CSE 250
Data Structures

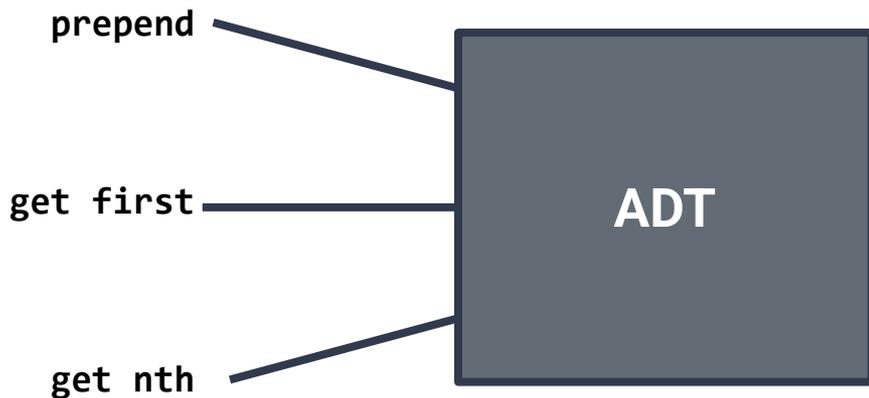Dr. Eric Mikida
epmikida@buffalo.edu
208 Capen Hall

Lec 05: Intro to Complexity

Announcements and Feedback

- AI Quiz, PA0, WA1 due Sunday @ 11:59PM
- PA1 releases Monday

Thought Experiment

Often, many data structures can satisfy a given ADT…how do you choose?

Thought Experiment

Data Structure 1
- Very fast **prepend**, **get first**
- Very slow **get nth**

Data Structure 2
- Very fast **get nth**, **get first**
- Very slow **prepend**

Data Structure 3
- Very fast **get nth**, **get first**
- Occasionally slow **prepend**

Which is better?

Thought Experiment

Data Structure 1 (LinkedList)
- Very fast **prepend**, **get first**
- Very slow **get nth**

Data Structure 2 (Array)
- Very fast **get nth**, **get first**
- Very slow **prepend**

Data Structure 3 (ArrayList…in reverse)
- Very fast **get nth**, **get first**
- Occasionally slow **prepend**

Which is better?

IT DEPENDS!

Thought Experiment

Data Structure 1 (LinkedList)
- Very **fast** prepend, `get first`
- Very **slow** `get nth`

Data Structure 2 (ArrayList)
- Very **fast** `get nth`, `get first`
- Very **slow** prepend

Data Structure 3 (ArrayList…in reverse)
- Very **fast** `get nth`, `get first`
- Occasionally **slow** prepend

What is "fast"? "slow"?

Attempt #1: Wall-clock time?

- What is fast?
  - 10s? 100ms? 10ns?
  - …it depends on the task
- Algorithm vs Implementation
  - Compare Grace Hopper's implementation to yours
- What machine are you running on?
  - Your old laptop? A lab machine? The newest, shiniest processor on the market?
- What bottlenecks exist? CPU vs IO vs Memory vs Network…

Attempt #1: Wall-clock time?

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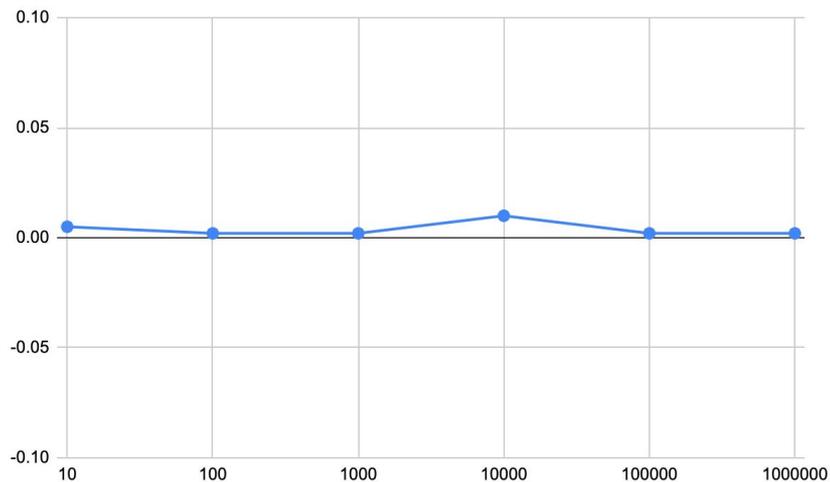
Wall-clock time is not terribly useful… 8

Analysis Checklist

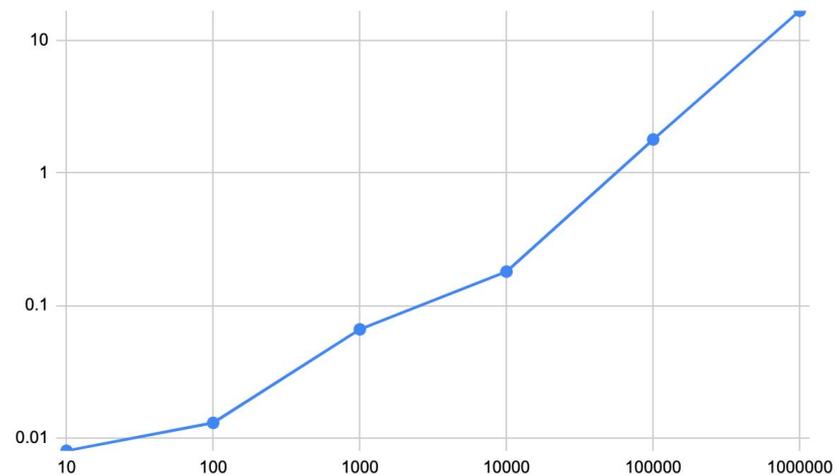1. Don't think in terms of wall-time, think in terms of "number of steps"

