CSE 331: Algorithms & Complexity

"Greedy Algorithms II"

Prof. Charlie Anne Carlson (She/Her)

Lecture 16

Wednesday October 3rd, 2025

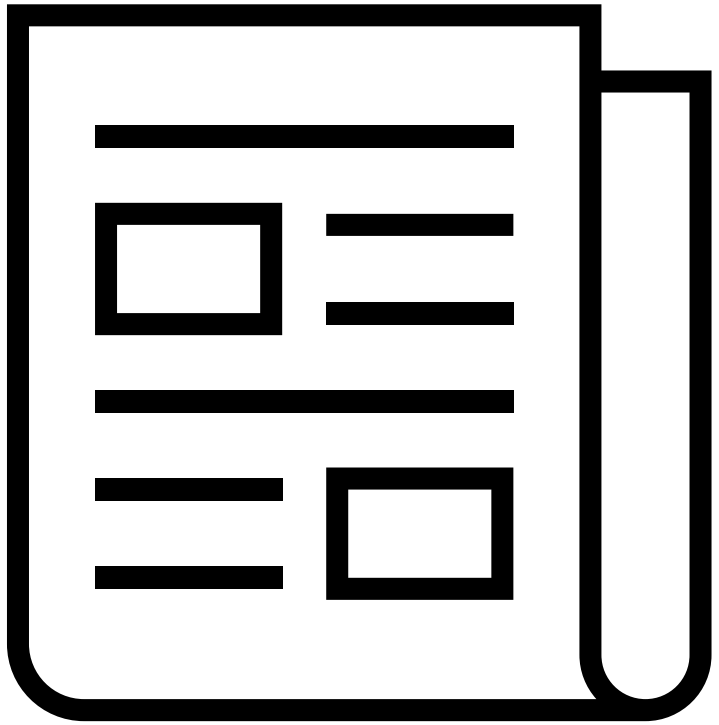
University at Buffalo®

Schedule

1. Course Updates
2. Greedy Algorithms
3. Interval Scheduling
4. Greedy Algorithm
5. Runtime Analysis

Course Updates

- HW 3 & Quiz #1 Grading
- HW 4 Out Soon
  - Not Due Next Week!
- Group Project
  - First Problems Oct 31st
- Sample Midterms Out
- Midterms Oct 6 and Oct 8



Midterm Advice

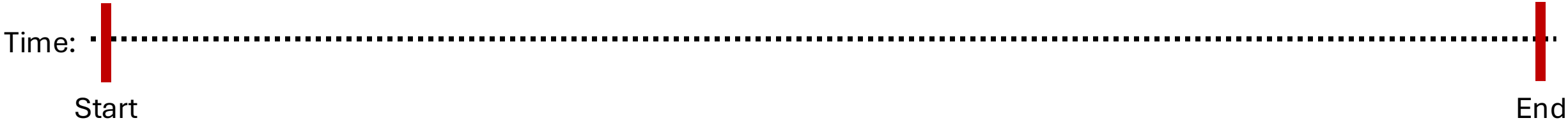
- Midterms Oct 6 and Oct 8.
- You get a reference sheet.
- Exams are in class during regular class time.
- Don’t panic!
  - Sleep, eat, drink water, and study what you can even it that isn’t everything!

What are Greedy Algorithm?

- Build solution one piece at a time.
- Only look at immediate information to make choices.
- Never go back on a decision.
- NOT ALWAYS THE BEST CHOICE!

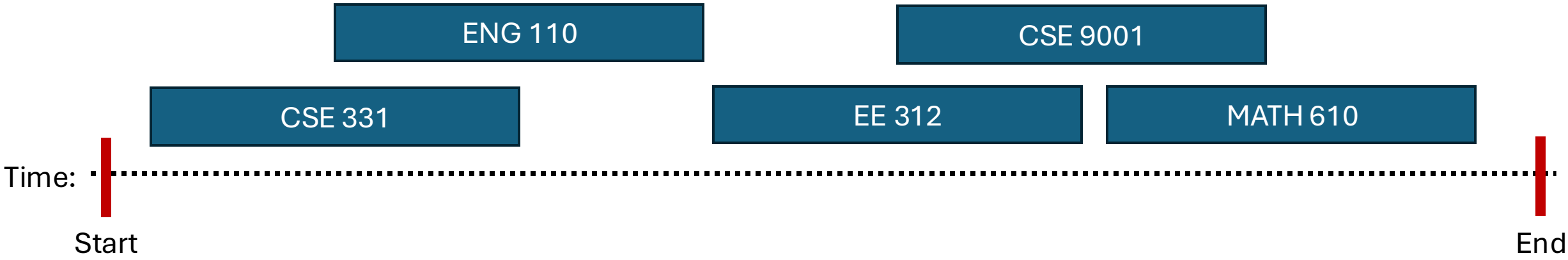
Interval Scheduling

- Consider an interval of time (e.g. Wednesday).



