Written Assignment #1

Due Date: Sunday, Feb 01 @ 11:59PM
Total Points: 30

Instructions

Answer all questions in this written assignment, showing work when required. All work must be your own, created solely by you and using only the allowed resources for the course (as stated in the syllabus). Your solutions may either be handwritten and scanned, or typeset. Submit your work as a PDF via AutoLab to the WA1 submission target.

You may submit as many times as you like, but **only your last submission will be graded** and should include the entire submission. You should view your submission after you upload it to make sure that it is not corrupted or malformed. Submissions that are rotated, upside down, or that do not load will not receive credit. Illegible or incomplete submissions will lose credit depending on what can be read. Ensure that your final submission contains all pages.

You are responsible for making sure your submission went through successfully.

Written submissions may be turned in up to one day late for a 50% penalty.

No grace day usage is allowed.

Questions

Simplify each of the following equations, $f_i(n)$. Your final result should be a sum-free equation in terms of n. **Show your work as a sequence of steps.** For each step, indicate the specific rule (see the provided cheatsheet below) that relates the current step to the previous one. All logarithms are base-2 and should be simplifed when possible.

Each function is worth 5 points, but no points will be awarded if work is not shown.

$$f_1(n) = \sum_{i=1}^{n} (n^3 + 6i)$$

$$f_2(n) = \sum_{i=3}^{n^3} \frac{n \cdot i}{6}$$

$$f_3(n) = \sum_{i=3}^{6} (i^2 + 1)$$

$$f_4(n) = \sum_{i=4}^{\log n} 3 \cdot 2^i$$

$$f_5(n) = \sum_{i=0}^{n} \sum_{j=0}^{\log i} 2^j$$

$$f_6(n) = \sum_{i=1}^{n^2} \sum_{j=1}^{\log n} i$$

Summation Rules

(S1) $\sum_{i=j}^{k} c = (k-j+1)c$

(S2) $\sum_{i=j}^{k} (cf(i)) = c\sum_{i=j}^{k} f(i)$

(S3) $\sum_{i=j}^{k} (f(i)+g(i)) = \sum_{i=j}^{k} f(i) + \sum_{i=j}^{k} g(i)$

(S4) $\sum_{i=j}^{k} f(i) = \sum_{i=l}^{k} f(i) - \sum_{i=l}^{j-1} f(i)$ for any $l < j$

(S5) $\sum_{i=j}^{k} f(i) = f(j) + f(j+1) + ... + f(k-1) + f(k)$

(S6) $\sum_{i=j}^{k} f(i) = f(j) + ... + f(l-1) + \sum_{i=l}^{k} f(i)$ for any $j < l \leq k$

(S7) $\sum_{i=j}^{k} f(i) = \left(\sum_{i=j}^{l} f(i)\right) + f(l+1) + ... + f(k)$ for any $j \leq l < k$

(S8) $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$

(S9) $\sum_{i=0}^{k} 2^i = 2^{k+1} - 1$

Log Rules

(L1) $\log(n^a) = a\log(n)$

(L2) $\log(an) = \log(a) + \log(n)$

(L3) $\log\left(\frac{n}{a}\right) = \log(n) - \log(a)$

(L4) $\log_b(n) = \frac{\log_c(n)}{\log_c(b)}$

(L5) $\log(2^n) = 2^{\log(n)} = n$