Let's do a quick demo…
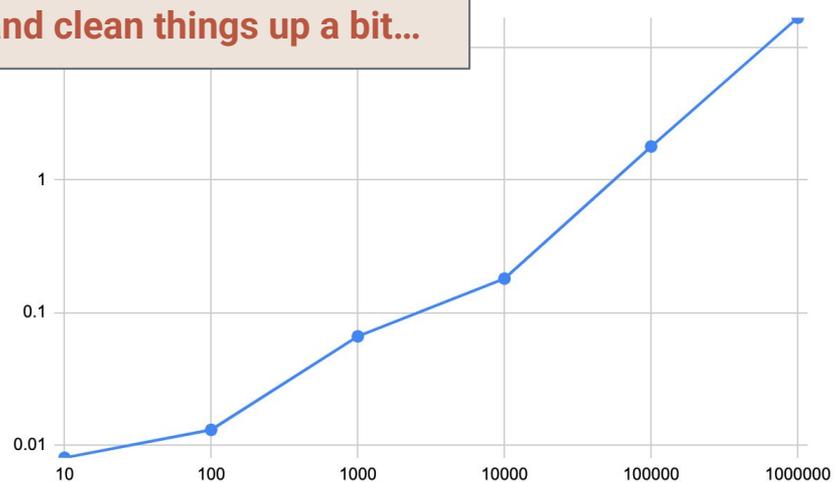
Comparing Random Access for Array vs List

Array

List

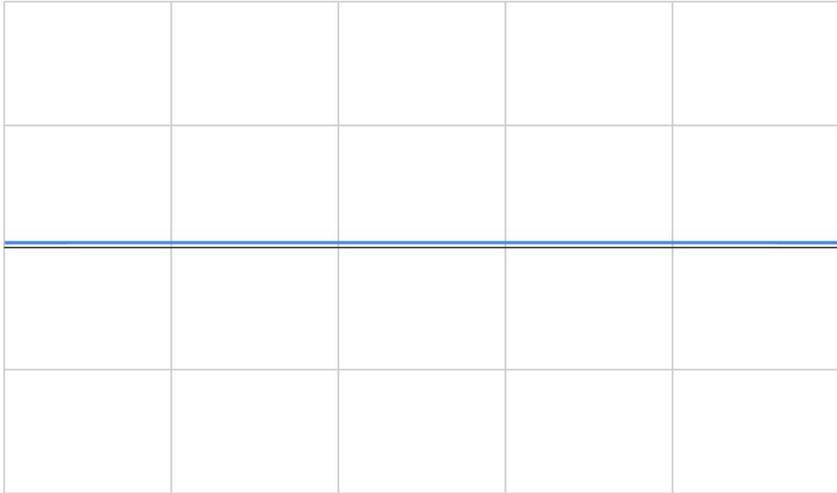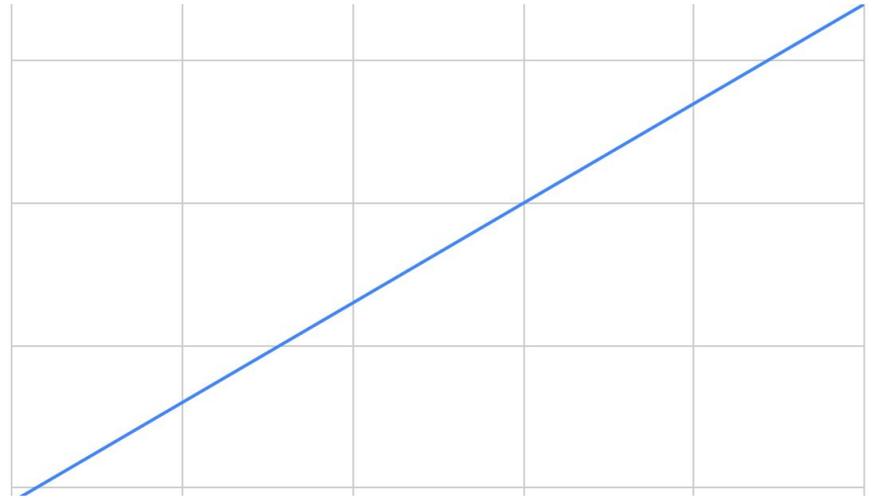# Comparing Random Access for Array vs List

Array

List

Let's ignore the specific numbers and clean things up a bit...