Interval Scheduling

- Consider an interval of time (e.g. Wednesday)
- Consider tasks that need to be completed during specific times (e.g. classes)



Interval Scheduling

- Consider an interval of time (e.g. Wednesday).
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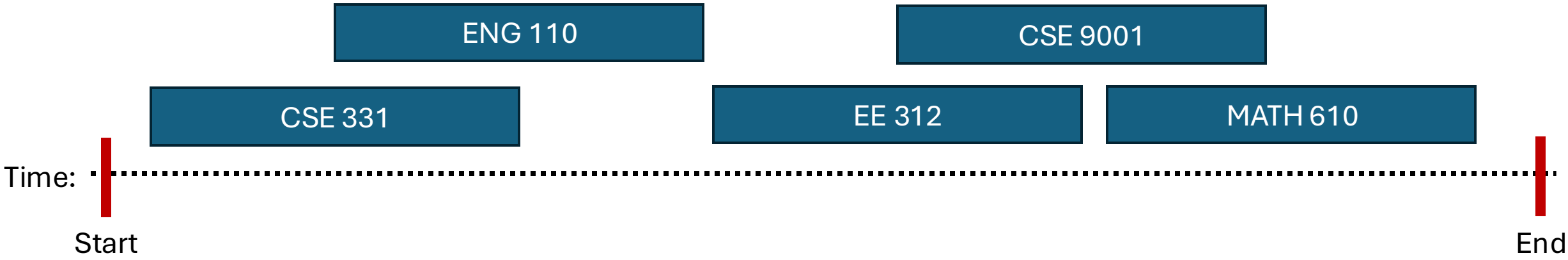
- We want to fit as many tasks as possible into the day such that no two overlap.



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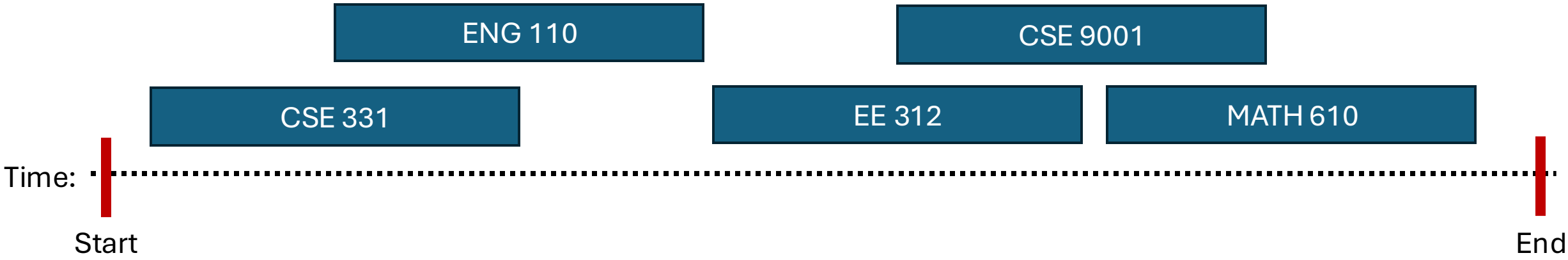

Interval Scheduling

- Consider an interval of time (e.g. Wednesday).
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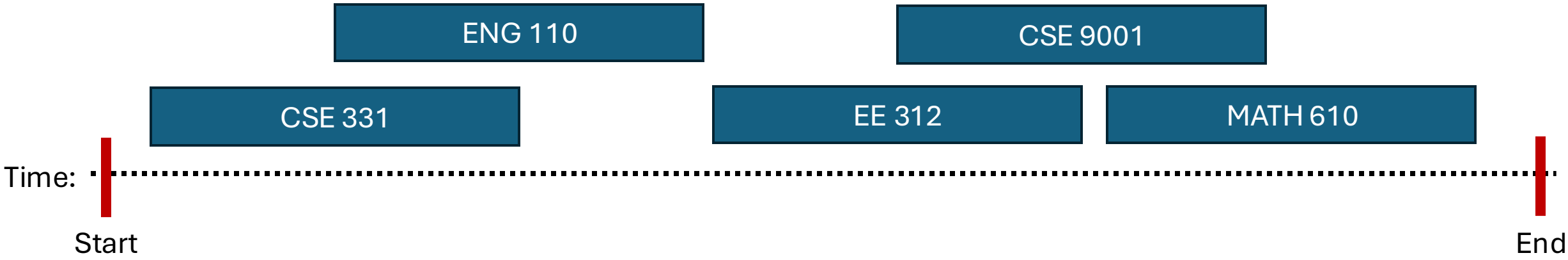
Q: Do we ever pick CSE 9001?



