CSE 250 Recitation

April 3 - 4: Binary Trees

Heaps

A heap is a **partially ordered** **complete** **binary tree**

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Every parent has at most two children

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Every leaf is in the two deepest layers and leaves are added from left to right

Every parent has at most two children

Heaps

A heap is a **partially ordered** **complete** **binary tree**

You can infer the order between parents and children, but not between siblings

Min Heap: parent ≤ children
Max Heap: parent ≥ children

Every leaf is in the two deepest layers and leaves are added from left to right

Every parent has at most two children

Binary Search Trees

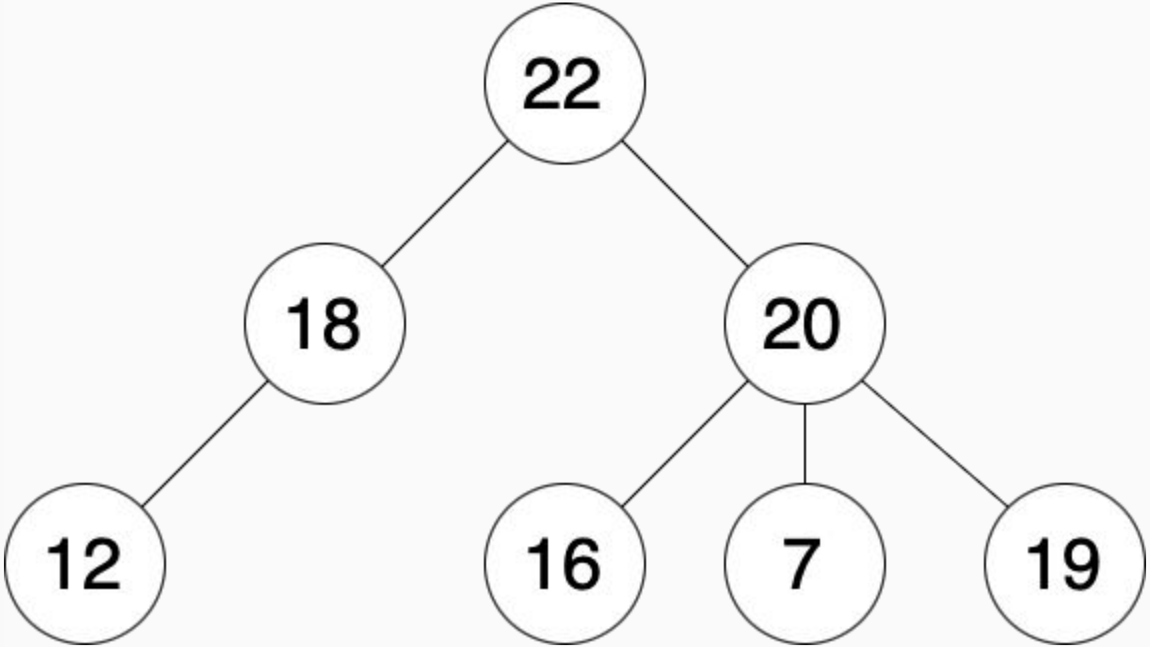
A binary search tree is a **binary tree** where:

- Every node in the right *subtree* of X is greater than X
- Every node in the left subtree of X is less than X

These two conditions mean that each node partitions the binary search tree into a lesser subtree and greater subtree