Comparing Random Access for Array vs List
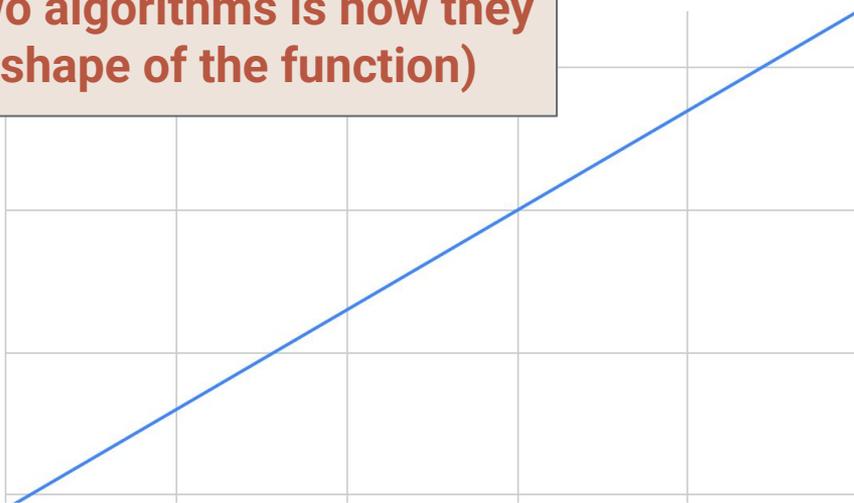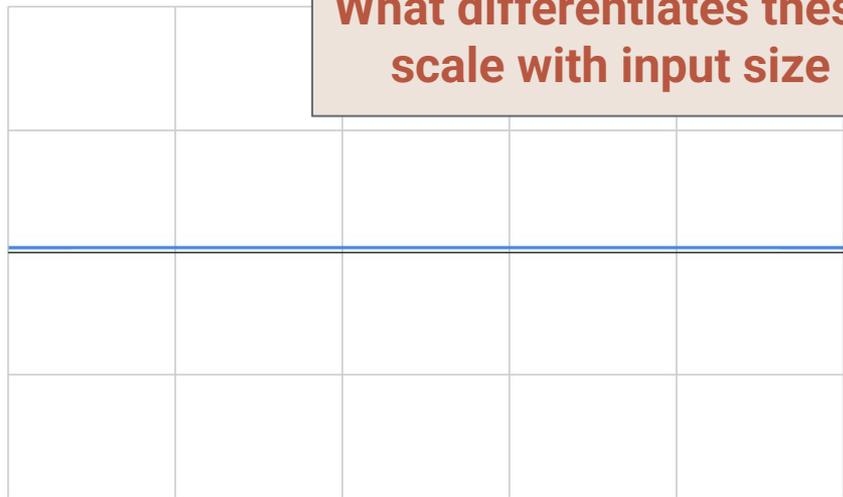
Array

List

Comparing Random Access for Array vs List

Array

List

What differentiates these two algorithms is how they scale with input size (the shape of the function)

Analysis Checklist

1. Don't think in terms of wall-time, think in terms of "number of steps"
2. **To give a useful solution, we should take "scale" into account**
   ○ **How does the runtime change as we change the size of the input?**

Counting Steps

```java
1  public void userFullName(User[] users, int id) {
2    User user = users[id];
3    String fullName = user.firstName + user.lastName;
4    return fullName;
5  }
```

Counting Steps

```java
1 public void userFullName(User[] users, int id) {
2   User user = users[id];
3   String fullName = user.firstName + user.lastName;
4   return fullName;
5 }
```

How many steps does this function take?

Counting Steps

```
1 public void userFullName(User[] users, int id) {
2   User user = users[id];
3   String fullName = user.firstName + user.lastName;
4   return fullName;
5 }
```

7 steps…ish? Maybe? What the heck is a step?

Counting Steps

```java
1  public void updateUsers(User[] users) {
2    x = 1;
3    for(User user : users) {
4      user.id = x;
5      x = x + 1;
6    }
7  }
```

Counting Steps

```
1  public void updateUsers(User[] users) {
2    x = 1;          ⟵
3    for(User user : users) {
4      user.id = x;
5      x = x + 1;
6    }
7  }
```

1

Counting Steps

```
1  public void updateUsers(User[] users) {
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3    for(User user : users) {   ←
4      user.id = x;
5      x = x + 1;
6    }
7  }
```

$$1 + \sum_{user \in users}$$

Counting Steps

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1  public void updateUsers(User[] users) {
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$$1 + \sum_{user \in users} 4$$

Counting Steps

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1  public void updateUsers(User[] users) {
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$$1 + \sum_{user \in users} 4 = 1 + 4 \cdot |\text{users}|$$
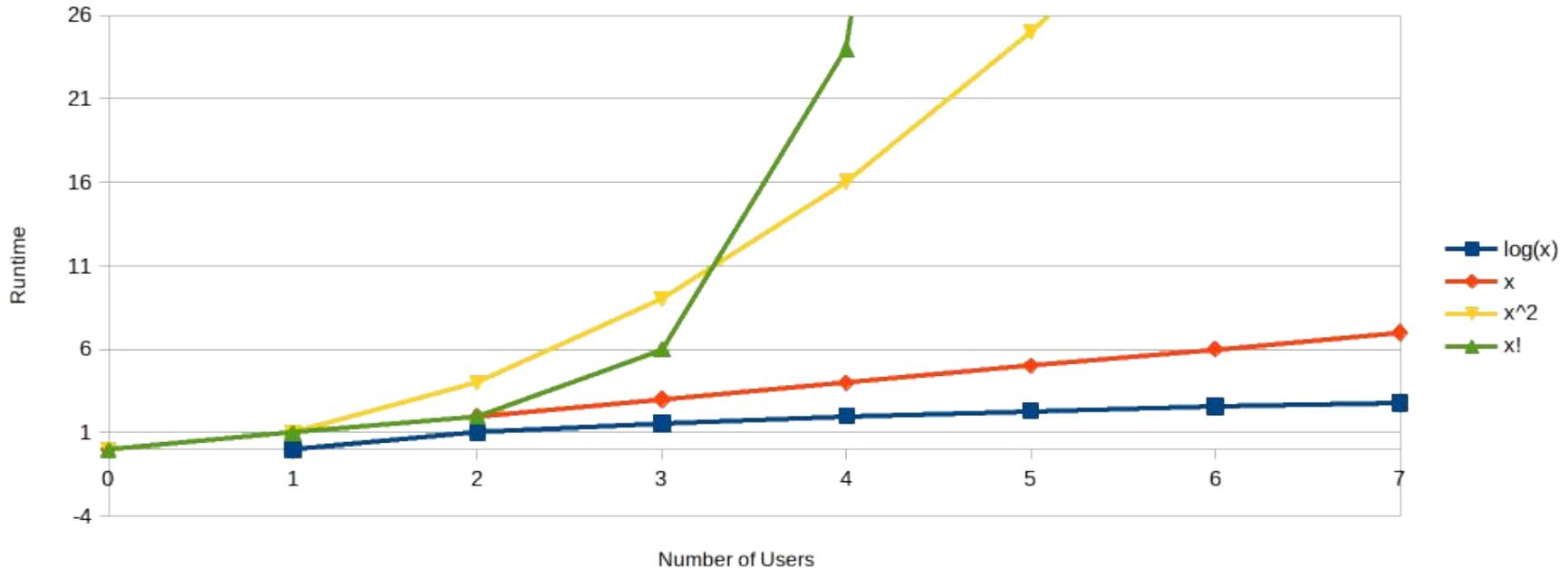
Steps to "Functions"