Interval Scheduling

- Consider an interval of time (e.g. Wednesday).
- Consider tasks that need to be completed during specific times (e.g. classes).
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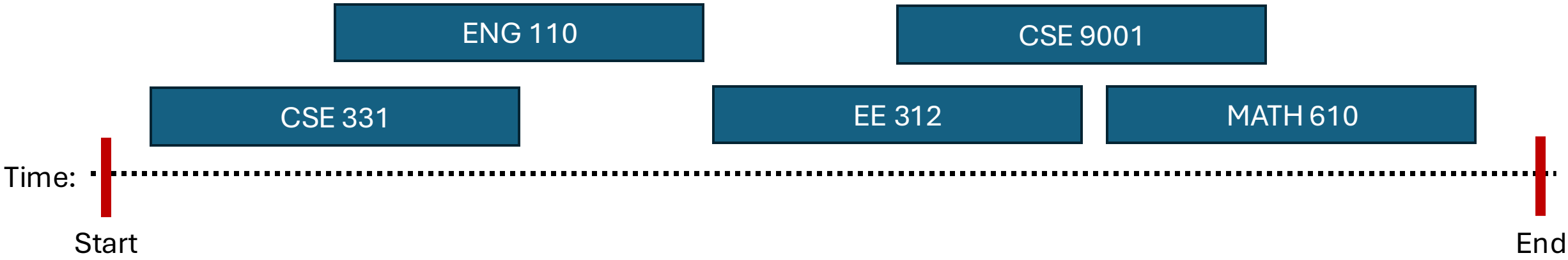
A: No, because it blocks two classes!



Optimal Solution #1

- Consider an interval of time (e.g. Wednesday).
- Consider tasks that need to be completed during specific times (e.g. classes).
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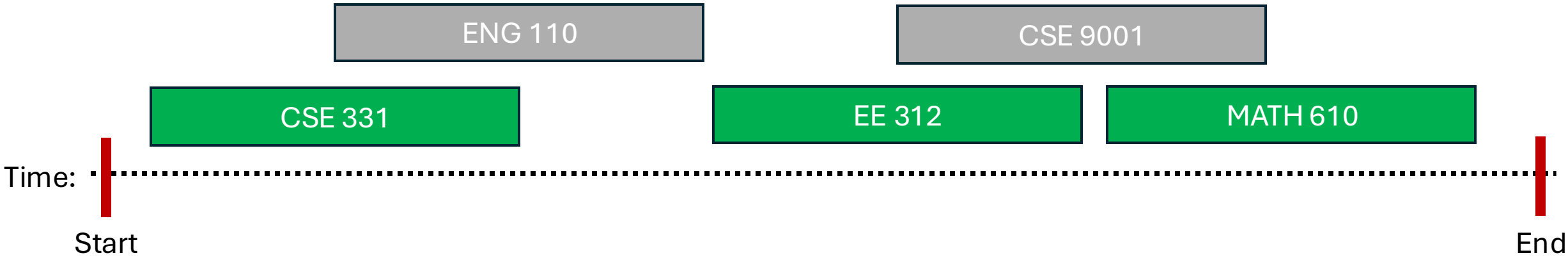

Optimal Solution #2

- Consider an interval of time (e.g. Wednesday).
- Consider tasks that need to be completed during specific times (e.g. classes).
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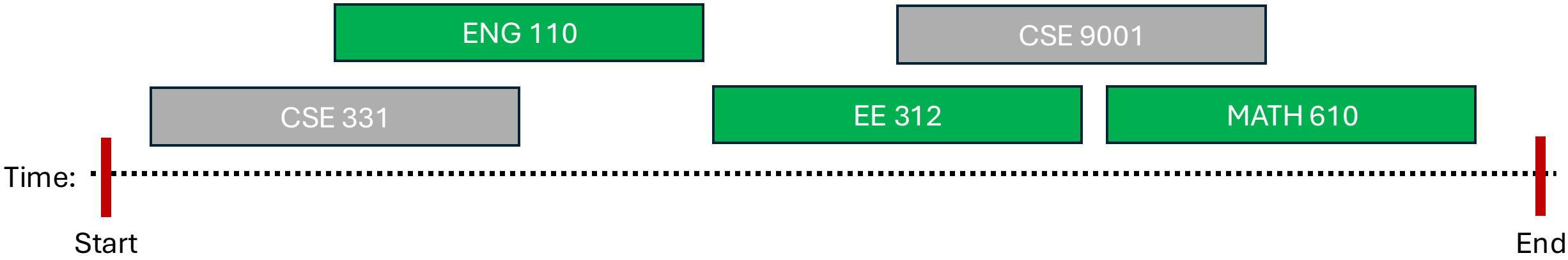


Interval Scheduling Problem (Support Page)

Interval Scheduling via examples

In which we derive an algorithm that solves the Interval Scheduling problem via a sequence of examples.

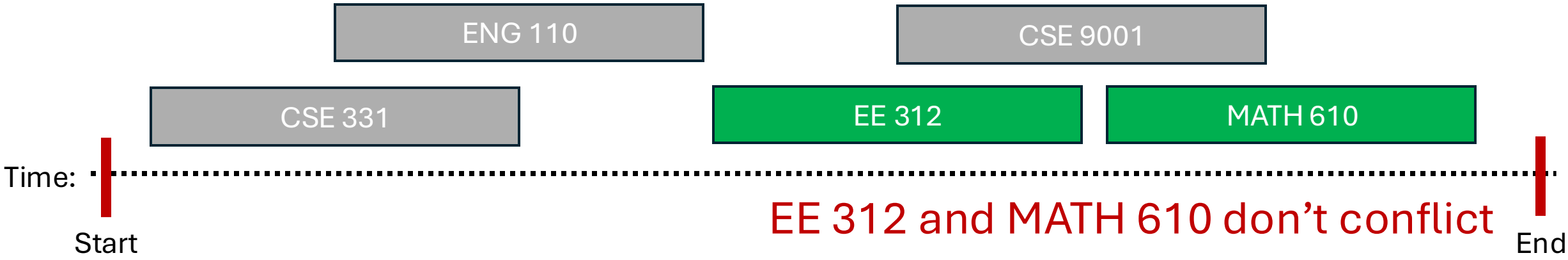
The problem

In these notes we will solve the following problem:

> **Interval Scheduling Problem**
>
> `Input:` An input of n intervals $[s(i), f(i))$, or in other words, $\{s(i), \ldots, f(i) - 1\}$ for $1 \leq i \leq n$ where i represents the intervals, $s(i)$ represents the start time, and $f(i)$ represents the finish time.
>
> `Output:` A schedule S of n intervals where no two intervals in S conflict, and the total number of intervals in S is maximized.

Interval Scheduling Problem

- **Input**: A set of n intervals with start and finish times.
  - For $1 \leq i \leq n$, $[s(i), f(i))$ where $s(i)$ and $f(i)$ are start and finish times of task i respectively.
- **Output**: A schedule (subset of intervals) S such that no two intervals in S conflict and the total number of intervals is maximized.



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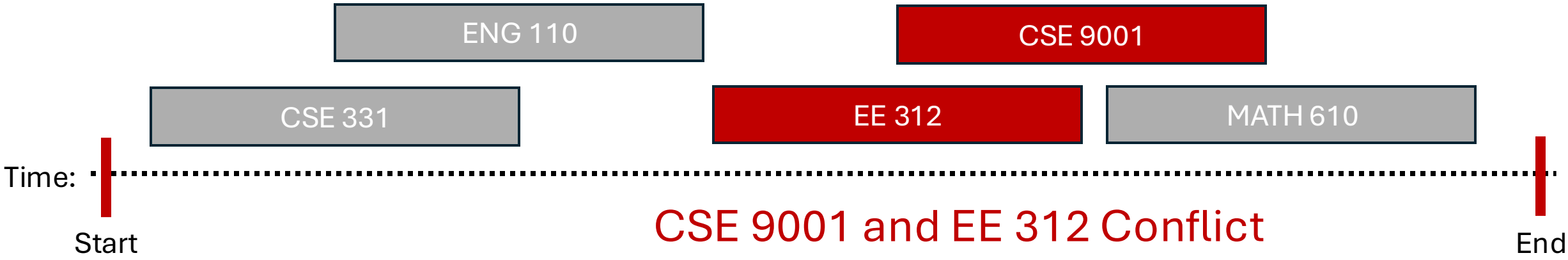
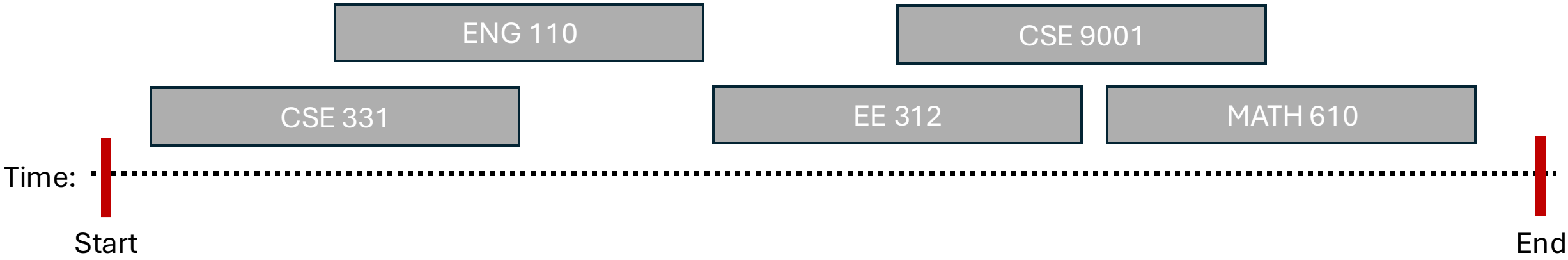


Q: How should we try to solve this?

- **Input**: A set of n intervals R with start and finish times.
  - For $1 \leq i \leq n$, $[s(i), f(i))$ where $s(i)$ and $f(i)$ are start and finish times of task i respectively.
- **Output**: A schedule (subset of intervals) S such that no two intervals in S conflict and the total number of intervals is maximized.



Q: How should we try to solve this?