Binary Tree Examples

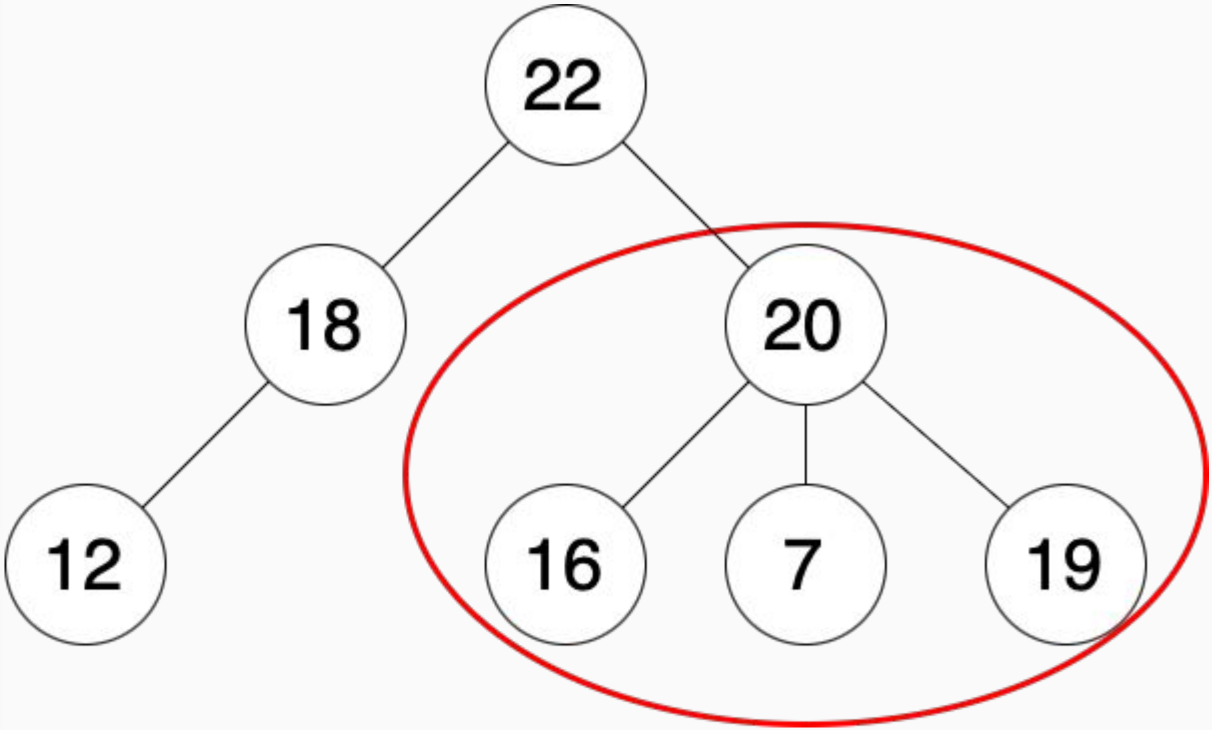
Is this a binary tree?



Binary Tree Examples

Is this a binary tree?

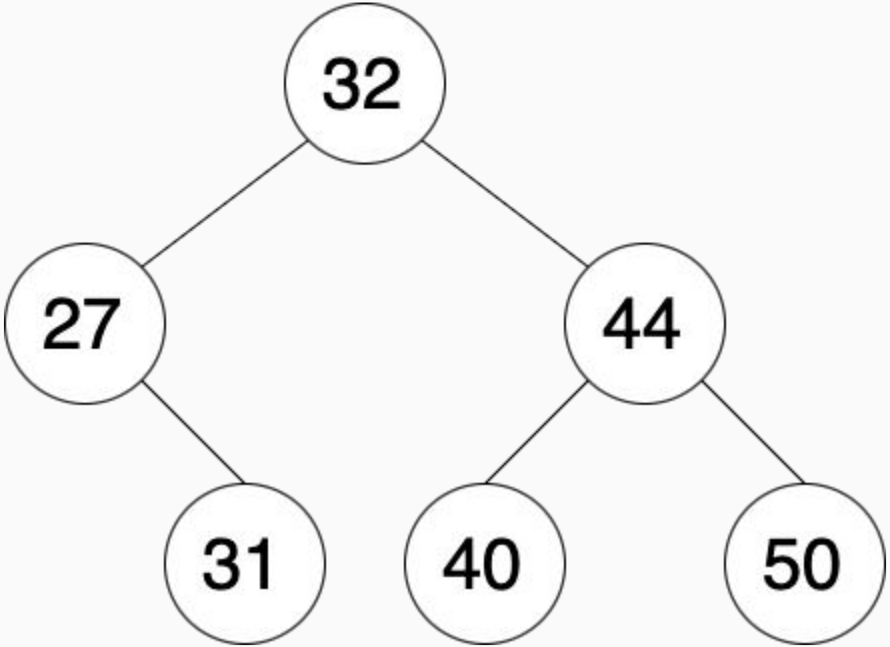
NO! Node 20 has > 2 children



Binary Tree Examples

Is this a binary tree?

Could this be a heap, binary search tree, or both?