Now that we have number of steps* in terms of summations…

…which we can simplify (like in WA1) into mathematical functions…
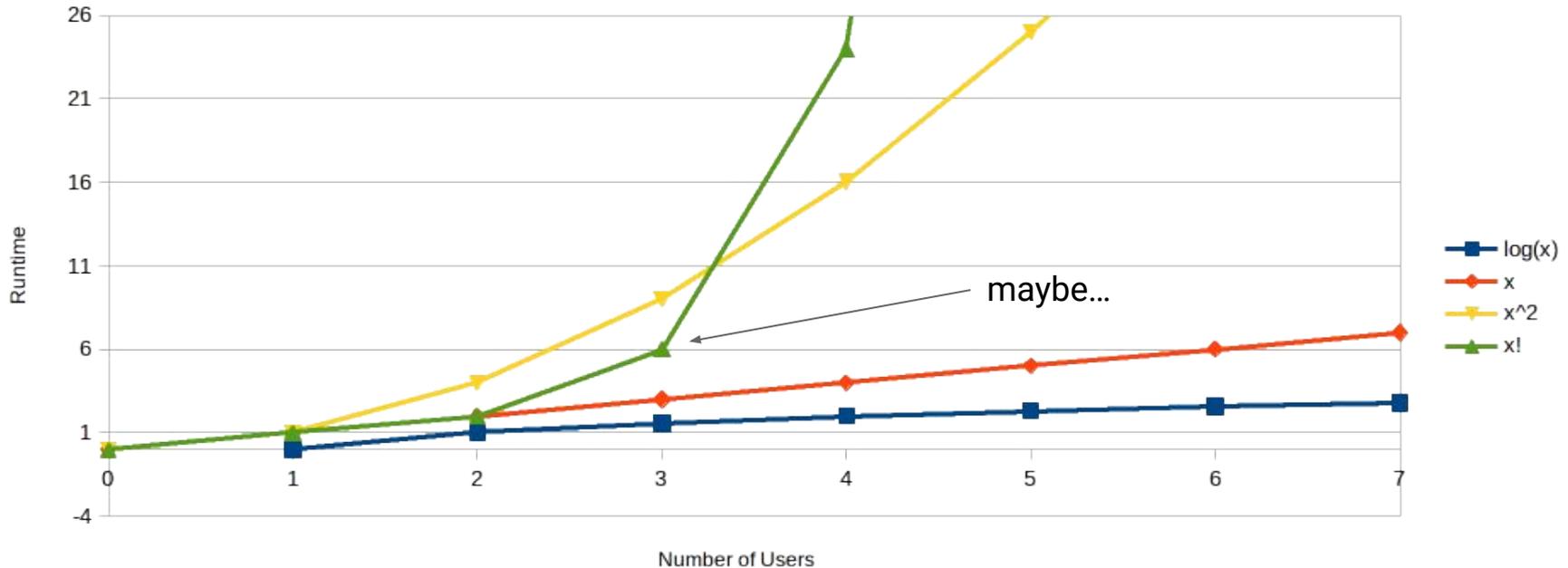
We can start analyzing runtime as a function

