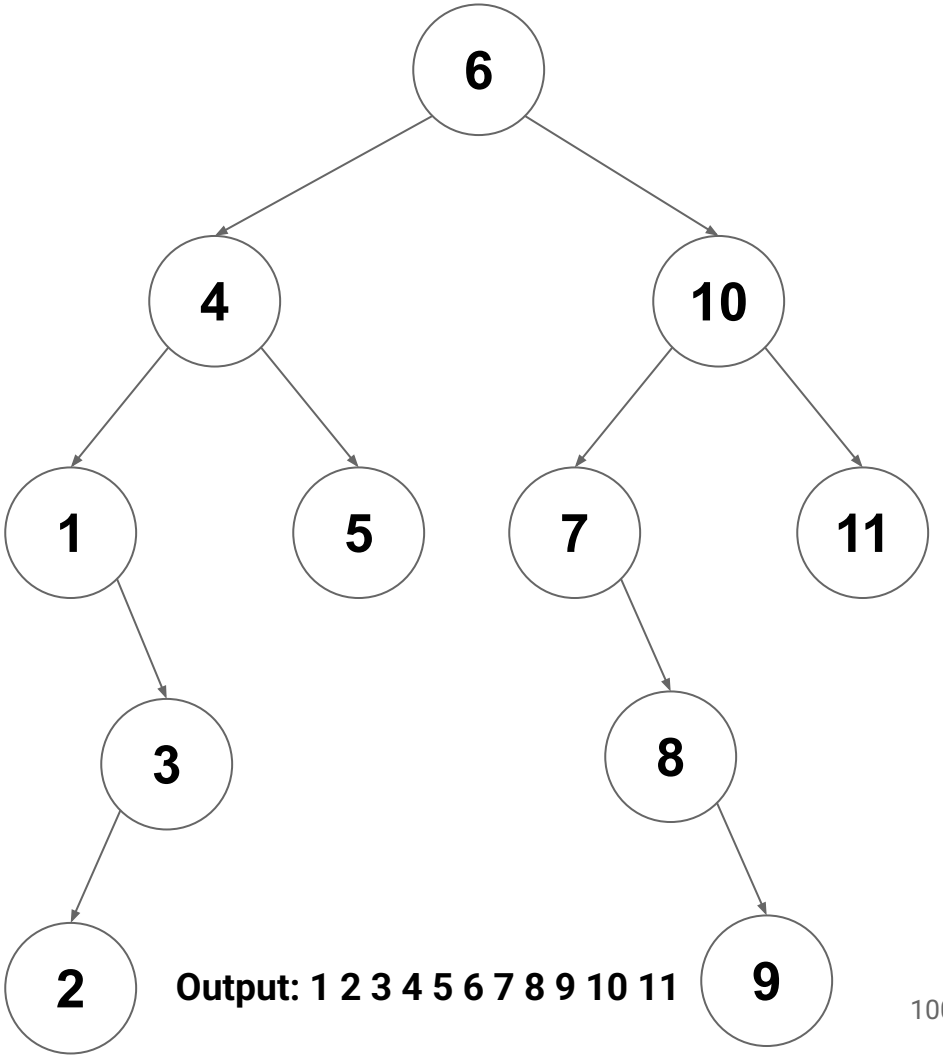


In-Order Traversal on a BST

`inorderVisit(6)`



In-Order Traversal on a BST



Tree Traversal: In-Order Iterator

```
1  class TreeIterator<T> implements Iterator<T> {
2    Stack<TreeNode<T>> toVisit;
3    TreeIterator() {
4      toVisit = new Stack<>();
5      pushLeft(root);
6    }
7    void pushLeft(Optional<TreeNode<T>> node) {
8      if (node.isPresent()) {
9        toVisit.push(node.get());
10       pushLeft(node.get().leftChild());
11     }
12   }
13   /* ... */
14 }
```

Tree Traversal: In-Order Iterator

```
1  class TreeIterator<T> implements Iterator<T> {
2    Stack<TreeNode<T>> toVisit;
3    TreeIterator() {
4      toVisit = new Stack<>();
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6    }
7    void pushLeft(Optional<TreeNode<T>> node) {
8      if (node.isPresent()) {
9        toVisit.push(node.get());
10       pushLeft(node.get().leftChild());
11     }
12   }
13   /* ... */
14 }
```

Keep track of what we need to visit in a stack

Recursively push all the left nodes (they are the smallest and therefore should be visited first)

Tree Traversal: In-Order Iterator

```
1  class TreeIterator<T> implements Iterator<T> {
2    Stack<TreeNode<T>> toVisit;
3    TreeIterator() {
4      toVisit = new Stack<>();
5      pushLeft(root);
6    }
7    void pushLeft(Optional<TreeNode<T>> node) {
8      if (node.isPresent()) {
9        toVisit.push(node.get());
10       pushLeft(node.get().leftChild());
11     }
12   }
13   /* ... */
14 }
```

Push the node, and then recursively push it's left children

Tree Traversal: In-Order Iterator

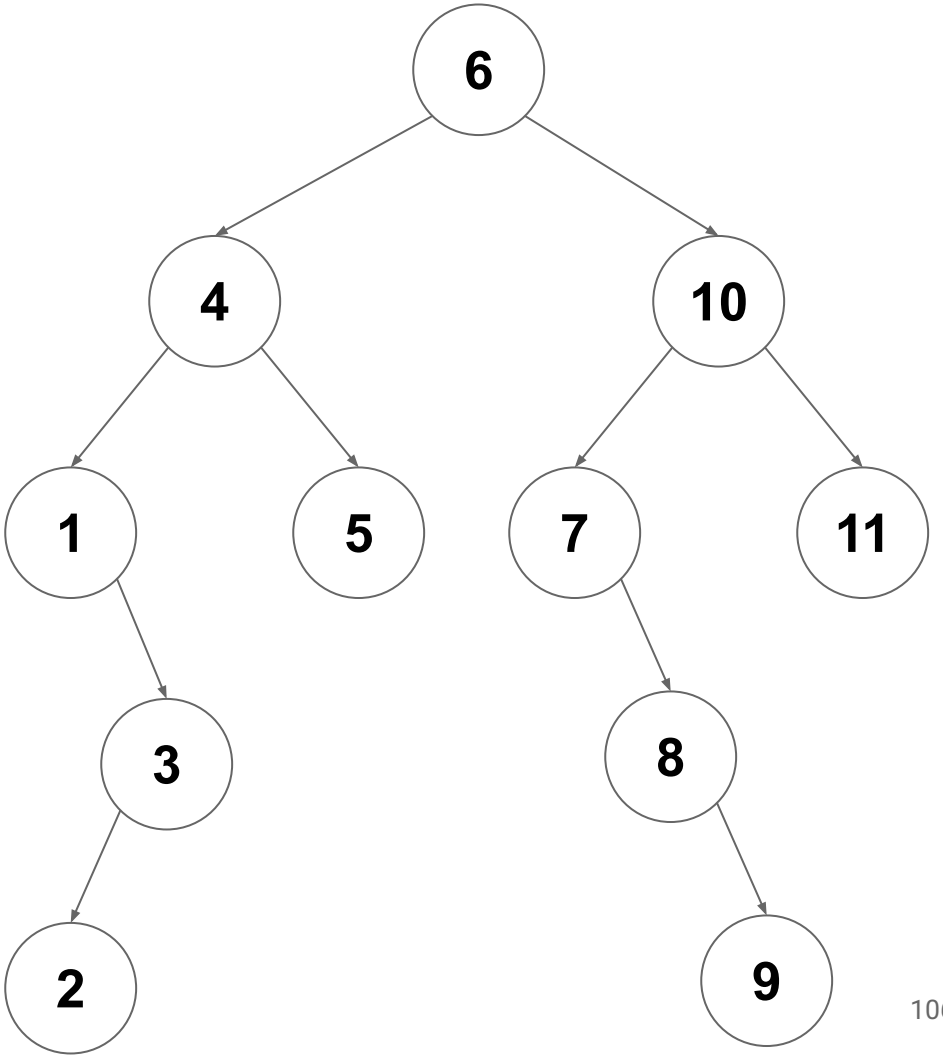
```
1  class TreeIterator<T> implements Iterator<T> {
2    /* ... */
3
4    boolean hasNext() { return !toVisit.isEmpty(); }
5
6    T next() {
7      TreeNode<T> nextNode = toVisit.pop();
8      pushLeft(nextNode.rightChild);
9      return nextNode.value;
10   }
11 }
```