Binary Tree Examples

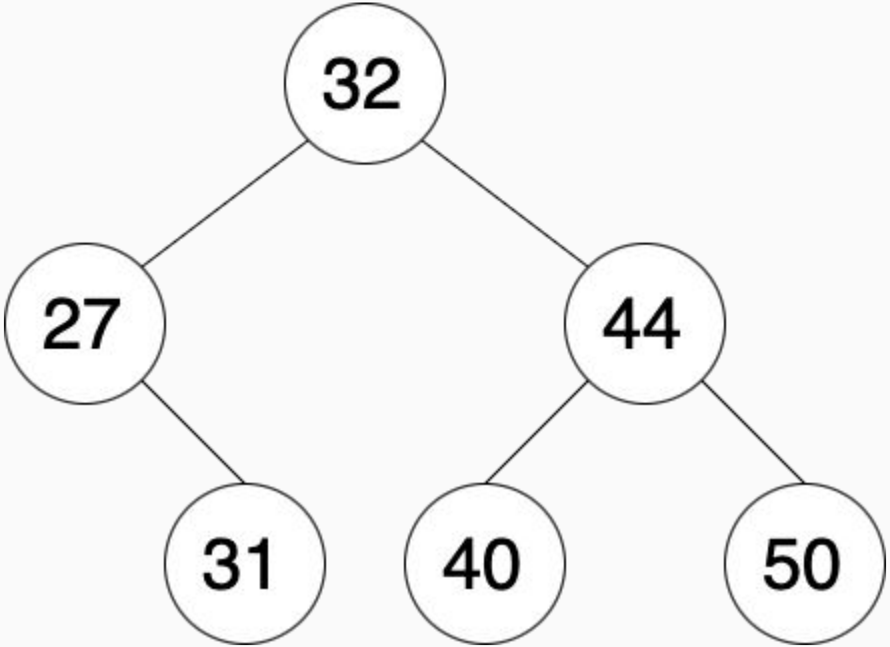
Is this a binary tree?

YES!

Could this be a heap, binary search tree, or both?

BST (every node partitions its subtrees)

Not a heap (not complete, incorrect ordering)



Binary Tree Examples

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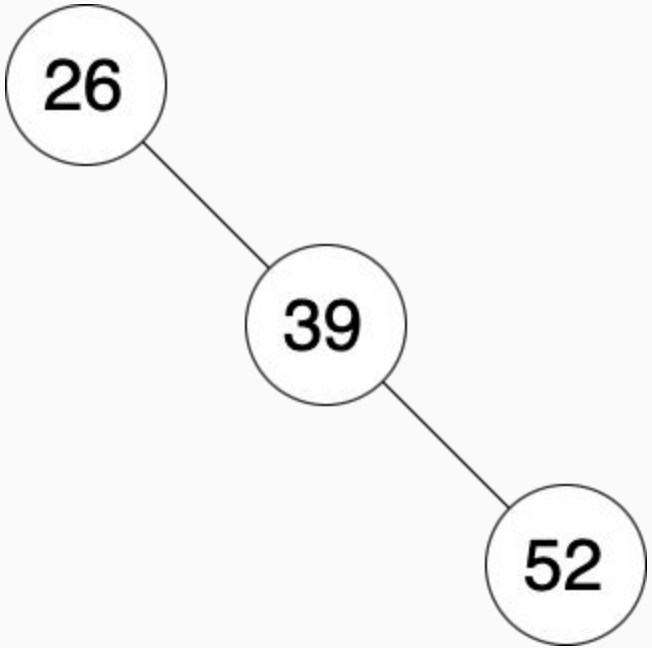
Could this be a heap, binary search tree, or both?



Binary Tree Examples

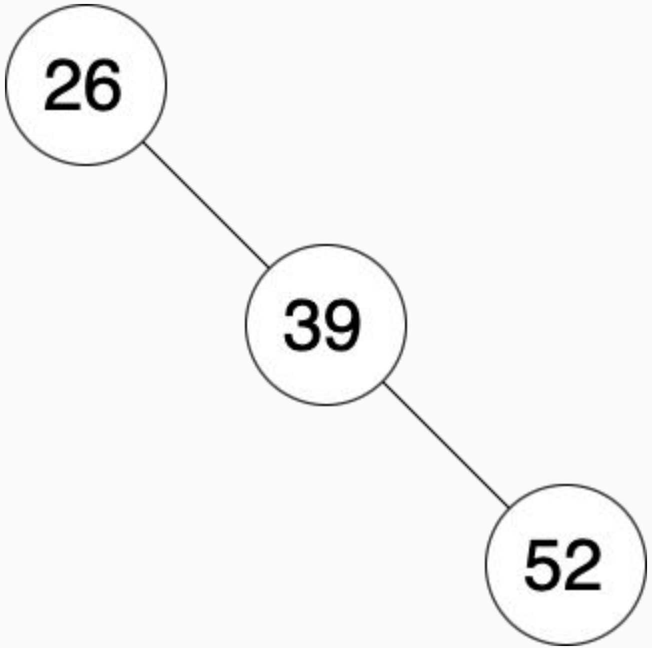
Is this a binary tree?