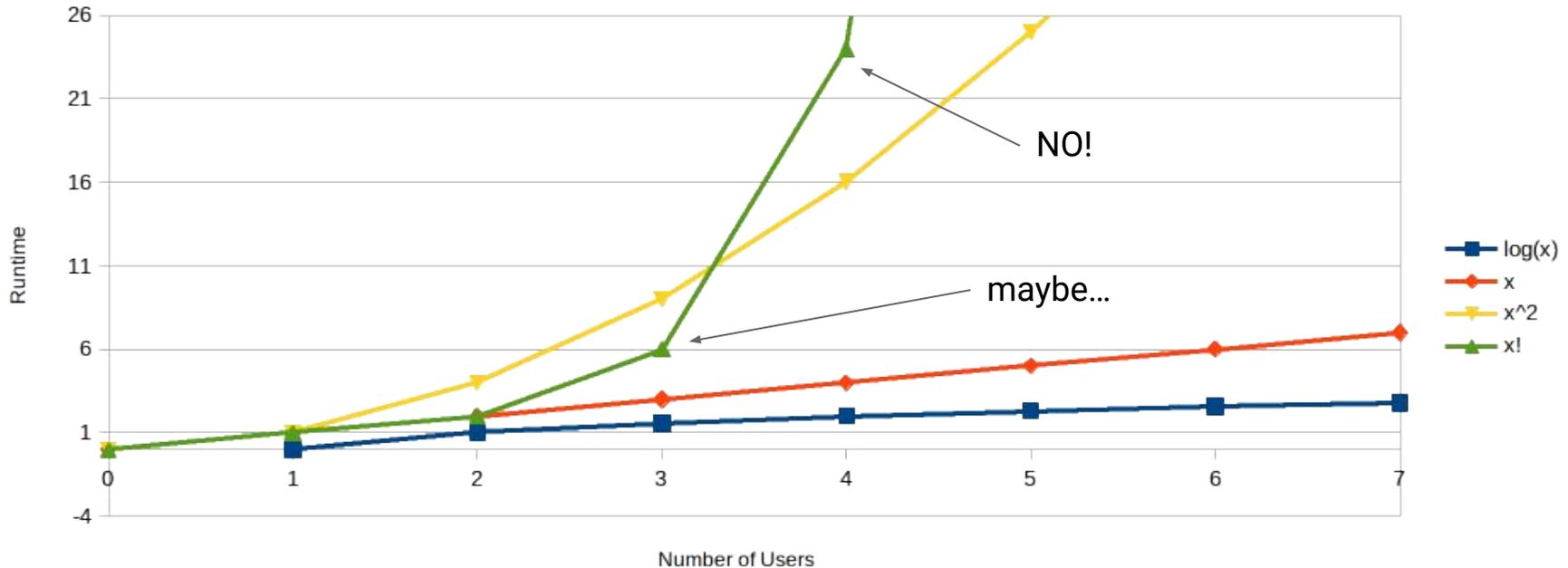*we'll give a better definition of what a "step" is later*

Runtime as a Function

Would you consider an algorithm that takes |Users|! number of steps?

Runtime as a Function

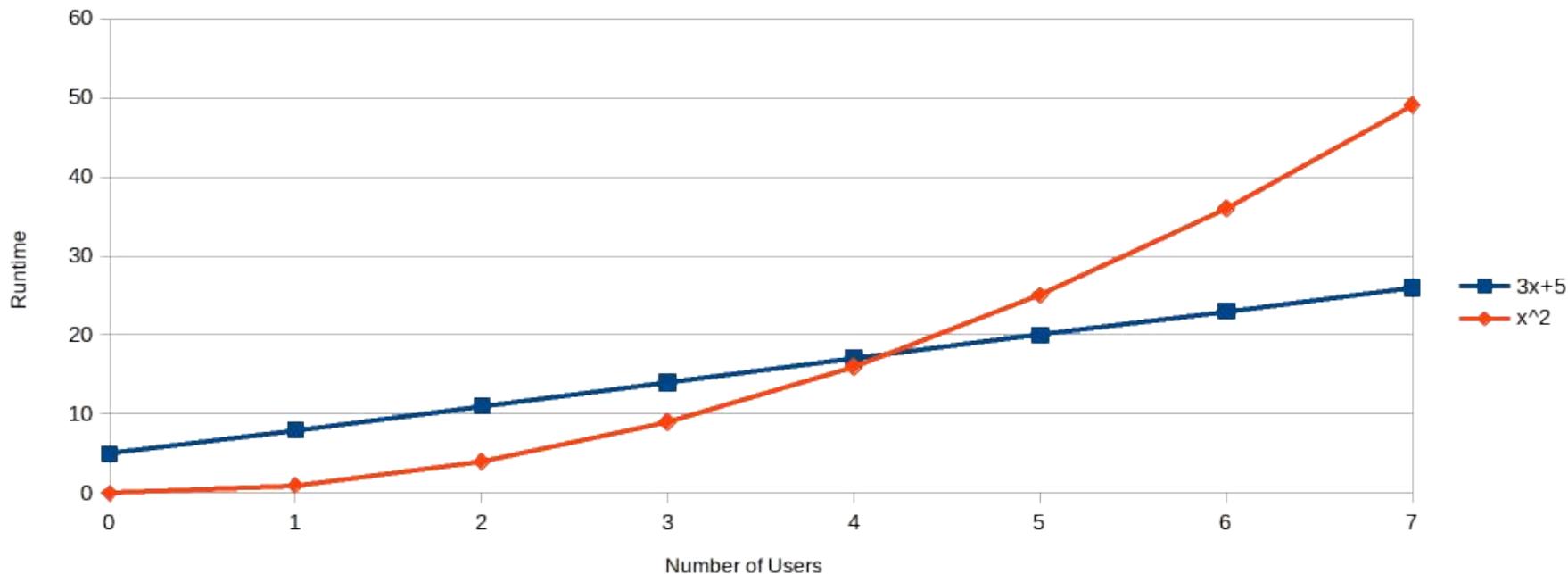**Would you consider an algorithm that takes |Users|! number of steps?**

Runtime as a Function

Would you consider an algorithm that takes |Users|! number of steps?