Tree Traversal: In-Order Iterator

```
1  class TreeIterator<T> implements Iterator<T> {
2    /* ... */
3
4    boolean hasNext() { return !toVisit.isEmpty(); }
5
6    T next() {
7      TreeNode<T> nextNode = toVisit.pop();
8      pushLeft(nextNode.rightChild);
9      return nextNode.value;
10   }
11 }
```

Pop the next node, then push the left children of it's right subtree

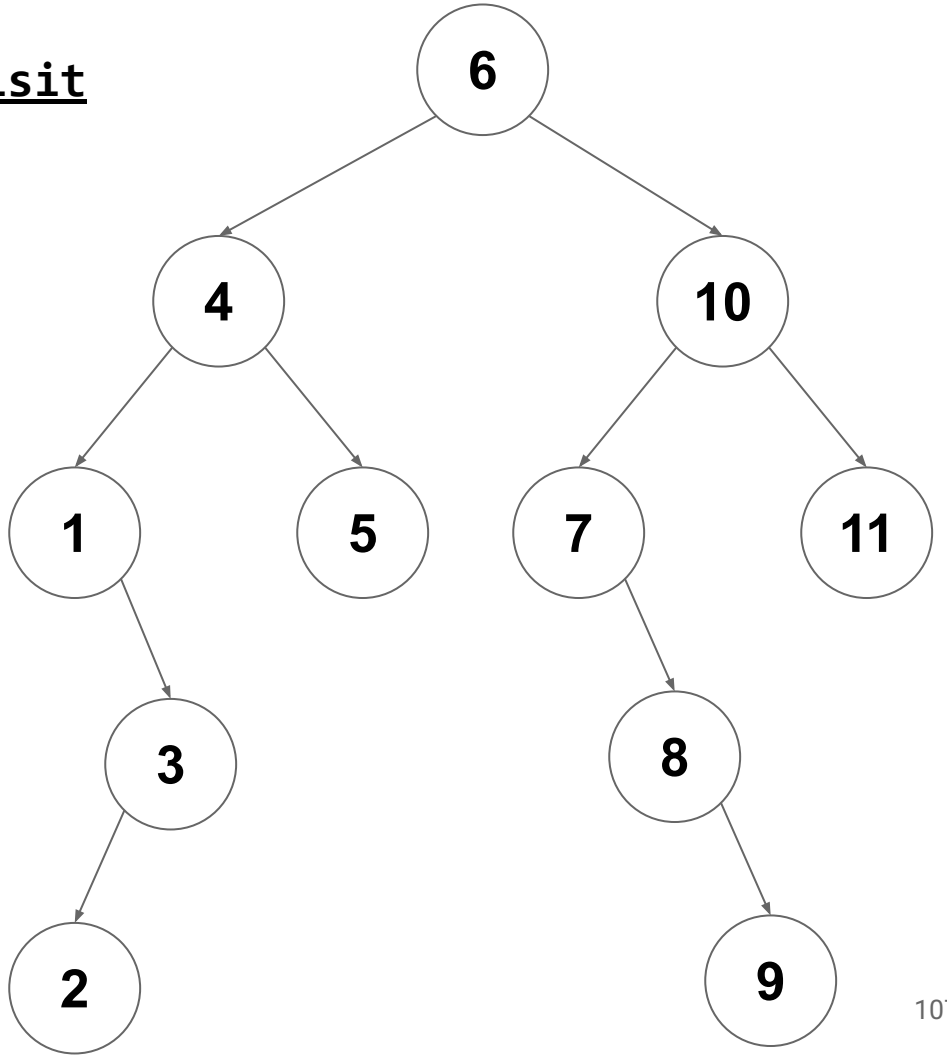
In-Order Traversal with an Iterator



In-Order Traversal with an Iterator

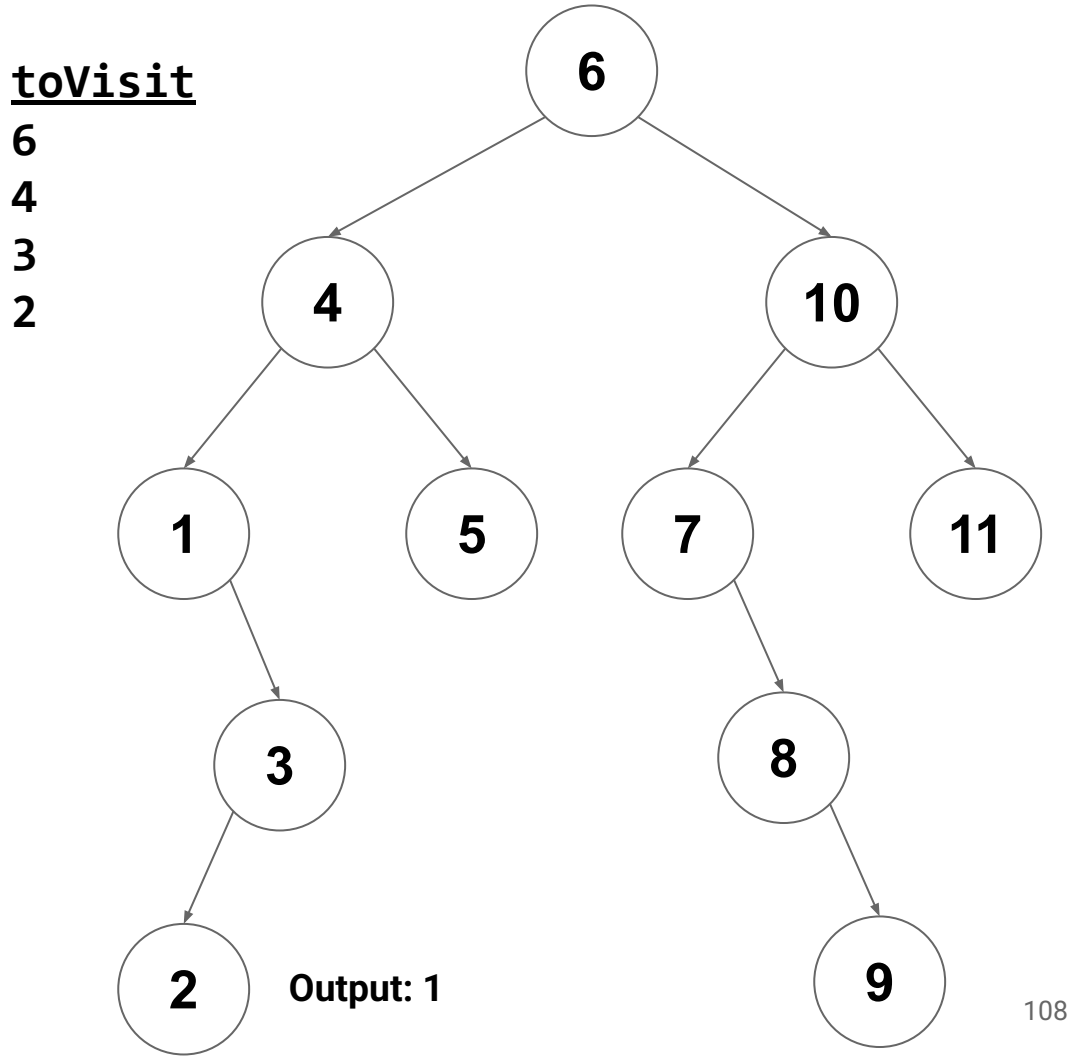
When we create the iterator, the **`toVisit`** stack is initialized