YES!

Could this be a heap, binary search tree, or both?

BST (every node partitions its subtrees)

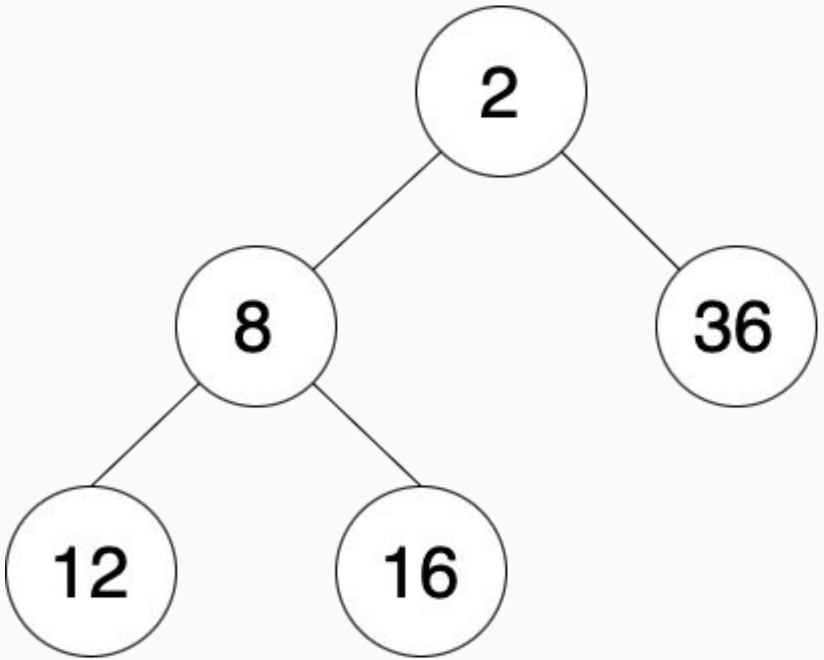
Not a heap (not complete)



Binary Tree Examples

Is this a binary tree?

Could this be a heap, binary search tree, or both?



Binary Tree Examples

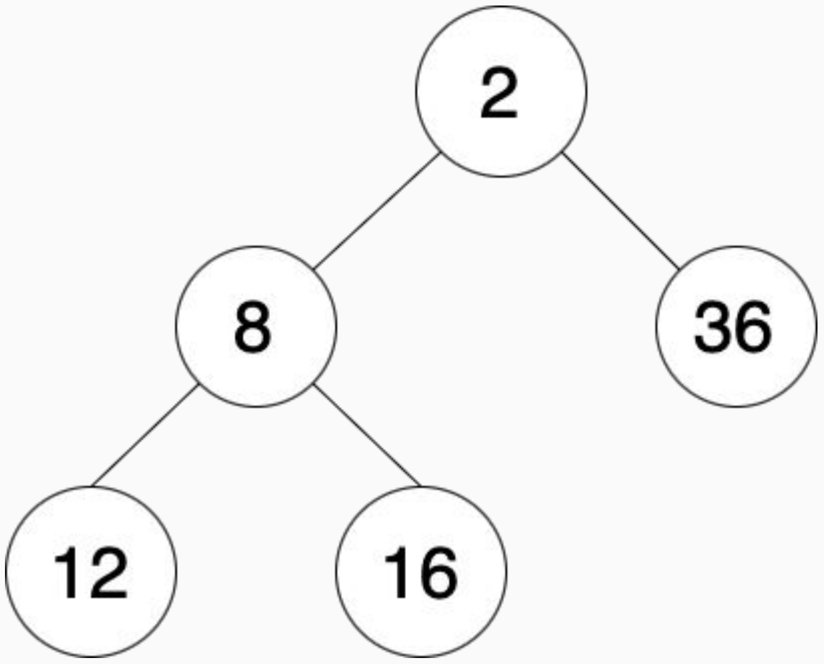
Is this a binary tree?

