CSE 250

Data Structures

Dr. Eric Mikida
epmikida@buffalo.edu
208 Capen Hall

Lec 03: Math Refresher

Announcements and Feedback

- Make sure you are on Piazza and AutoLab
- Academic Integrity Quiz due 2/1 @ 11:59PM **(MUST GET 100%)**
- PA0 due 2/1 @ 11:59PM **(MUST GET 100%)**
- WA1 out now, due 2/1 @ 11:59PM
- Office hours and recitations start this week

Today's Topics

- Summations
- Logarithms

Summations

The general form of a summation:

$$\sum_{i=j}^{k} f(i) = f(j) + f(j+1) + ... + f(k)$$

i is the summation variable

j and ***k*** are variables representing summation bounds

f*(*i*)** is an arbitrary function of ***i (note this doesn't mean it has to include an ***i***)

Useful Tricks

If ***f***(***i***) = ***c*** and ***c*** is a constant (with respect to ***i***)

$$\sum_{i=j}^{k} c = \underbrace{(c + c + ... + c)}_{(k - j + 1) \text{ times}}$$

Useful Tricks

If ***f*(*i*)** = ***c*** and ***c*** is a constant (with respect to ***i***)

$$\sum_{i=j}^{k} c = (c + c + ... + c)$$

$$= (k - j + 1) \cdot c$$

Therefore summations of a constant can immediately be written in a closed form! (ie a form without any summations)

Useful Tricks

If ***c*** is a constant and ***f*(*i*)** is a function of ***i***:

$$\sum_{i=j}^{k} cf(i) = (cf(j) + cf(j+1) + ... + cf(k))$$

Useful Tricks

If ***c*** is a constant and ***f***(***i***) is a function of ***i***:

$$\sum_{i=j}^{k} cf(i) = (cf(j) + cf(j+1) + ... + cf(k))$$

$$= c(f(j) + f(j+1) + ... + f(k))$$

Useful Tricks

If ***c*** is a constant and ***f*(*i*)** is a function of ***i***:

$$\sum_{i=j}^{k} cf(i) = (cf(j) + cf(j+1) + ... + cf(k))$$

$$= c(f(j) + f(j+1) + ... + f(k))$$

$$= c\sum_{i=j}^{k} f(i)$$

Therefore constants can be pulled out of the summation!

Useful Tricks

If ***f*(*i*)** and ***g*(*i*)** are functions of ***i***:

$$\sum_{i=j}^{k}(f(i) + g(i)) = (f(j) + g(j)) + (f(j+1) + g(j+1)) + ... + (f(k) + g(k))$$

Useful Tricks

If ***f*(*i*)** and ***g*(*i*)** are functions of ***i***:

$$\sum_{i=j}^{k}(f(i)+g(i)) = (f(j)+g(j)) + (f(j+1)+g(j+1)) + \ldots + (f(k)+g(k))$$

$$= (f(j)+f(j+1)+\ldots+f(k)) + (g(j)+g(j+1)+\ldots+g(k))$$

Useful Tricks

If ***f*(*i*)** and ***g*(*i*)** are functions of ***i***:

$$\sum_{i=j}^{k}(f(i) + g(i)) = (f(j) + g(j)) + (f(j+1) + g(j+1)) + ... + (f(k) + g(k))$$

$$= (f(j) + f(j+1) + ... + f(k)) + (g(j) + g(j+1) + ... + g(k))$$

$$= \left(\sum_{i=j}^{k} f(i)\right) + \left(\sum_{i=j}^{k} g(i)\right)$$

Therefore a summation of a function with multiple terms can be split into multiple simpler summations

Useful Tricks

If ***j < l < k***:

$$\sum_{i=j}^{k} f(i) = f(j) + ... + f(k)$$

Useful Tricks

If ***j* < *l* < *k***:

$$\sum_{i=j}^{k} f(i) = f(j) + ... + f(k)$$

$$= f(j) + ... + f(l-1) + f(l) + ... + f(k)$$

Useful Tricks

If ***j* < *l* < *k***:

$$\sum_{i=j}^{k} f(i) = f(j) + ... + f(k)$$

$$= f(j) + ... + f(l-1) + f(l) + ... + f(k)$$

$$= \left(\sum_{i=j}^{l-1} f(i)\right) + \left(\sum_{i=l}^{k} f(i)\right)$$

Therefore we can group terms of the summation to turn one summation into two summations with different bounds…AND…