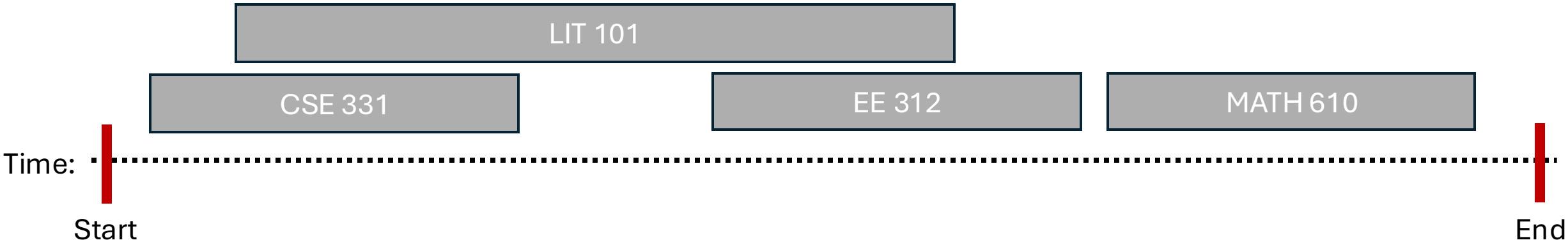
A: Let’s try to generate some examples and see what works and what doesn’t.

Build an Algorithm

- Basic Algorithm Outline:
  - S is empty
  - While R is not empty:
    - Pick i in R
    - Add i to S
    - Remove i from R

Build an Algorithm

- Basic Algorithm Outline:
  - S is empty
  - While R is not empty:
    - Pick i in R
    - Add i to S You might add conflicts!
    - Remove i from R



Build an Algorithm

- Basic Algorithm Outline:
  - S is empty
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    - Pick i in R
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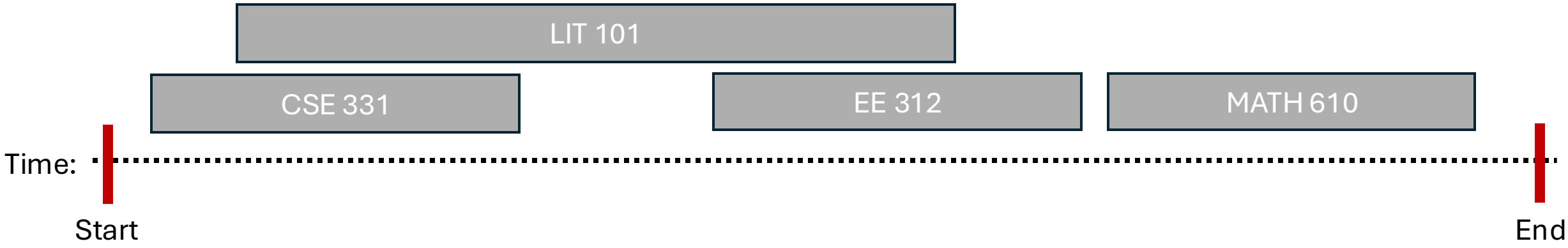

Build an Algorithm

- Basic Algorithm Outline:
  - S is empty
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    - Pick i in R Q: How do we do this?
    - Add i to S
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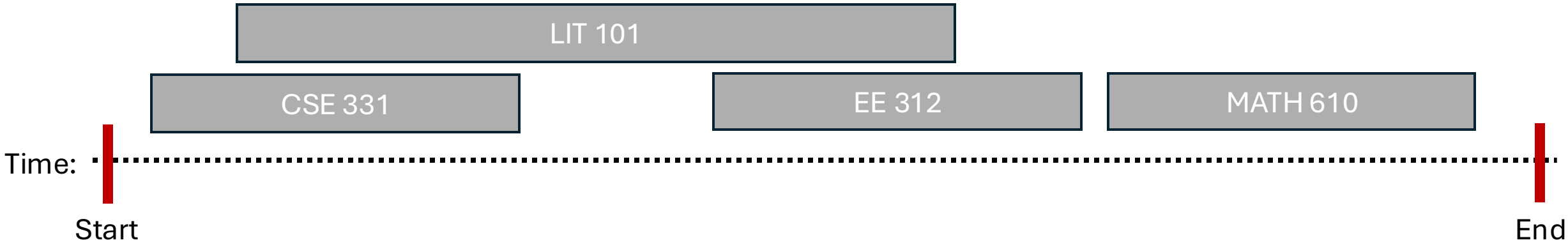

Build a Greedy Algorithm

- Basic Algorithm Outline:
  - S is empty
  - While R is not empty:
    - Pick i in R that minimizes v(i)
    - Add i to S
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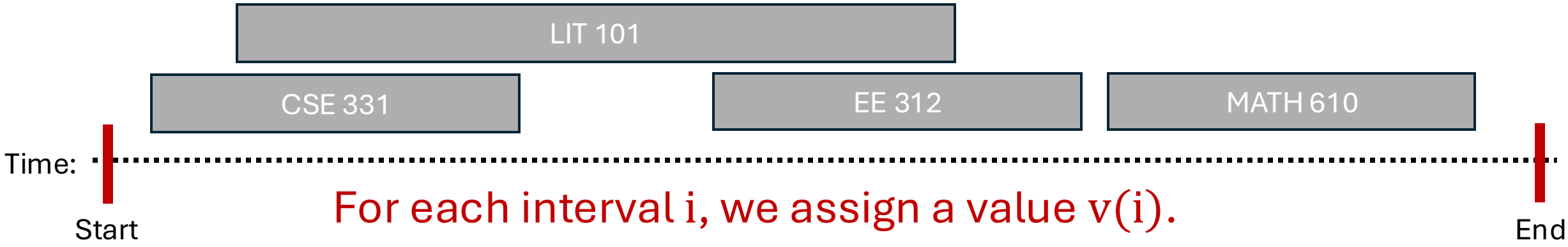

For each interval i, we assign a value v(i).