`toVisit`
6
4
1



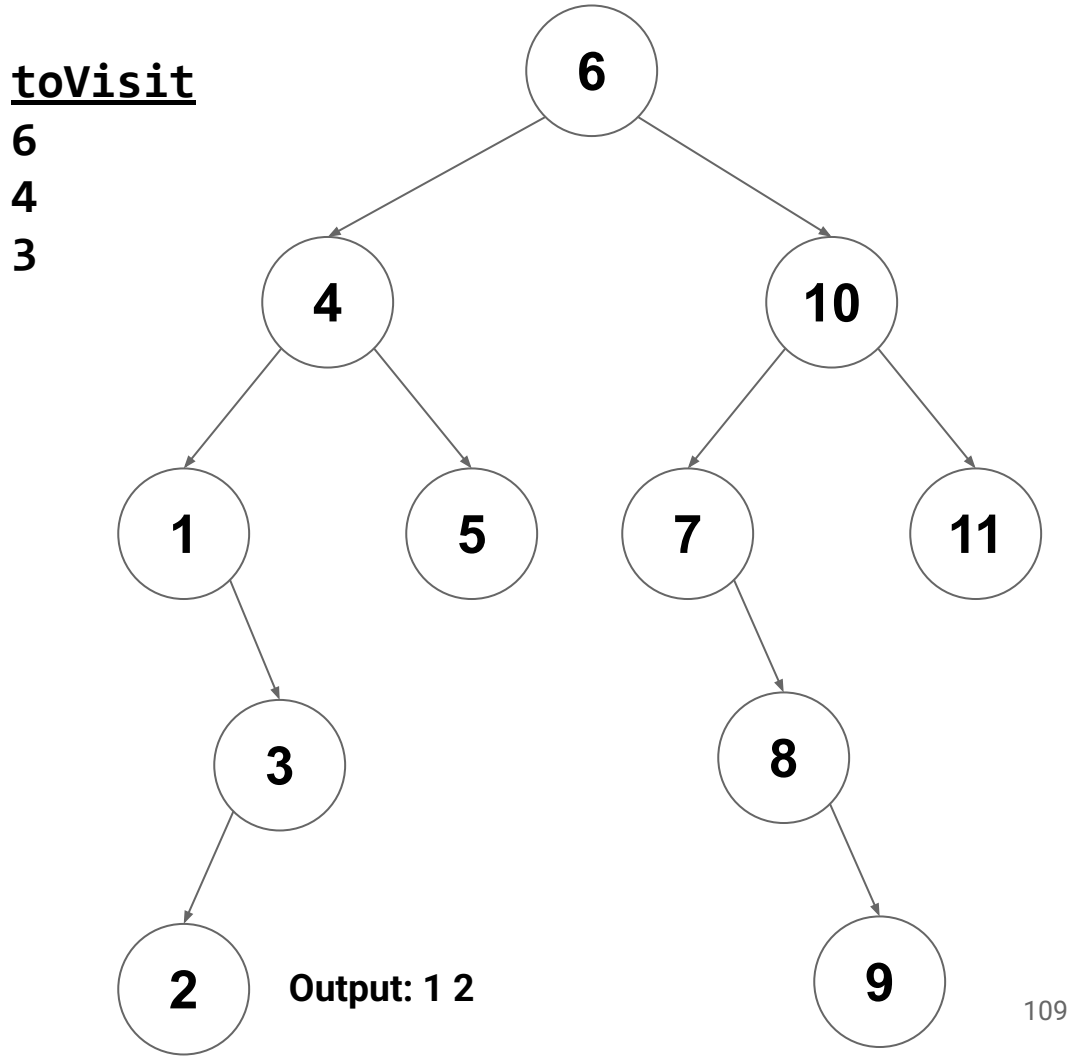
In-Order Traversal with an Iterator

`next` pops the stack (1), and calls `pushLeft` on the right subtree of 1



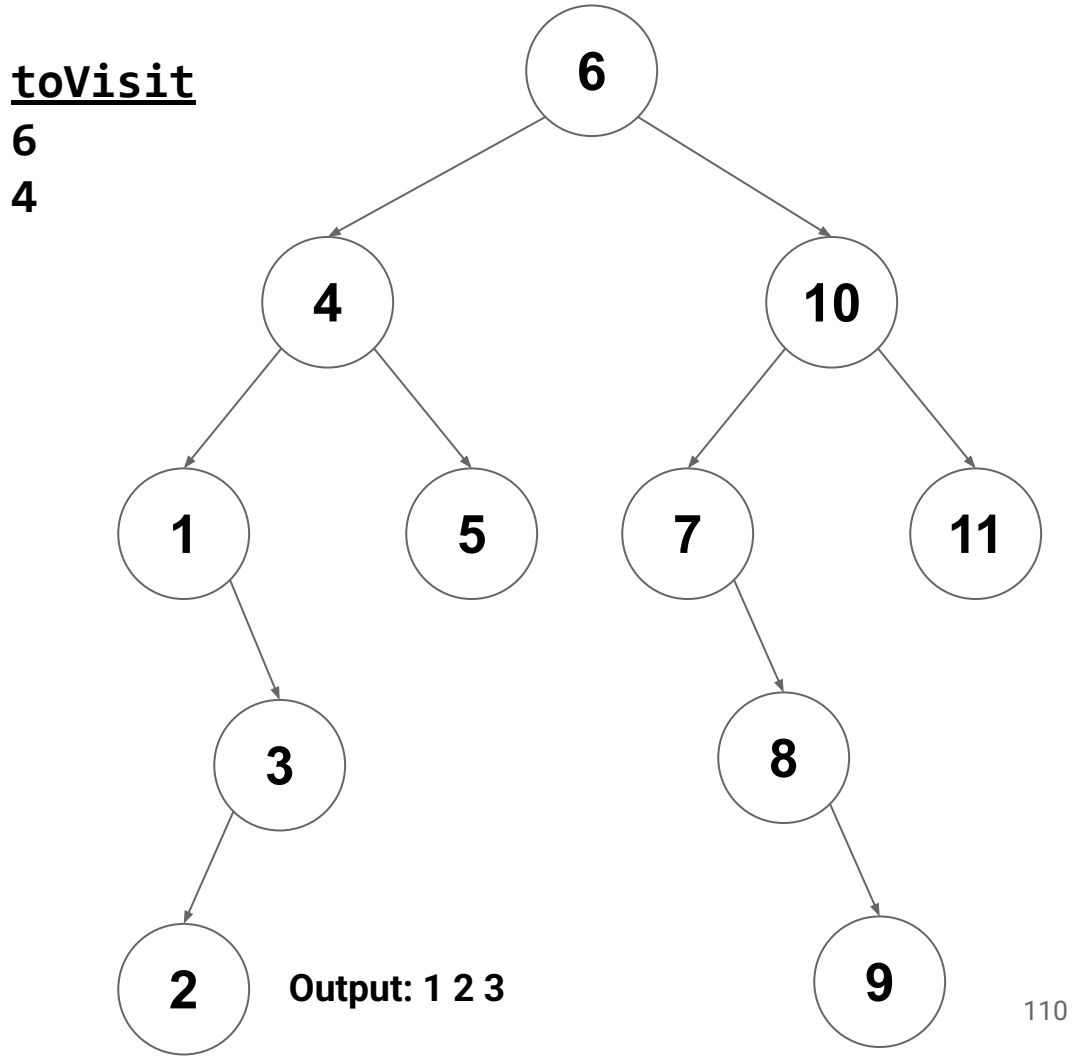