Useful Tricks

If ***j* < *l* < *k***:

$$\left(\sum_{i=j}^{k} f(i)\right) = \left(\sum_{i=j}^{l-1} f(i)\right) + \left(\sum_{i=l}^{k} f(i)\right)$$

Useful Tricks

If ***j* < *l* < *k***:

$$\left(\sum_{i=j}^{k} f(i)\right) = \boxed{\left(\sum_{i=j}^{l-1} f(i)\right)} + \left(\sum_{i=l}^{k} f(i)\right)$$

Subtract to other side

Useful Tricks

If ***j* < *l* < *k***:

$$\left(\sum_{i=j}^{k} f(i)\right) = \left(\sum_{i=j}^{l-1} f(i)\right) + \left(\sum_{i=l}^{k} f(i)\right)$$

$$\left(\sum_{i=j}^{k} f(i)\right) - \left(\sum_{i=j}^{l-1} f(i)\right) = \left(\sum_{i=l}^{k} f(i)\right)$$

…by moving terms around, we can adjust the bounds of the summation

Series

Some common closed form solutions:

$$\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$$

$$\sum_{i=0}^{k} 2^i = 2^{k+1} - 1$$

Note that the lower bounds for these summations are NOT variables… they are specific constants.

Summary

- The previous tricks will always be provided on WAs and exams
- Usually the goal will be to reduce some complicated summation to a simpler form without a summation
  - Some of the tricks get rid of summations
  - Some allow you to manipulate summations/bounds so that you can apply tricks that get rid of summations
- Be cognizant of what variables are constant ***with respect to the summation variable*** and which one aren't

Logarithms

$$a \cdot n = \underbrace{a + a + ... + a}_{\textbf{\textit{a}} \text{ added together } \textbf{\textit{n}} \text{ times}}$$

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$$a \cdot n = \underbrace{a + a + \ldots + a}_{\text{a added together n times}}$$

a added together ***n*** times

$$a^n = \underbrace{a \cdot a \cdot \ldots \cdot a}$$

a multiplied together ***n*** times

Logarithms

The inverse operation is division

$$\underbrace{a \cdot n = a + a + ... + a}$$

a added together ***n*** times

$$a^n = \underbrace{a \cdot a \cdot ... \cdot a}$$

a multiplied together ***n*** times

The inverse operation is ???

Logarithms

The inverse operation is division

$$a \cdot n = \underbrace{a + a + \ldots + a}$$

a added together ***n*** times

$$a^n = \underbrace{a \cdot a \cdot \ldots \cdot a}$$

a multiplied together ***n*** times

The inverse operation is $\mathbf{log_a}$

Logarithms

$$\log_a(b) = \text{the number of times you multiply a together to get b}$$

$$\log_2(32) = 5$$

$$\log_3(27) = 3$$

$$\log_2(\frac{1}{8}) = -3$$

$$\log_2(2^{10}) = 10$$

Logarithms

Logarithm is the inverse exponent

$$b^{\log_b(n)} = n = \log_b(b^n)$$

Product Rule

Let's say $\boldsymbol{n = a \cdot b}$

How are $\mathbf{log_2(\boldsymbol{n})}$, $\mathbf{log_2(\boldsymbol{a})}$, and $\mathbf{log_2(\boldsymbol{b})}$ related?

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How are $\mathbf{log_2(\boldsymbol{n})}$, $\mathbf{log_2(\boldsymbol{a})}$, and $\mathbf{log_2(\boldsymbol{b})}$ related?

$$a = 2 \cdot \ldots \cdot 2 \qquad\qquad b = 2 \cdot \ldots \cdot 2$$

Product Rule

Let's say $\boldsymbol{n = a \cdot b}$

How are $\mathbf{log_2(}\boldsymbol{n}\mathbf{)}$, $\mathbf{log_2(}\boldsymbol{a}\mathbf{)}$, and $\mathbf{log_2(}\boldsymbol{b}\mathbf{)}$ related?

$$a = \underbrace{2 \cdot \ldots \cdot 2}_{\log_2(a) \text{ times}} \qquad b = \underbrace{2 \cdot \ldots \cdot 2}_{\log_2(b) \text{ times}}$$

Product Rule

Let's say ***n* = *a* · *b***

How are $\log_2(n)$, $\log_2(a)$, and $\log_2(b)$ related?