Runtime as a Function

Which is better? 3x|Users|+5 or |Users|^2

Analysis Checklist

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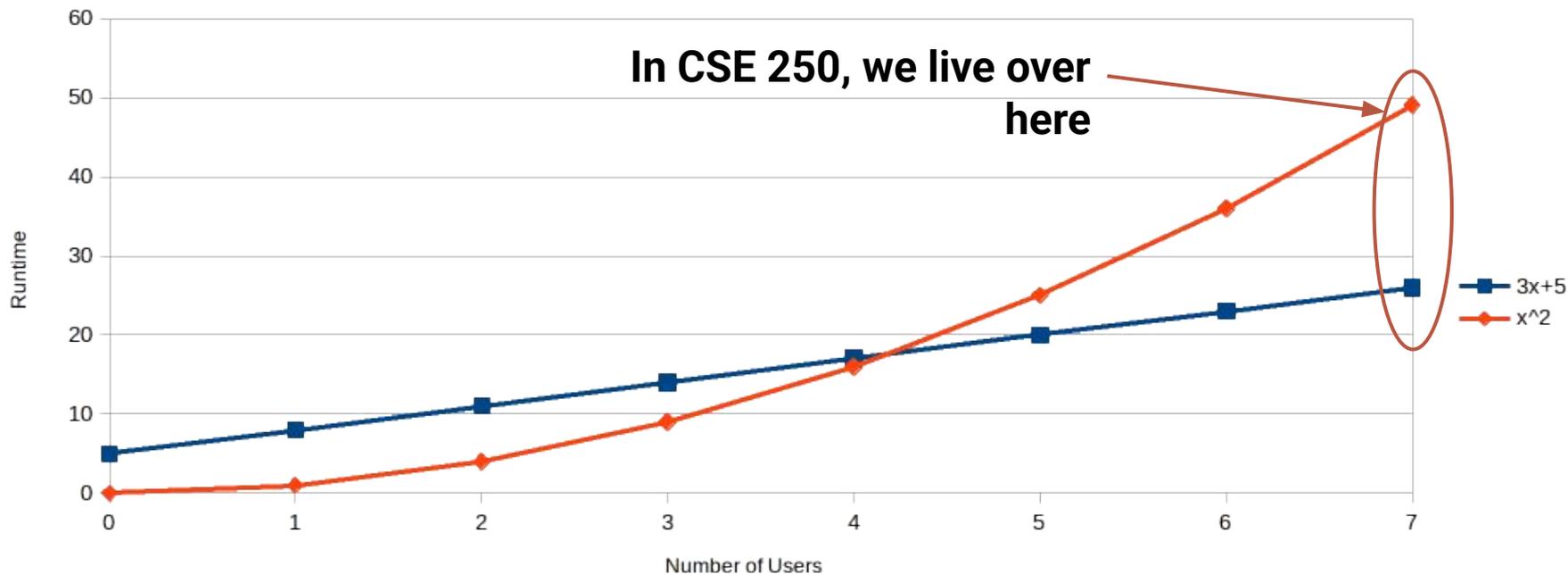   - How does the runtime change as we change the size of the input?

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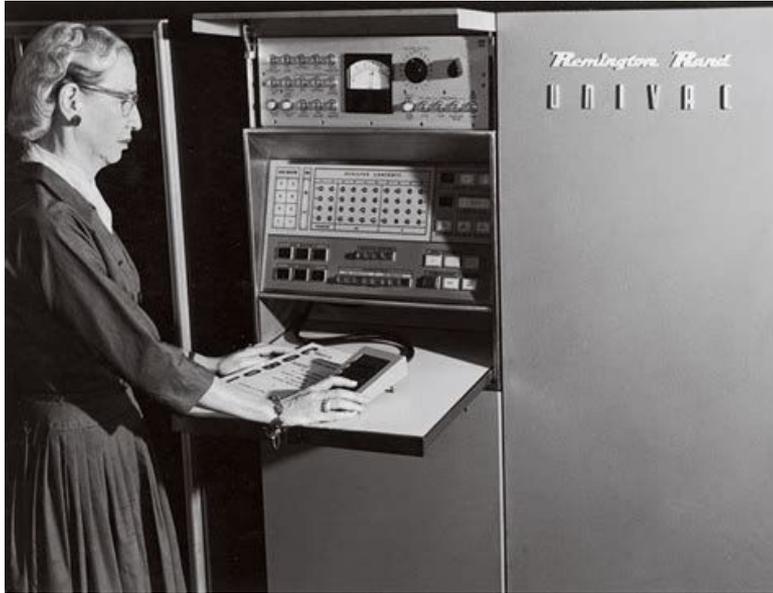3. **Focus on "large" inputs**
   - **Rank functions based on how they behave at large scales**

