CSE 250
Data Structures

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Capen 208

Lecture 13
Binary Search

Announcements

- WA2 out now, due Sunday @ 11:59PM
- WA1 grades have been released
- Midterm info and practice exams will be posted by tonight
- PA1 has completed
  - Amnesty policy reminder
  - Imposter Syndrome

Sets and Bags (...so far)

	`LinkedList`	`ArrayList`
`Set.add`	$O(n)$	$O(n)$
`Set.contains`	$O(n)$	$O(n)$
`Set.remove`	$O(n)$	$\Theta(n)$
`Bag.add`	$O(1)$	$O(n)$, Amortized $\Theta(1)$
`Bag.contains`	$\Theta(n)$	$\Theta(n)$
`Bag.remove`	$O(n)$	$\Theta(n)$

Potential Improvements

Does order matter for `Sets`/`Bags`?

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Does order matter for `Sets`/`Bags`?

No! **`Sets`** and **`Bags`** are unordered...
...so we are not restricted to storing data in any particular order!

Can we take advantage of this?

Set Implementation (w/ArrayList)

```
ArrayList<T> data


remove(elem):
  idx = 0
  while idx < data.size():
    if data[idx] == elem:
      data.remove(idx)
      return true
    idx = idx + 1
  return false
```

Runtime: $\Theta(n)$

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What makes `remove` so expensive?

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What makes `remove` so expensive?

We have to shift over elements to fill the hole *while keeping everything in the same order*!

Can we improve on this?

Set Implementation (w/ArrayList)

```
ArrayList<T> data


remove(elem):
  idx = 0, last = data.size()-1
  while idx < data.size():
    if data[idx] == elem:
      data[idx] = data[last]
      data.remove(last)
      return true
    idx = idx + 1
  return false
```

Runtime: $\Theta(n)$

What makes `remove` so expensive?

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Can we improve on this?

Fill the hole with the last element!