In-Order Traversal with an Iterator

`next` pops the stack (2) and pushes the right subtree (nothing)



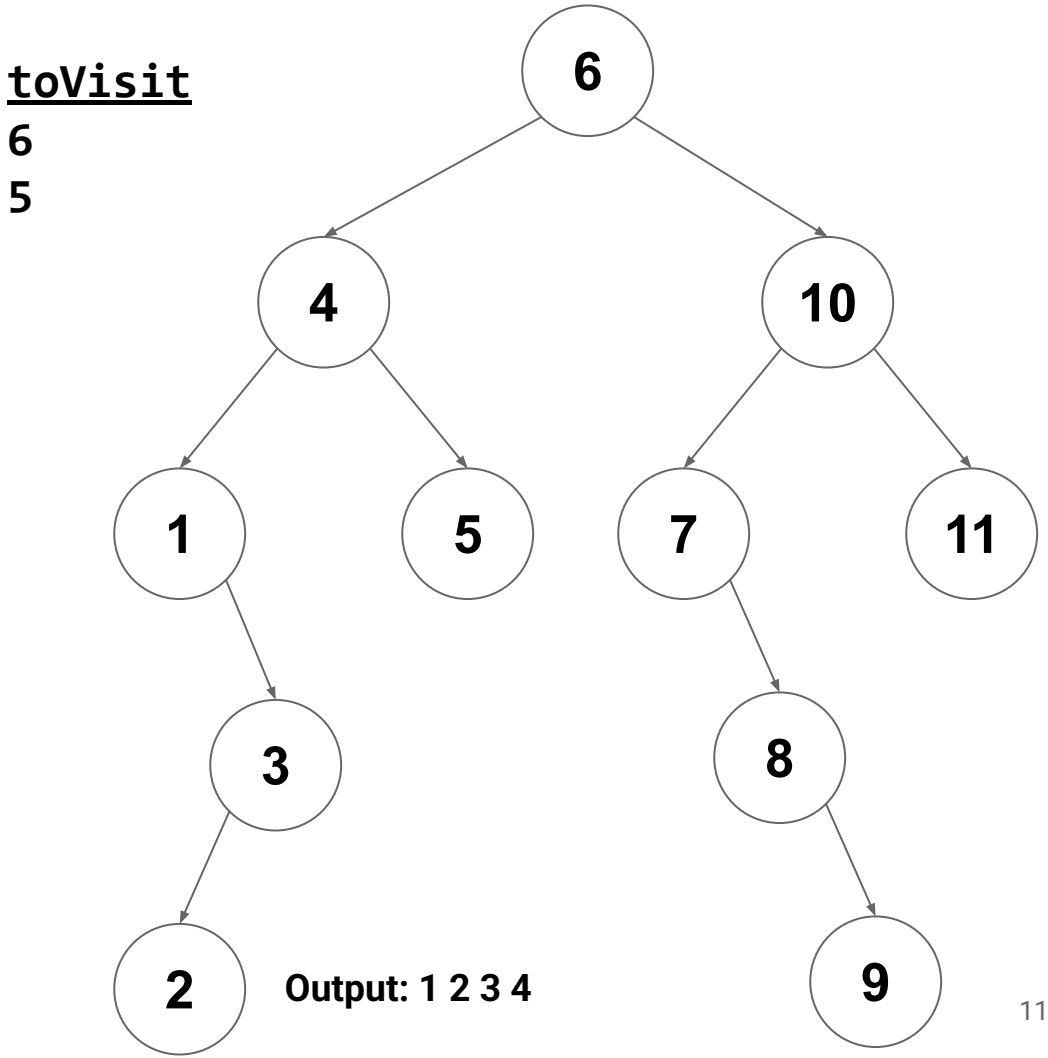
In-Order Traversal with an Iterator

`next` pops the stack (3) and pushes the right subtree (nothing)