YES!

Could this be a heap, binary search tree, or both?

Min Heap (complete and every parent is ≤ its children)

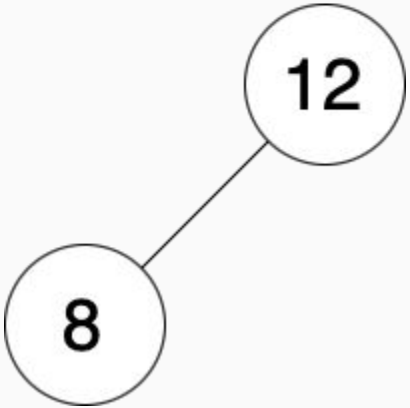
Not a BST (every left child is greater than its parent)



Binary Tree Examples

Is this a binary tree?

Could this be a heap, binary search tree, or both?



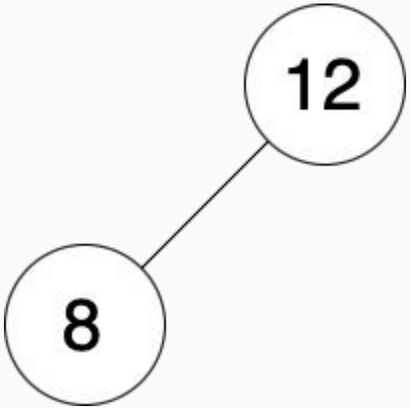
Binary Tree Examples

Is this a binary tree?

YES!

Could this be a heap, binary search tree, or both?

Could be either a BST or a Max Heap!



Exercise (Part 1)

Draw a Min Heap containing the values {1, 2, 3, 4, 6, 7, 8, 9}

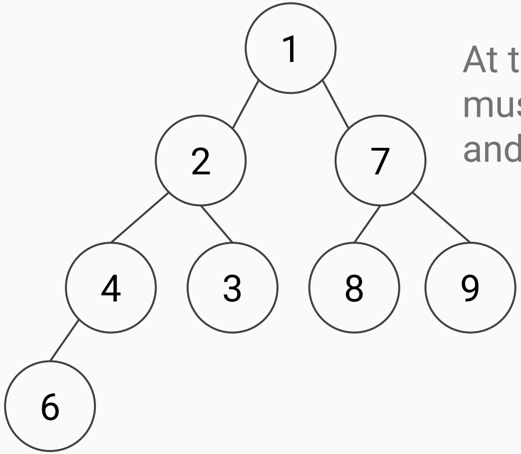
Draw a BST containing the values {1, 2, 3, 4, 6, 7, 8, 9}

Trade papers with a neighbor and verify that their trees are valid

- If they are not, discuss and fix with your neighbor
- If they are, keep your neighbors paper for the next exercise
- If you are unsure, ASK!

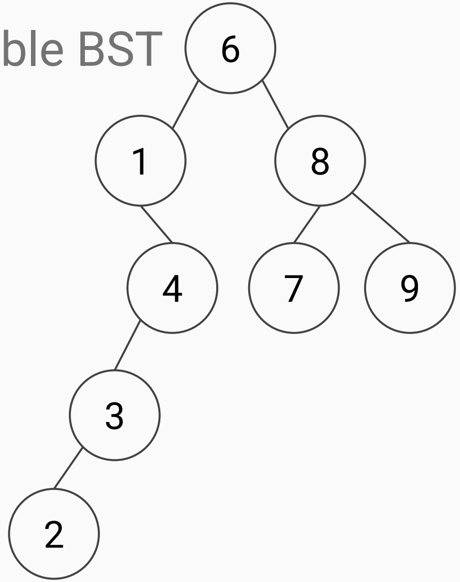
Example

One possible Heap



At the very least, your Heap must have this structure, and must have 1 at the root!