Q: What should we pick for $v(i)$?

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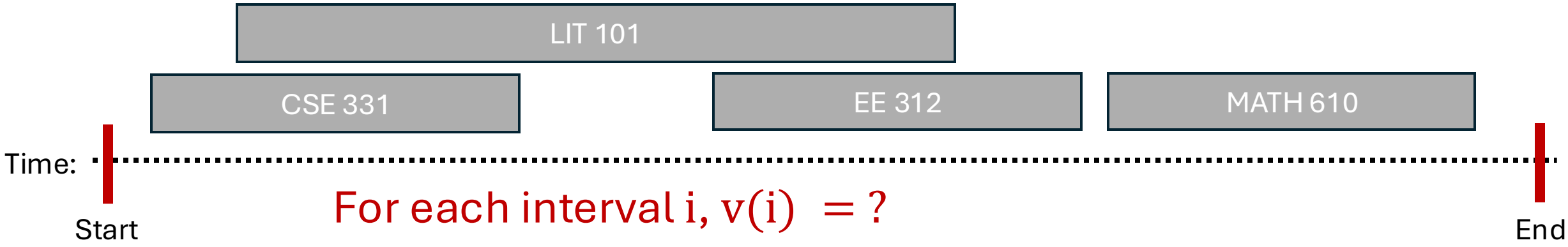

Attempt I: Interval Length (Okay)

- Basic Algorithm Outline:
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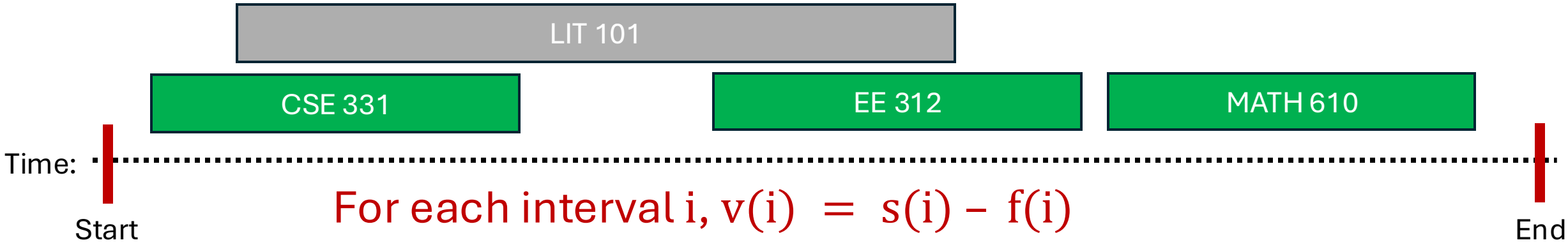

For each interval i, $v(i) = s(i) - f(i)$

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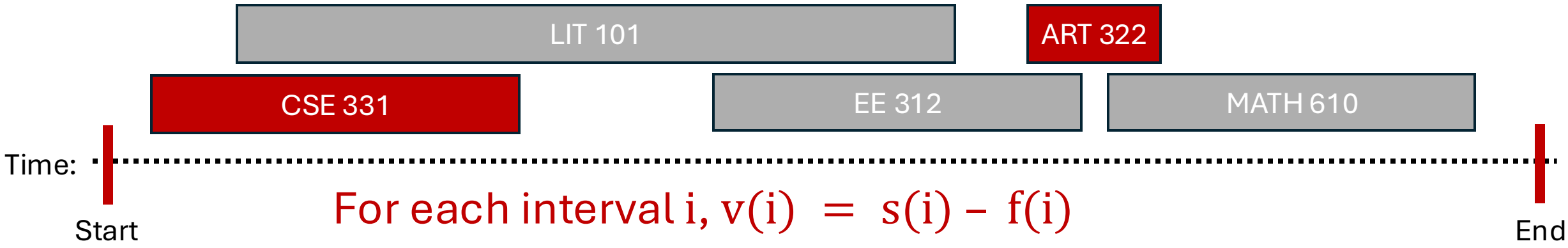

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Attempt II: Start Time (Okay)

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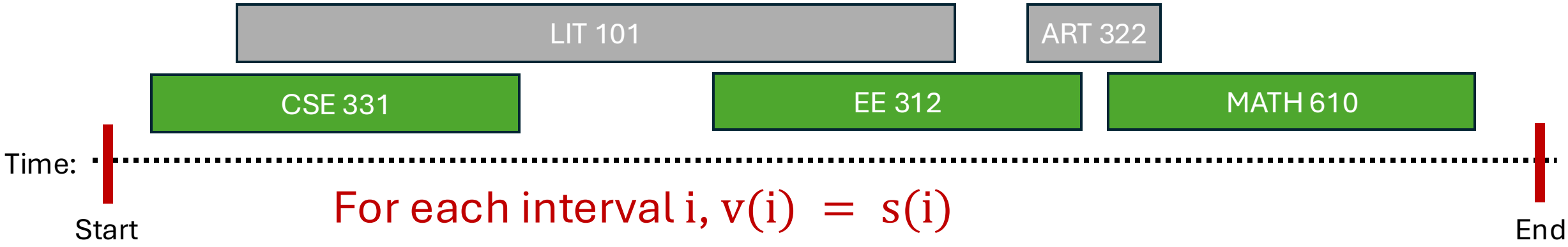