Runtime as a Function
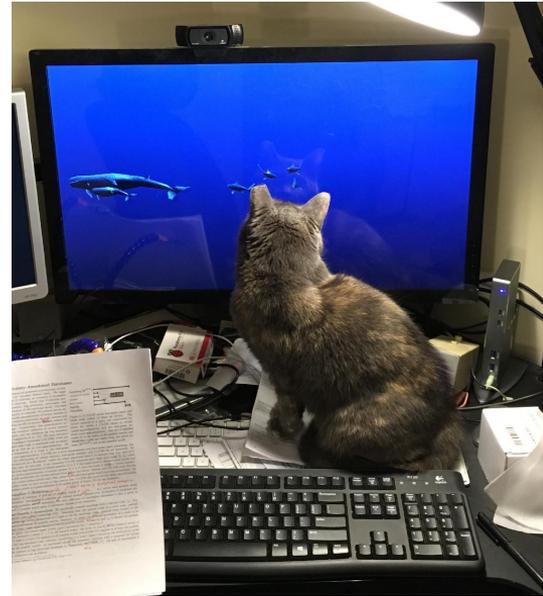

In CSE 250, we live over here

Legend:
- 3x+5
- x^2

Which is better? $3x|Users|+5$ or $|Users|^2$

Goal: Ignore implementation details

vs

Seasoned Pro Implementation

Error 23: Cat on Keyboard

Goal: Ignore execution environment

vs

Intel i9

Motorola 68000

Images from openclipart.org, used with permission
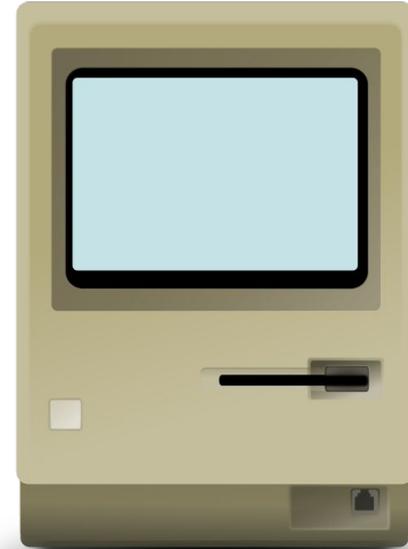
Goal: Judge the Algorithm Itself

- How fast is a step? Don't care
  - Only count number of steps
- Can this be done in two steps instead of one?
  - "3 steps per user" vs "some number of steps per user"
  - Sometimes we don't care...sometimes we do

Analysis Checklist

1. Don't think in terms of wall-time, think in terms of "number of steps"
2. To give a useful solution, we should take "scale" into account
   ○ How does the runtime change as we change the size of the input?
3. Focus on "large" inputs
   ○ Rank functions based on how they behave at large scales

Analysis Checklist

1. Don't think in terms of wall-time, think in terms of "number of steps"
2. To give a useful solution, we should take "scale" into account
   - How does the runtime change as we change the size of the input?
3. Focus on "large" inputs
   - Rank functions based on how they behave at large scales
4. **Decouple algorithm from infrastructure/implementation**
   - **Asymptotic notation…?**

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Wall-clock time is not terribly useful… 37

Attempt #2: Growth Functions

Not a function in code...but a mathematical function:

$$T(n)$$

n: The "size" of the input

    ie: number of users,rows, pixels, etc

T(n): The number of "steps" taken for input of size n

    ie: 20 steps per user, where n = |Users|, is 20 x n

Some Basic Assumptions:

Problem sizes are non-negative integers

$$n \in \{0, 1, 2, 3, ...\} = \{0\} \cup \mathbb{Z}^+$$

We can't reverse time…(obviously)

$$T(n) > 0$$

Smaller problems aren't harder than bigger problems

$$n_1 < n_2 \Rightarrow T(n_1) \leq T(n_2)$$

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$$n_1 < n_2 \Rightarrow T(n_1) \leq T(n_2)$$

$T: \{0\} \cup \mathbb{Z}^+ \rightarrow \mathbb{R}^+$

T is non-decreasing

First Problem...

We are still implementation dependent...

$$T_1(n) = 19n$$

$$T_2(n) = 20n$$

First Problem...

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Does 1 extra step per element really matter...?

Is this just an implementation detail?

First Problem...

We are still implementation dependent...