One possible BST



Exercise (Part 2)

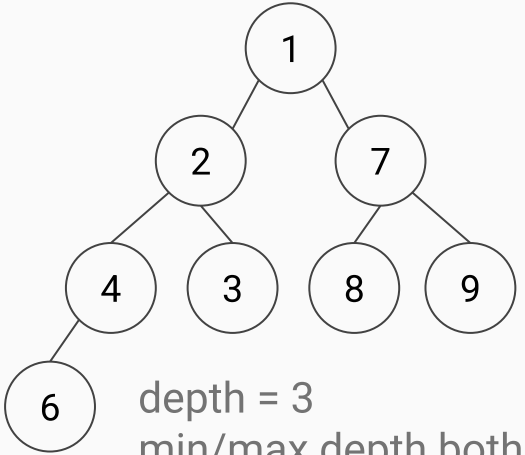
For each tree on your neighbors paper answer the following:

- What is the depth of the tree?
- What is the minimum/maximum depth the tree could be?
- Write out the array representation of the Heap
- Perform a left rotation around 6 on the BST (if possible)
    - How did it affect the depth of the BST?

Swap papers back with your neighbor and verify their answers, ask questions if you are unsure!

Example

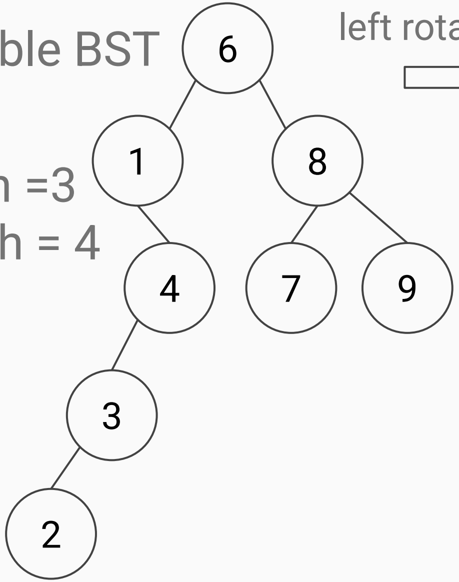
One possible Heap



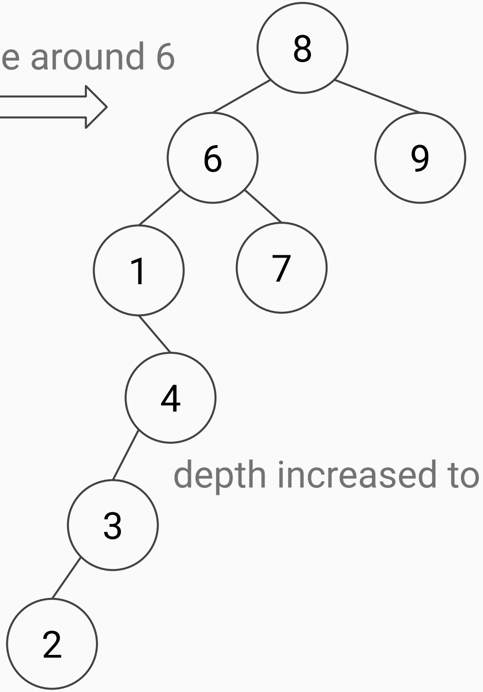
depth = 3
min/max depth both 3
[1,2,7,4,3,8,9,6]

One possible BST

depth = 4
min depth =3
max depth = 4



left rotate around 6



depth increased to 5

Exercise (Part 3)

On the paper in front of you:

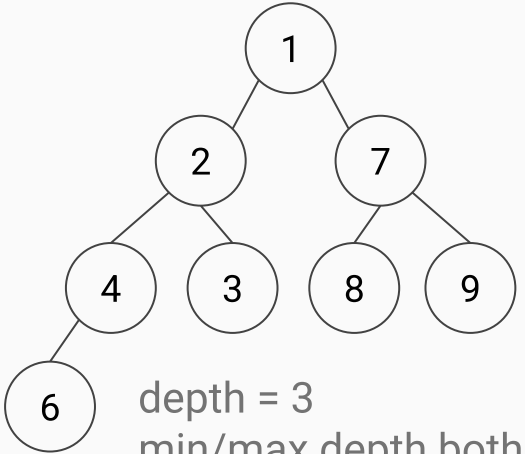
- Draw the heap that would result from inserting 5 into the heap
- Draw the BST that would result from inserting 5 into the BST

For each:

- How did the insertion affect the depth of the tree?
- How many comparisons did you have to do to perform the insertion?
- What is the maximum number of comparisons needed to:
    - Insert any value into the tree in front of you (and give a value that would cause this)
    - Insert 5 into any arbitrary (but valid) tree containing nodes {1,2,3,4,6,7,8,9}
    - Insert any value into any arbitrary (but valid tree) in terms of n?