$$a = 2 \cdot \ldots \cdot 2 \qquad\qquad b = 2 \cdot \ldots \cdot 2$$

$$n = \underbrace{2 \cdot \ldots \cdot 2}_{\log_2(n) \text{ times}} = \underbrace{2 \cdot \ldots \cdot 2}_{\log_2(a) \text{ times}} \cdot \underbrace{2 \cdot \ldots \cdot 2}_{\log_2(b) \text{ times}}$$

Product Rule

Let's say $n = a \cdot b$

How are $\log_2(n)$, $\log_2(a)$, and $\log_2(b)$ related?

$$a = 2 \cdot \ldots \cdot 2 \qquad\qquad b = 2 \cdot \ldots \cdot 2$$

$$n = \underbrace{2 \cdot \ldots \cdot 2}_{\log_2(n) \text{ times}} = \underbrace{2 \cdot \ldots \cdot 2}_{\log_2(a) \text{ times}} \cdot \underbrace{2 \cdot \ldots \cdot 2}_{\log_2(b) \text{ times}}$$

$$\log_2(n) = \log_2(ab) = \log_2(a) + \log_2(b)$$

Exponent Rule

$$\log_2(a^n) = \log_2(a \cdot ... \cdot a)$$

Exponent Rule

$$\log_2(a^n) = \log_2(\overbrace{a \cdot \ldots \cdot a}^{\textbf{\textit{n}}\text{ times}})$$

Exponent Rule

$$\log_2(a^n) = \log_2(\overbrace{a \cdot \ldots \cdot a}^{n \text{ times}})$$

$$= \log_2(a) + \log_2(\overbrace{a \cdot \ldots \cdot a}^{n-1 \text{ times}})$$

Exponent Rule

$$\log_2(a^n) = \log_2(\overbrace{a \cdot \ldots \cdot a}^{\textbf{\textit{n}}\text{ times}})$$

$$= \log_2(a) + \log_2(\overbrace{a \cdot \ldots \cdot a}^{\textbf{\textit{n}-1}\text{ times}})$$

$$= \log_2(a) + \log_2(a) + \log_2(\overbrace{a \cdot \ldots \cdot a}^{\textbf{\textit{n}-2}\text{ times}})$$

Exponent Rule

$$\log_2(a^n) = \log_2(\overbrace{a \cdot \ldots \cdot a}^{n \text{ times}})$$

$$= \log_2(a) + \log_2(\overbrace{a \cdot \ldots \cdot a}^{n\text{-1 times}})$$

$$= \log_2(a) + \log_2(a) + \log_2(\overbrace{a \cdot \ldots \cdot a}^{n\text{-2 times}})$$

$$= \underbrace{\log_2(a) + \ldots + \log_2(a)}_{n \text{ times}}$$

Exponent Rule

$$\begin{aligned}
\log_2(a^n) &= \log_2(a \cdot ... \cdot a) \\
&= \log_2(a) + \log_2(a \cdot ... \cdot a) \\
&= \log_2(a) + \log_2(a) + \log_2(a \cdot ... \cdot a) \\
&= \log_2(a) + ... + \log_2(a) \\
&= n \cdot \log_2(a)
\end{aligned}$$

Division Rule

$$\log_2(\frac{a}{b}) = \log_2(a \cdot \frac{1}{b})$$

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$$= \log_2(a) + \log_2(b^{-1})$$

Division Rule

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$$= \log_2(a) + \log_2(\frac{1}{b})$$

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$$= \log_2(a) - \log_2(b)$$

Change of Base

$$b^m = n$$

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$$m = \frac{\log_c(n)}{\log_c(b)}$$

Change of Base

$$b^m = n$$

$$\log_c(b^m) = \log_c(n)$$

$$m \cdot \log_c(b) = \log_c(n)$$

$$m = \frac{\log_c(n)}{\log_c(b)}$$

$$\log_b(n) = \frac{\log_c(n)}{\log_c(b)}$$

Summary

Exponent Rule	$\log(n^a) = a\log(n)$
Product Rule	$\log(ab) = \log(a) + \log(b)$
Division Rule	$\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$
Change of Base	$\log_b(n) = \frac{\log_c(n)}{\log_c(b)}$
Inverse	$b^{\log_b(n)} = \log_b(b^n) = n$

** for this class, always assume base 2 unless otherwise stated **