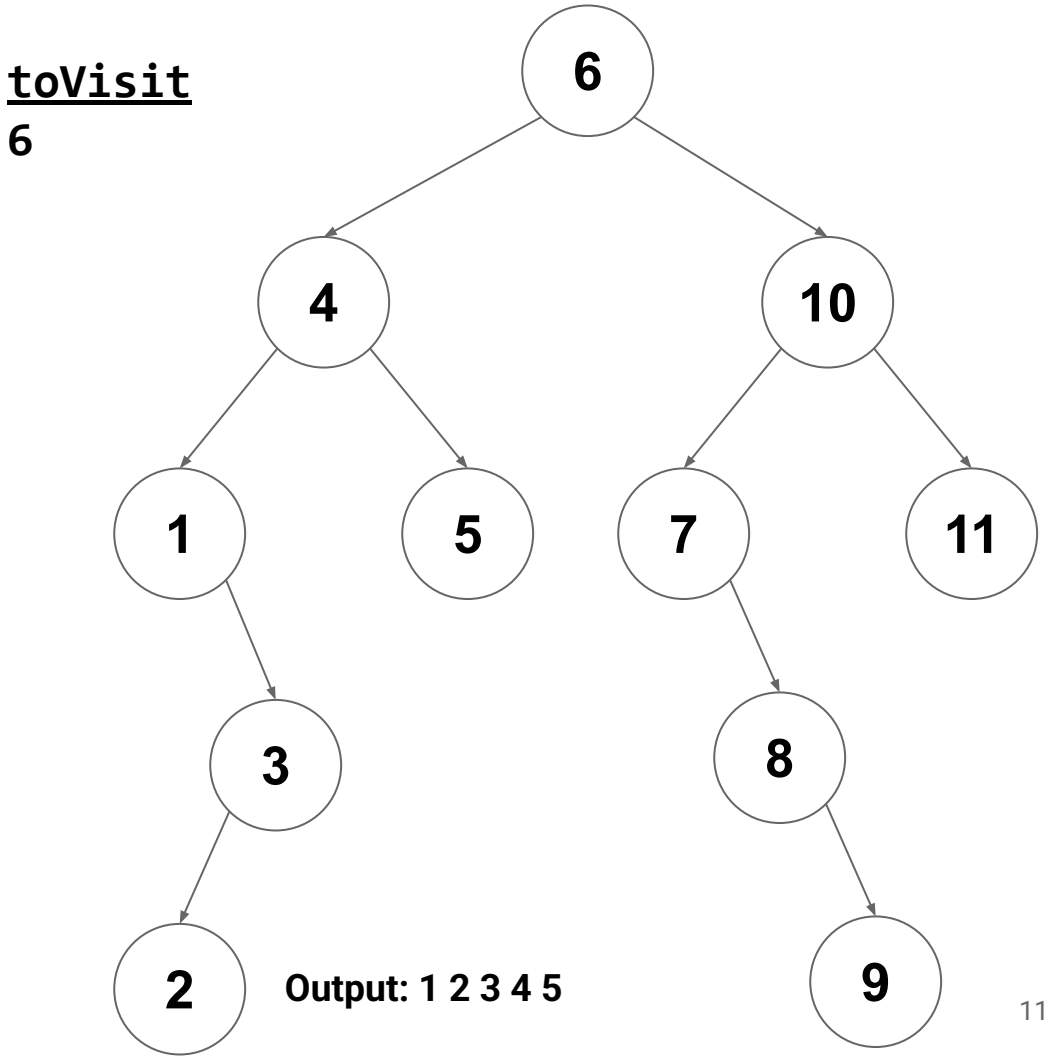
In-Order Traversal with an Iterator

`next` pops the stack (4) and pushes the right subtree



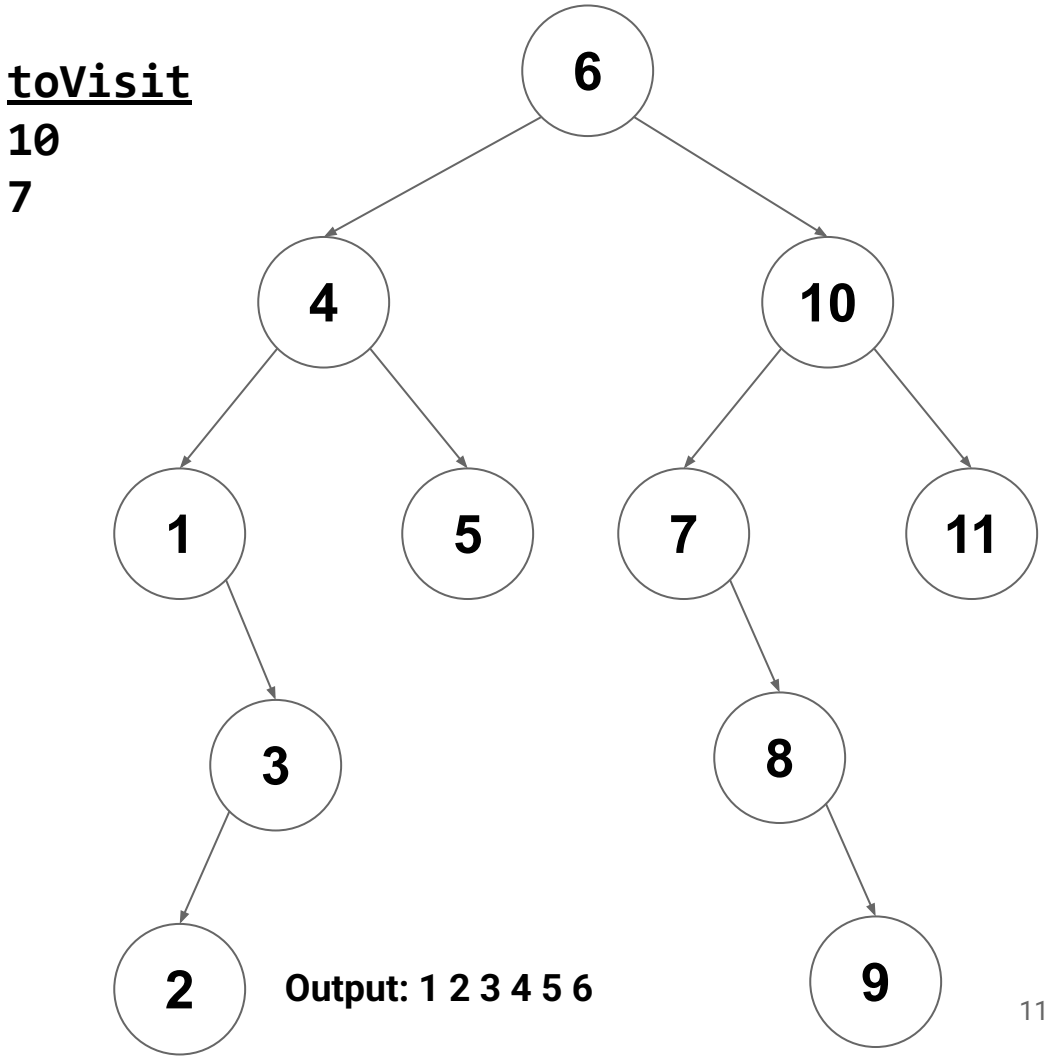
In-Order Traversal with an Iterator

`next` pops the stack (5) and pushes the right subtree (nothing)