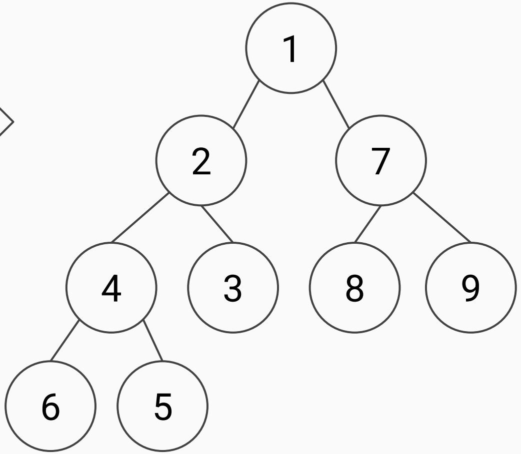
Example

One possible Heap



depth = 3
min/max depth both 3
[1,2,7,4,3,8,9,6]

Inserting 5



Comparisons required: 1

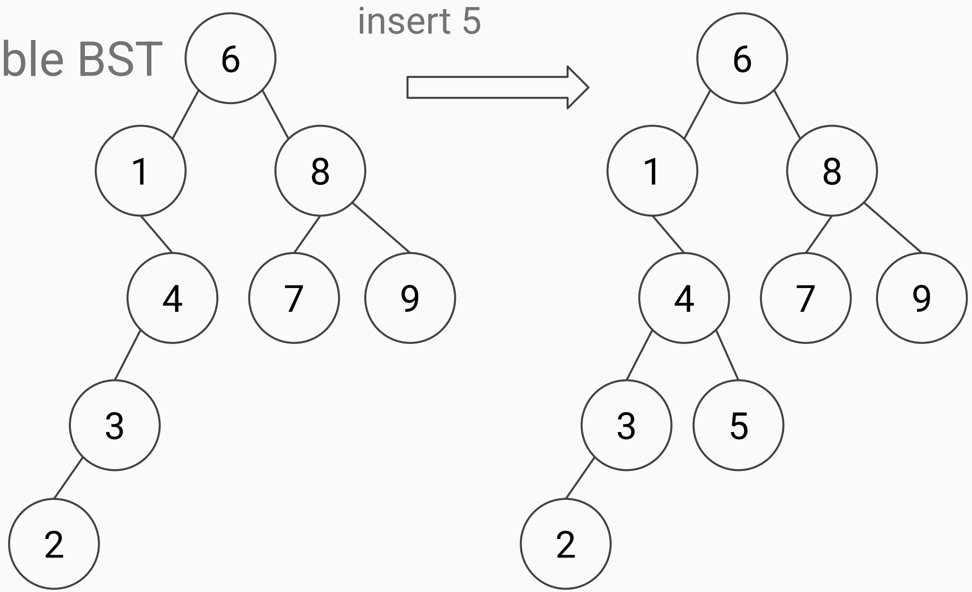
Max comparisons for this tree: 3 (for example when inserting 1)

Max comparisons to insert 5 into any heap with these values: 3

Max comparisons in general: log(n)

Example

One possible BST



Comparisons required: 3

Max comparisons for this tree: 5 (for example when inserting 1.5)

Max comparisons to insert 5 into any BST with these values: 8

Max comparisons in general: n

Bonus Questions

1. What are the tight bounds on inserting ***n*** elements into a Min Heap?
    a. What if the values are in **ascending order**?
    b. What if the values are in **descending order**?
2. What are the bounds on inserting ***n*** elements into a BST?
    a. What if the values are in **ascending order**?
    b. What if the values are in **descending order**?

Bonus Questions

1. What are the tight bounds on inserting ***n*** elements into a Min Heap? $O(n \log n)$, $\Omega(n)$
   a. What if the values are in **ascending order**? $\Theta(n)$
   b. What if the values are in **descending order**? $\Theta(n \log n)$
2. What are the bounds on inserting ***n*** elements into a BST? $O(n^2)$, $\Omega(n \log n)$
   a. What if the values are in **ascending order**? $\Theta(n^2)$
   b. What if the values are in **descending order**? $\Theta(n^2)$