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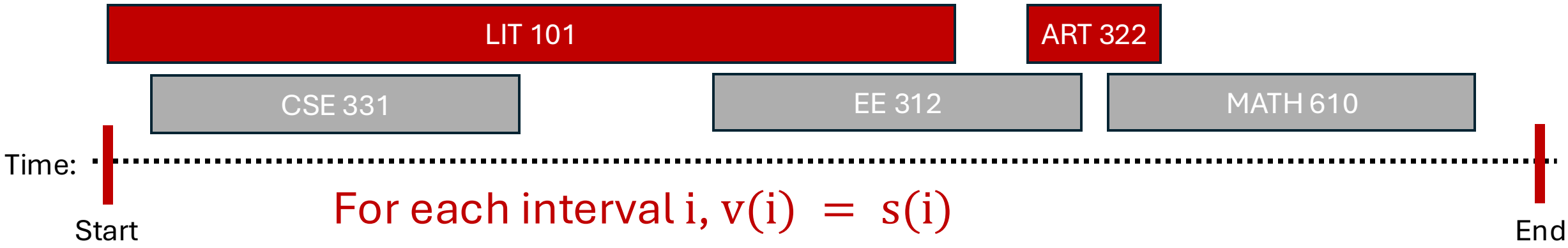

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Attempt III: Min Conflicts (Okay)

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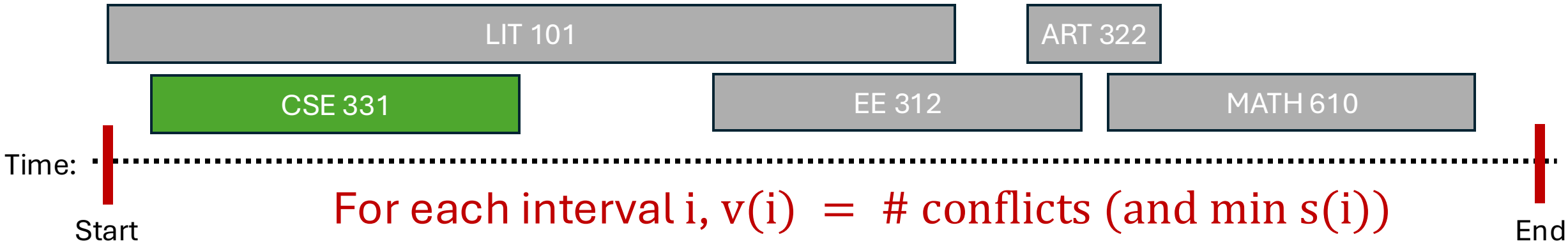

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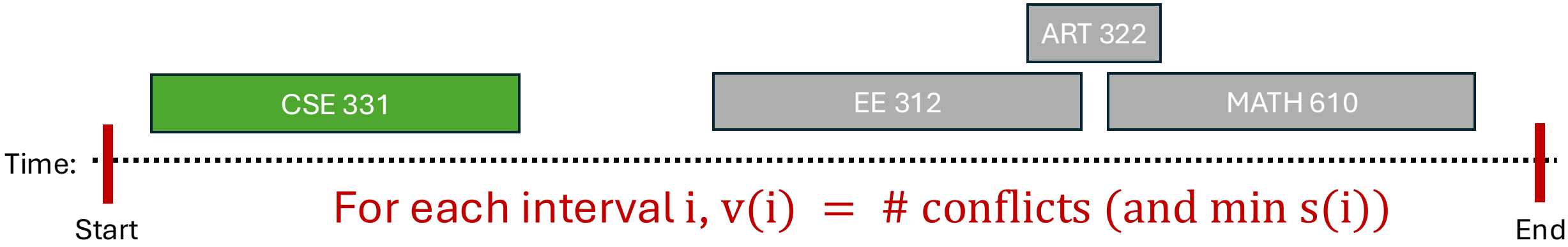

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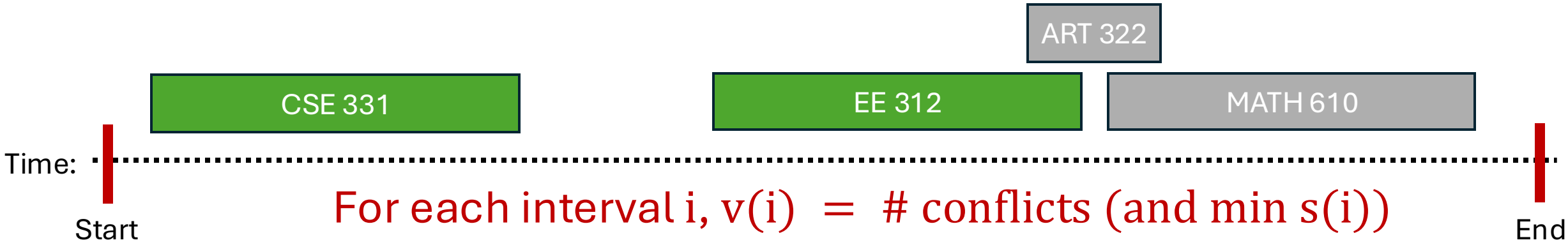