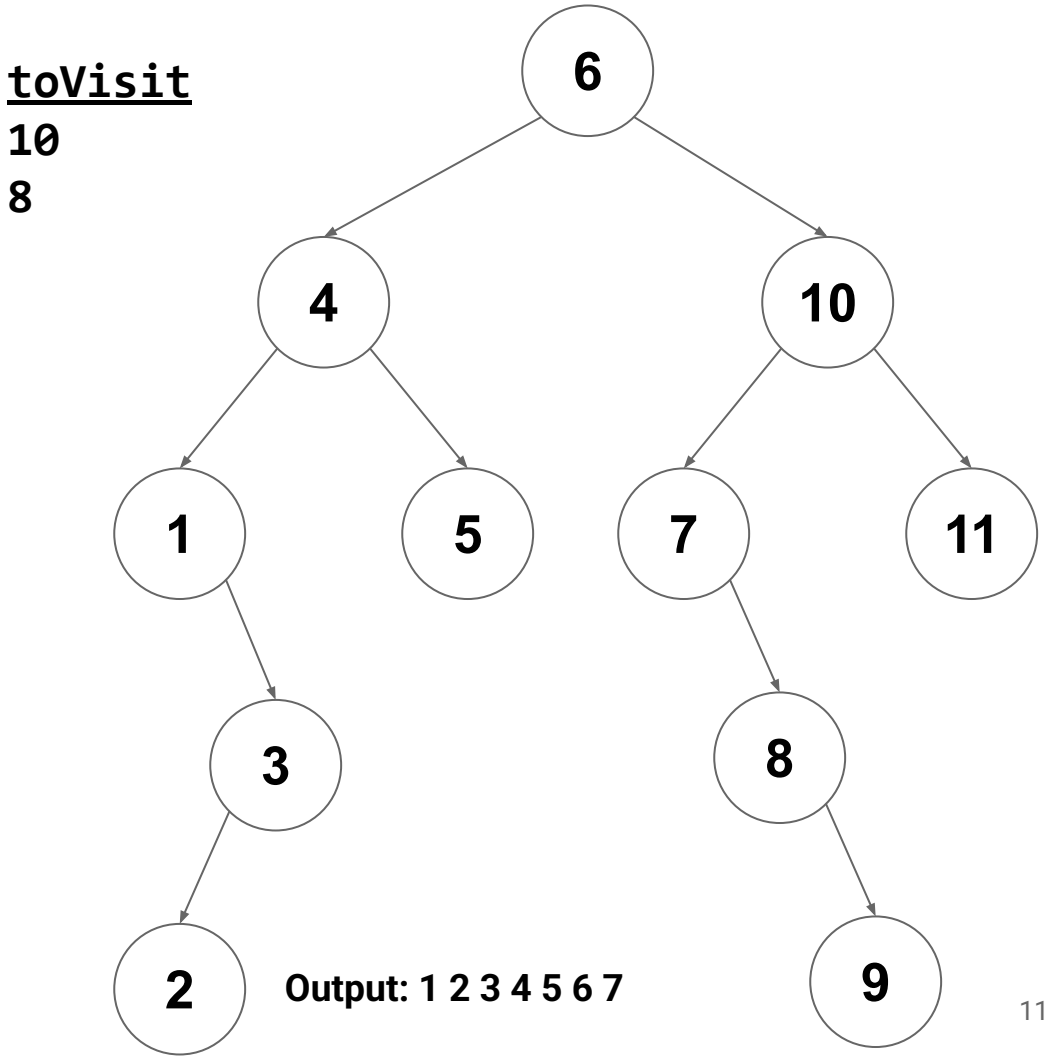
In-Order Traversal with an Iterator

`next` pops the stack (6) and pushes the right subtree (10 7)



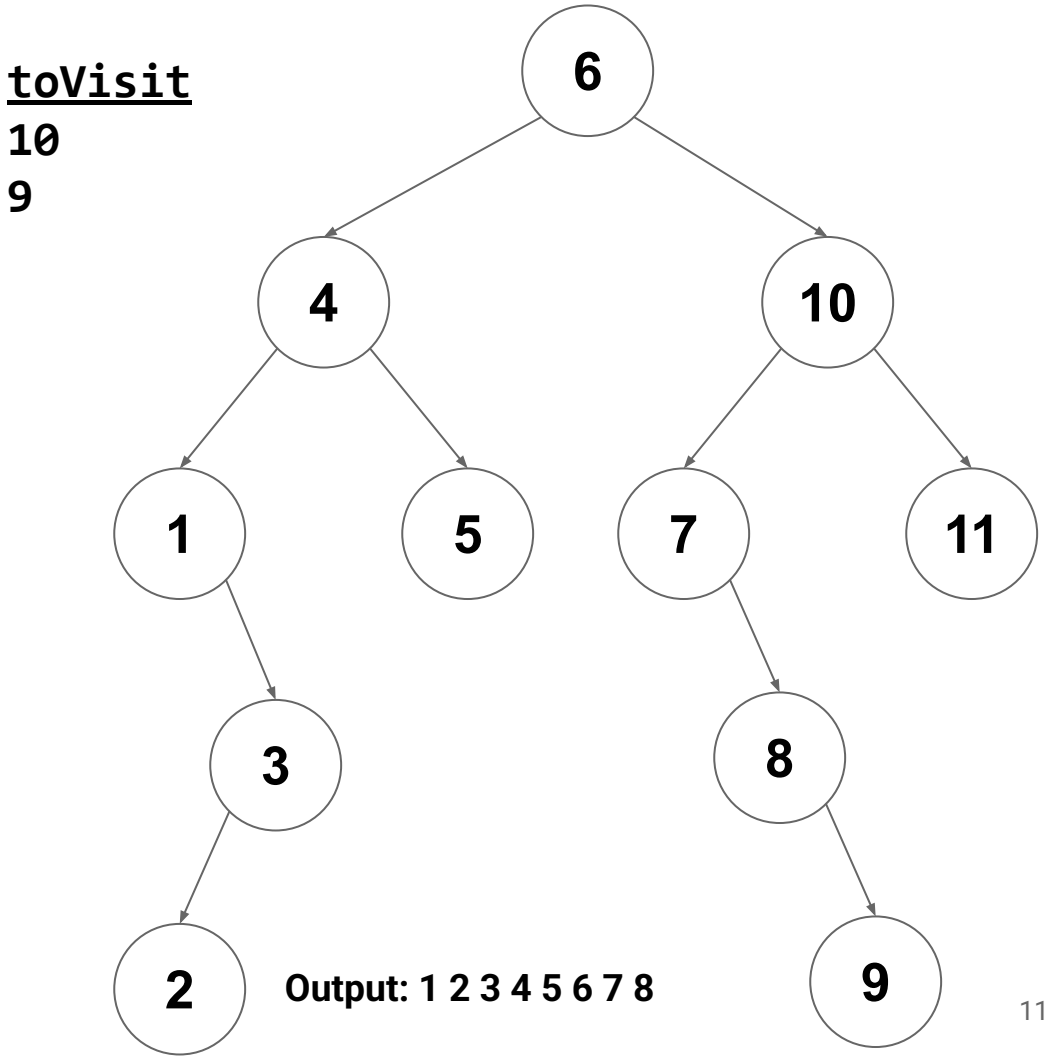
In-Order Traversal with an Iterator

`next` pops the stack (7) and pushes the right subtree (8)