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remove(elem):
  idx = 0, last = data.size()-1
  while idx < data.size():
    if data[idx] == elem:
      data[idx] = data[last]
      data.remove(last)
      return true
    idx = idx + 1
  return false
```

Runtime: ~~$\Theta(n)$~~ $O(n), \Omega(1)$

Runtime now becomes $O(n), \Omega(1)$

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Going Further

This was just a minor improvement...can we go further?

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First we need to understand more about searching...

Searching by Value

Consider searching for a value in an `Array` (or `ArrayList`)...

How long does that search take?

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This is called a **Linear Search** (it takes linear time)

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Could we improve on this if we knew our list had certain characteristics?

What if it was sorted?

Binary Search

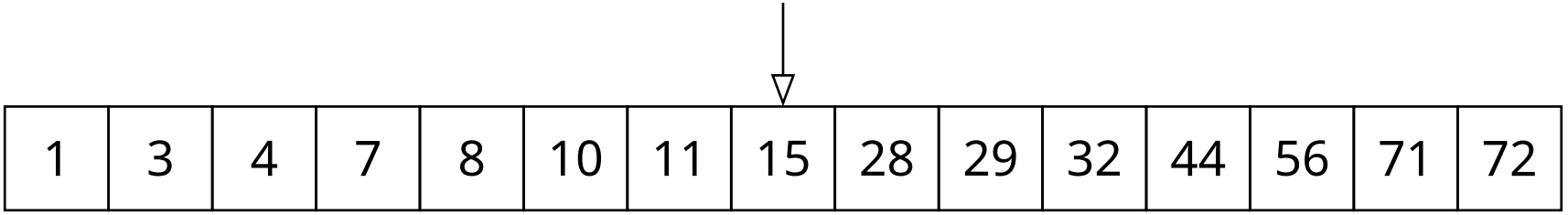
1	3	4	7	8	10	11	15	28	29	32	44	56	71	72

Searching For: 56
Current Index:

Imagine searching this (or a much larger) **`Array`** for a value, ie 56.

Where would ***you*** start?

Binary Search

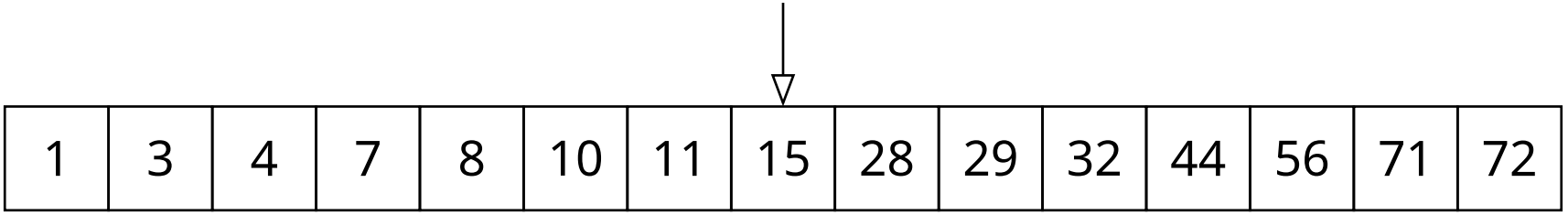


Searching For: 56
Current Index: 7

Imagine searching this (or a much larger) **`Array`** for a value, ie 56.

Where would ***you*** start? The middle!

Binary Search



Searching For: 56

Current Index: 7

What parts of the array can you rule out?

Binary Search

Searching For: 56

Current Index:

What parts of the array can you rule out? The whole left half!

Binary Search

Searching For: 56

Current Index: 11

Now just repeat on this new, smaller array!

Binary Search

Searching For: 56

Current Index: 13

Now just repeat on this new, smaller array!

Binary Search

Searching For: 56
Current Index: 12

Now just repeat on this new, smaller array!

FOUND IT!

Binary Search Runtime

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We cut the search space in half each time...What is the worst case scenario?

(AKA how many times might we have to halve the space?)

At most $\log(n)$ times!

Linear vs Binary Search

Linear Search:

- Removes 1 element from consideration each step, $O(n)$
- Does not require list to be sorted
- Does not require constant time random access

Binary Search:

- Removes half of the elements from consideration each step, $O(\log(n))$
- Requires list to be sorted
- Requires constant time random access (binary search on a **`LinkedList`** is $O(n)$)

$O(n)$ vs $O(\log(n))$

How big is the difference between $O(n)$ and $O(\log(n)$?

Imagine you have a sorted `Array` of size 1024:

- How many steps would a linear search take?

$O(n)$ vs $O(\log(n))$

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Imagine you have a sorted `Array` of size 1024:

- How many steps would a linear search take? **1024** (in the worst case)
- How many steps would a binary search take?

$O(n)$ vs $O(\log(n))$

How big is the difference between $O(n)$ and $O(\log(n)$?

Imagine you have a sorted `Array` of size 1024:

- How many steps would a linear search take? **1024** (in the worst case)
- How many steps would a binary search take? **10** (in the worst case)

That's a pretty big difference! ...but there's more...

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Imagine we just doubled the size of our `Array` to hold 2048 elements:

- Now how many steps does linear search take?

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- How many steps would a linear search take? **1024** (in the worst case)
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Imagine we just doubled the size of our `Array` to hold 2048 elements:

- Now how many steps does linear search take? **2048** (literally double the steps!)
- What about binary search?

$O(n)$ vs $O(\log(n))$

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Imagine you have a sorted `Array` of size 1024:

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- How many steps would a binary search take? **10** (in the worst case)

That's a pretty big difference! ...but there's more...

Imagine we just doubled the size of our `Array` to hold 2048 elements:

- Now how many steps does linear search take? **2048** (literally double the steps!)
- What about binary search? **11** (only added one extra step!)

$O(\log(n)$ scales MUCH better than $O(n)$

Implications for Set

Can we take advantage of Binary Search in our Set/Bag implementations?

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Can we take advantage of Binary Search in our Set/Bag implementations?

Let's store our data in a sorted **`ArrayList`**!

Set Implementation w/Sorted `ArrayList`

`boolean contains(elem):`