$$T_1(n) = 19n$$

$$T_2(n) = 20n$$

$$T_3(n) = 2n^2$$

T_1 and T_2 are much more "similar" to each other than they are to T_3

First Problem…

We are still implementation dependent…

$$T_1(n) = 19n$$

$$T_2(n) = 20n$$

$$T_3(n) = 2n^2$$

T_1 and T_2 are much more "similar" to each other than they are to T_3

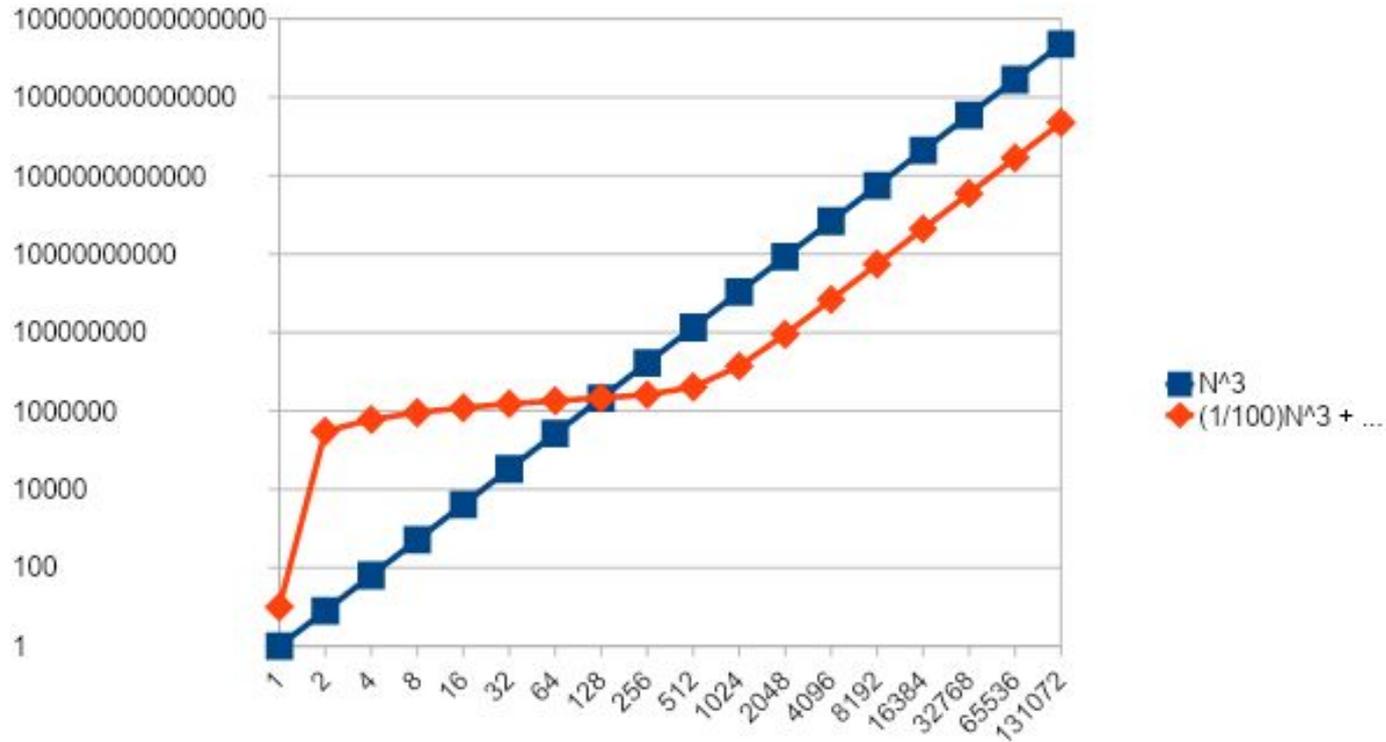*How do we capture this idea formally?*

How Do We Capture Behavior at Scale?
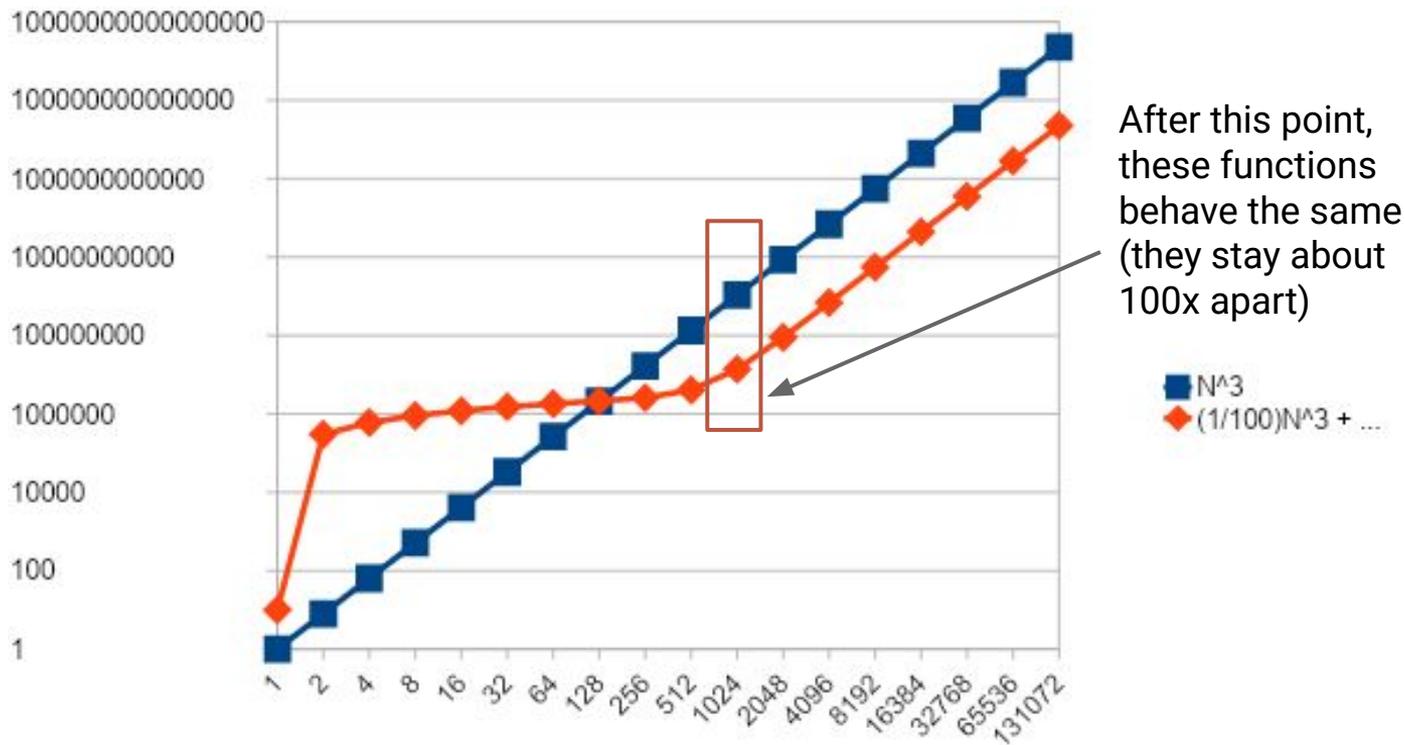
Consider the following two functions:

$$\frac{1}{100}n^3 + 10n + 1000000 \log(n)$$

$$n^3$$

How Do We Capture Behavior at Scale?

How Do We Capture Behavior at Scale?

After this point, these functions behave the same (they stay about 100x apart)

N^3
(1/100)N^3 + …

Attempt #3: Asymptotic Analysis

We want to organize runtimes (growth functions) into different *Complexity Classes*

Within the same complexity class, runtimes "behave the same"/"have the same shape" (at scale)