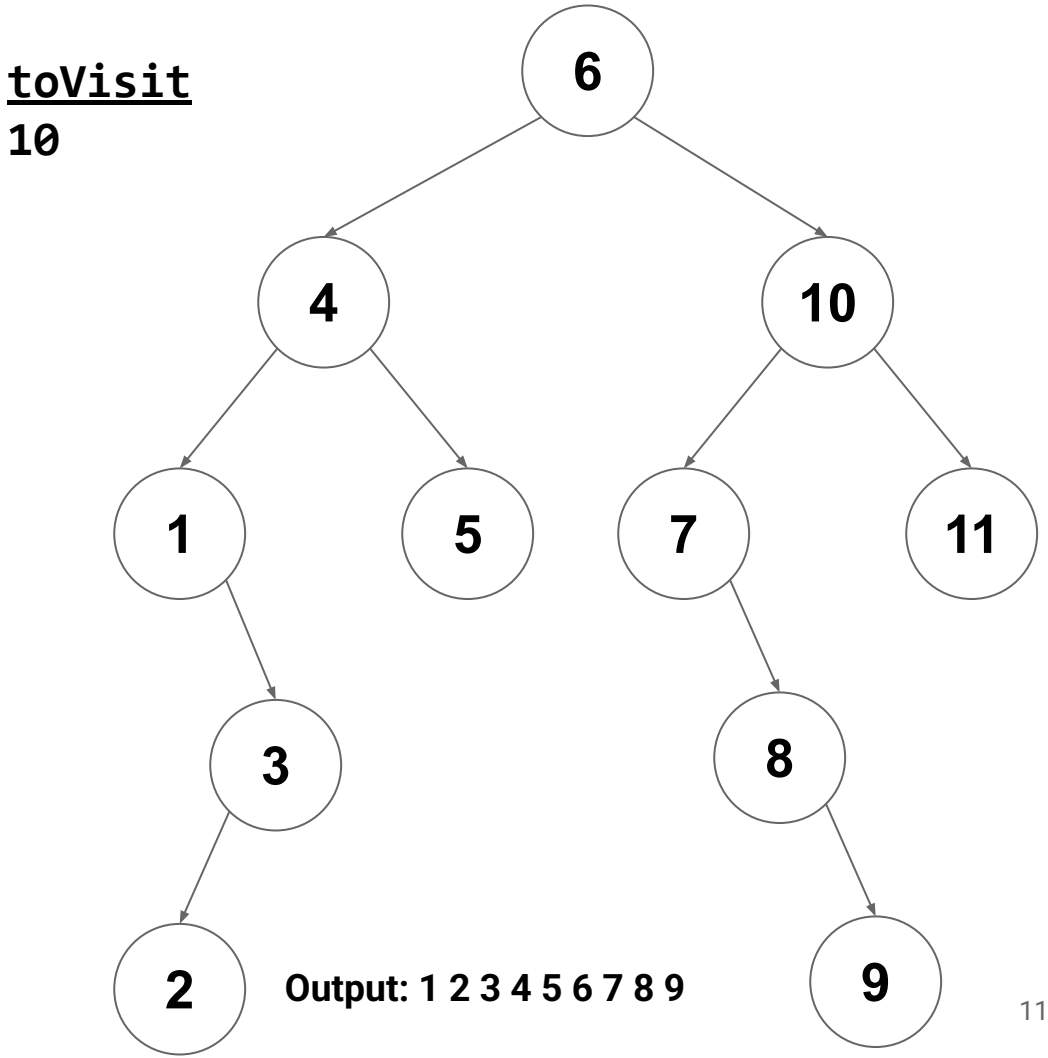
In-Order Traversal with an Iterator

`next` pops the stack (8) and pushes the right subtree (9)



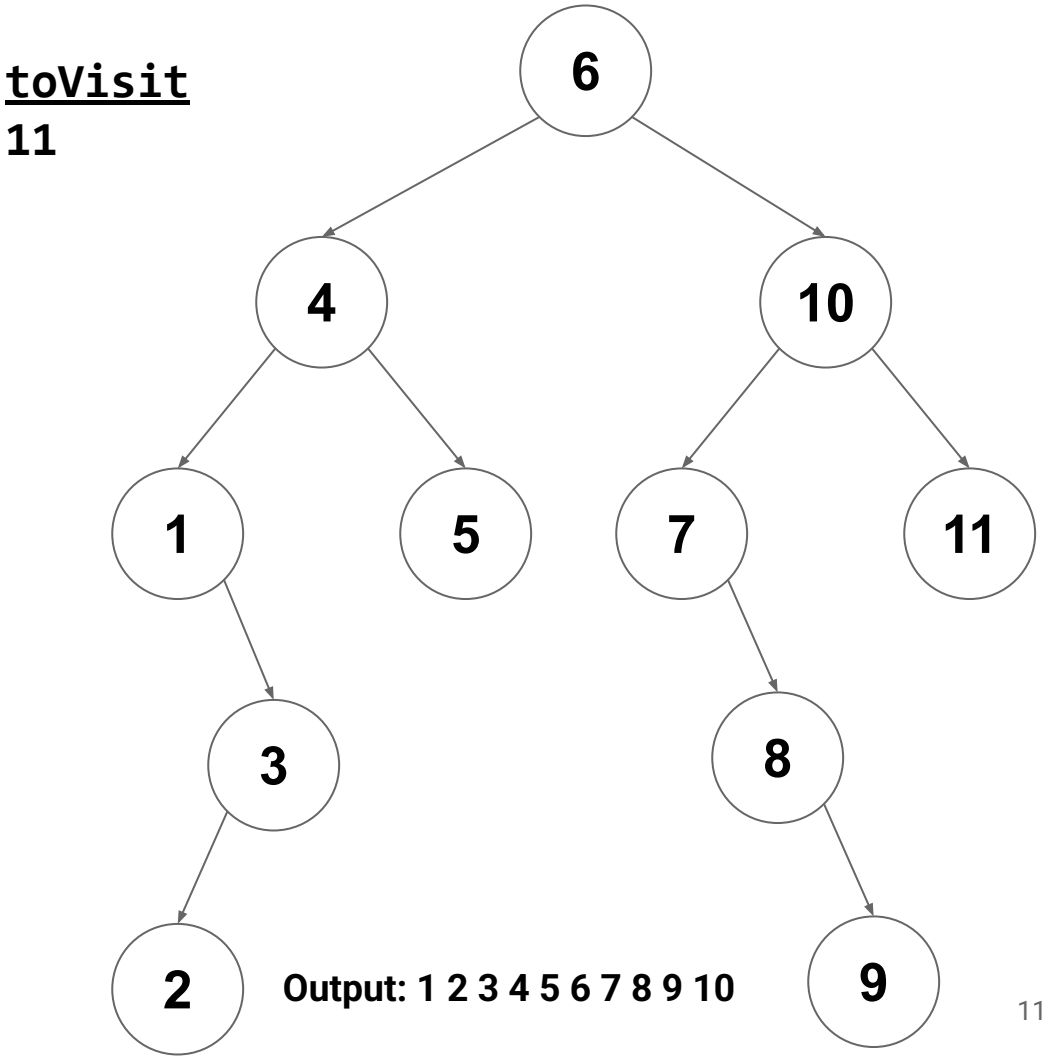
In-Order Traversal with an Iterator

`next` pops the stack (9) and pushes the right subtree (nothing)