1. Use binary search to find **`elem`**	$O(\log(n))$, $\Omega(1)$

Total: $O(\log(n))$, $\Omega(1)$

Our **`contains`** implementation can now just use Binary Search, great!

What about **`add`**/**`remove`**?

Set Implementation w/Sorted `ArrayList`

`void add(elem):`

1. Use binary search to find index of **`elem`**	$O(\log(n))$, $\Omega(1)$
2. If **`elem`** is not there, insert it at that index	$O(n)$, $\Omega(1)$

Total: $O(n)$, $\Omega(1)$

Binary Search requires data to always be sorted!

- We can find the insertion point quickly 😊
- We need to shift elements over to make room and maintain order 😔

Set Implementation w/Sorted `ArrayList`

`boolean remove(elem):`

1. Use binary search to find index of **`elem`**	$O(\log(n))$, $\Omega(1)$
2. If **`elem`** is there, remove it by index	$O(n)$, $\Omega(1)$

Total: $O(n)$, $\Omega(???)$

Binary Search requires data to always be sorted!

- We can find the element to remove quickly 😊
- We need to shift elements over to fill the hole and maintain order 😔

What is the best case runtime?

Set Implementation w/Sorted `ArrayList`

`boolean remove(elem):`

1. Use binary search to find index of **`elem`**	$O(\log(n))$, $\Omega(1)$
2. If **`elem`** is there, remove it by index	$O(n)$, $\Omega(1)$

Total: $O(n)$, $\Omega(\log(n))$

Binary Search requires data to always be sorted!

- We can find the element to remove quickly 😊
- We need to shift elements over to fill the hole and maintain order 😔

What is the best case runtime? $\Omega(\log(n))$

- If we find the element fast, we have to shift over more elements
- If the element is near the end, it will take $\log(n)$ steps to find it

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`Bag.add`	$O(1)$	$O(n)$, Amortized $\Theta(1)$	???
`Bag.contains`	$\Theta(n)$	$\Theta(n)$	???
`Bag.remove`	$O(n)$	$O(n)$	???

What about the **Bag** methods?

Sets and Bags (...so far)

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`Set.contains`	$O(n)$	$O(n)$	$O(\log(n))$
`Set.remove`	$O(n)$	$O(n)$	$O(n)$
`Bag.add`	$O(1)$	$O(n)$, Amortized $\Theta(1)$	$O(n)$
`Bag.contains`	$\Theta(n)$	$\Theta(n)$	$O(n)$
`Bag.remove`	$O(n)$	$O(n)$	$O(n)$

Only change is that **`contains`** needs to count the number of occurrences

Which ADT/Data Structure is Best?

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ADT:

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ADT: `List`

- We need to be able to add/remove data, so we cannot use **`Sequence`**
- Order matters, so we cannot use **`Set`** or **`Bag`**

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Step 2: Choose the Data Structure that best implements the needed operations

- Append:
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- Append: **`ArrayList`** is $O(n)$, **`LinkedList`** is $O(1)$
- Remove: **`ArrayList`** is $O(n)$, **`LinkedList`** is $O(1)$, since index is alway 0

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Data Structure: **`LinkedList`**

- Both of our required operations are asymptotically faster

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- What operations do we need to solve the problem? append, access by position
- What characteristics do we need the data to have? ordered, variable size

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ADT: **`List`**

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- Order matters, so we cannot use **`Set`** or **`Bag`**

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- Access by Index: **`ArrayList`** is $O(1)$, **`LinkedList`** is $O(n)$

Data Structure: **`ArrayList`**

- We perform many adds in a row, and the amortized runtime of both **`ArrayList`** and **`LinkedList`** is $\Theta(1)$
- **`ArrayLists`** give much faster access by index

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- What operations do we need to solve the problem? add, search
- What characteristics do we need the data to have? unique, search by value

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- What operations do we need to solve the problem? add, search
- What characteristics do we need the data to have? unique, search by value

ADT: Set

- Search by value, not position – don't need to maintain order
- Unique elements

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- Add:
- Search by Value:

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Step 2: Choose the Data Structure that best implements the needed operations

- Add: **`ArrayList`** is $O(n)$, **`LinkedList`** is $O(n)$
- Search by Value: **`ArrayList`** is $O(n)$ but $O(\log(n))$ if sorted, **`LinkedList`** is $O(n)$

Data Structure:

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Data Structure: Sorted **`ArrayList`**

- We can take advantage of Binary Search to find movies quickly
- Adding is slow, but scenario focused on fast search