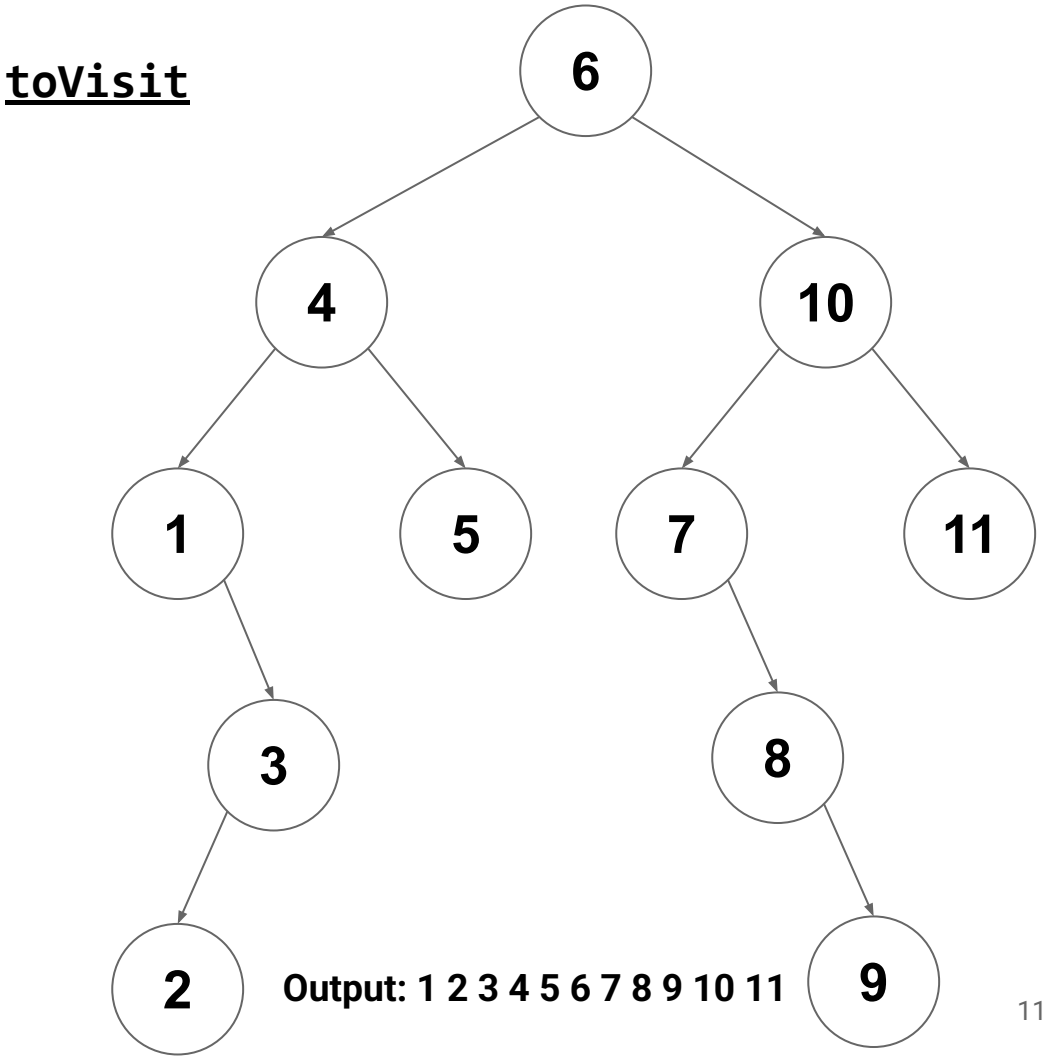
In-Order Traversal with an Iterator

`next` pops the stack (10) and pushes the right subtree (11)



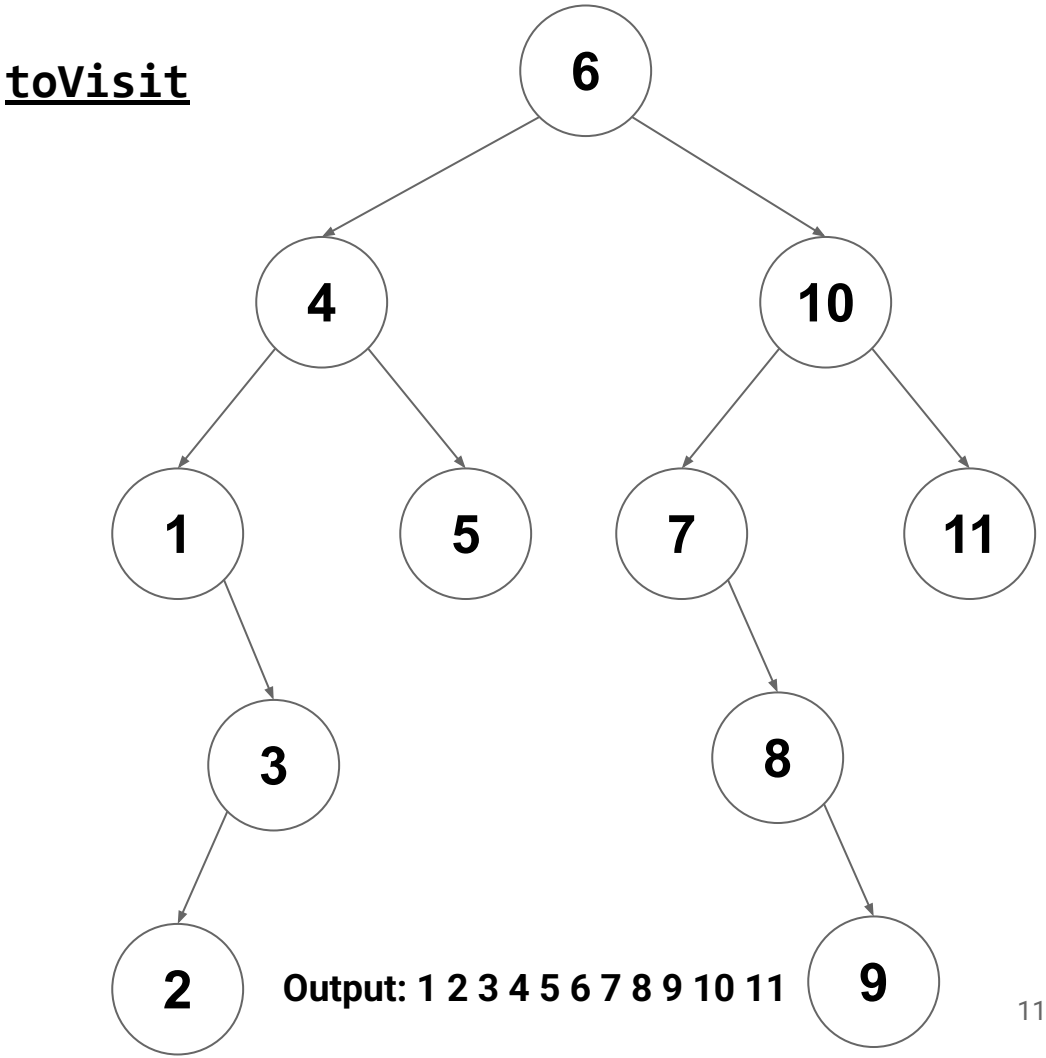
In-Order Traversal with an Iterator

`next` pops the stack (11) and pushes the right subtree (nothing)



In-Order Traversal with an Iterator

Our **`toVisit`** stack is empty, so **`isEmpty`** will now be true



Complexity

```
1  TreeIterator() {
2    toVisit = new Stack<>();
3    pushLeft(root);
4  }
```

What is our worst-case runtime to initialize the iterator?

Complexity

```
1  TreeIterator() {
2    toVisit = new Stack<>();
3    pushLeft(root);
4  }
```

What is our worst-case runtime to initialize the iterator? ***O(d)***

Complexity

```
1  TreeIterator() {
2    toVisit = new Stack<>();
3    pushLeft(root);
4  }
```

What is our worst-case runtime to initialize the iterator? ***O(d)***

(we may have to push as many as ***d*** *nodes onto the stack)*

Complexity

```
1  T next() {
2    TreeNode<T> nextNode = toVisit.pop();
3    pushLeft(nextNode.rightChild);
4    return nextNode.value;
5  }
```

What is our worst-case runtime to call **`next`***?*

Complexity

```
1  T next() {
2    TreeNode<T> nextNode = toVisit.pop();
3    pushLeft(nextNode.rightChild);
4    return nextNode.value;
5  }
```

What is our worst-case runtime to call **`next`***?* ***O*(*d*)**

(we may have to push as many as ***d*** *nodes onto the stack)*

Complexity

*What is the worst-case complexity to visit ALL **n** nodes?*

Complexity

*What is the worst-case complexity to visit ALL **n** nodes?*

Each node is at the top of the stack exactly once:

Complexity

*What is the worst-case complexity to visit ALL **n** nodes?*

Each node is at the top of the stack exactly once:

- One push ***O*(1)**

Complexity

*What is the worst-case complexity to visit ALL **n** nodes?*

Each node is at the top of the stack exactly once:

- One push ***O*(1)**
- One pop ***O*(1)**

Complexity

*What is the worst-case complexity to visit ALL **n** nodes?*

Each node is at the top of the stack exactly once:

- One push ***O*(1)**
- One pop ***O*(1)**

Total: *O*(*n*)