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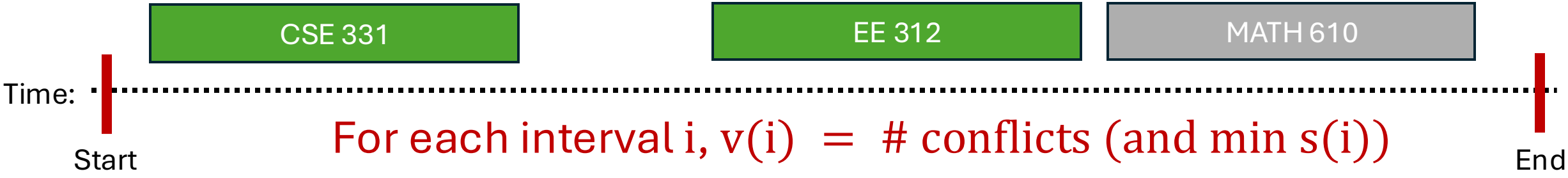

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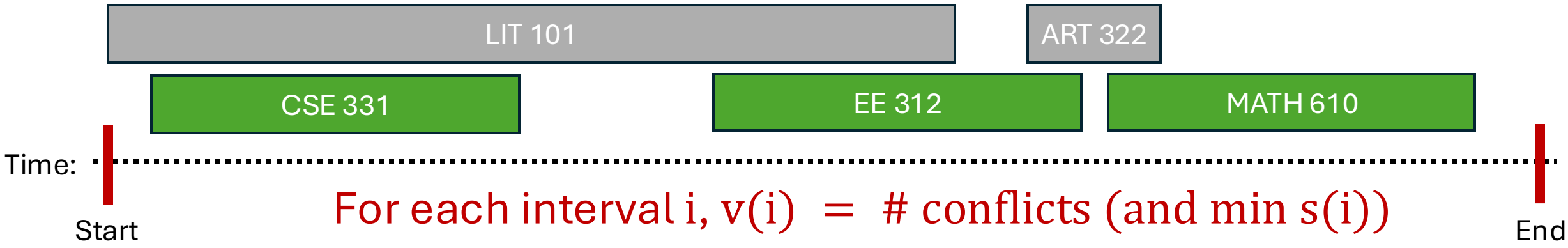
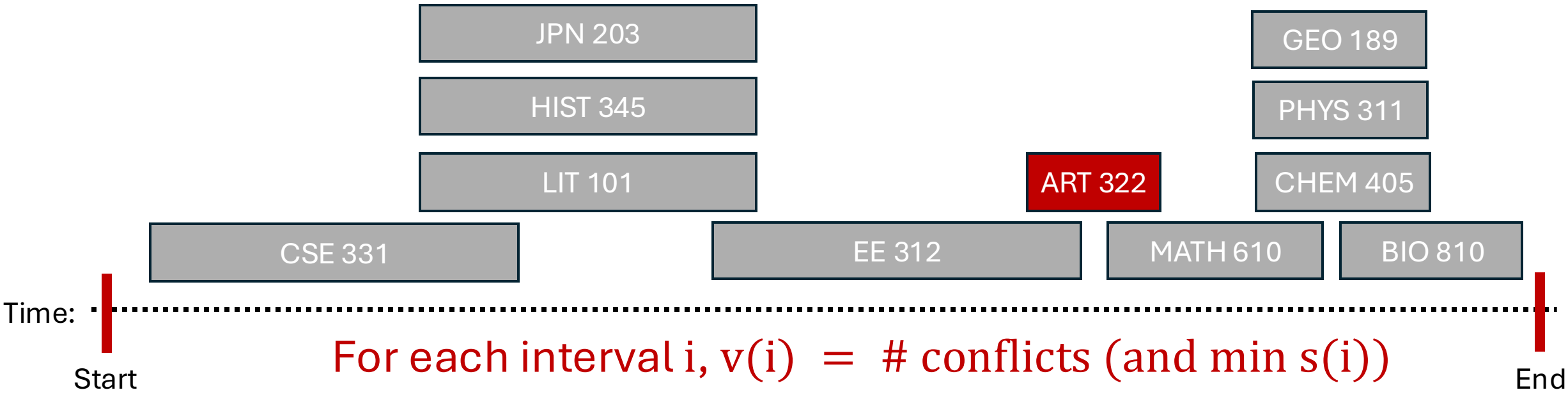


For each interval i, v(i) = # conflicts (and min s(i))

Attempt III: Min Conflicts (Oh no!)



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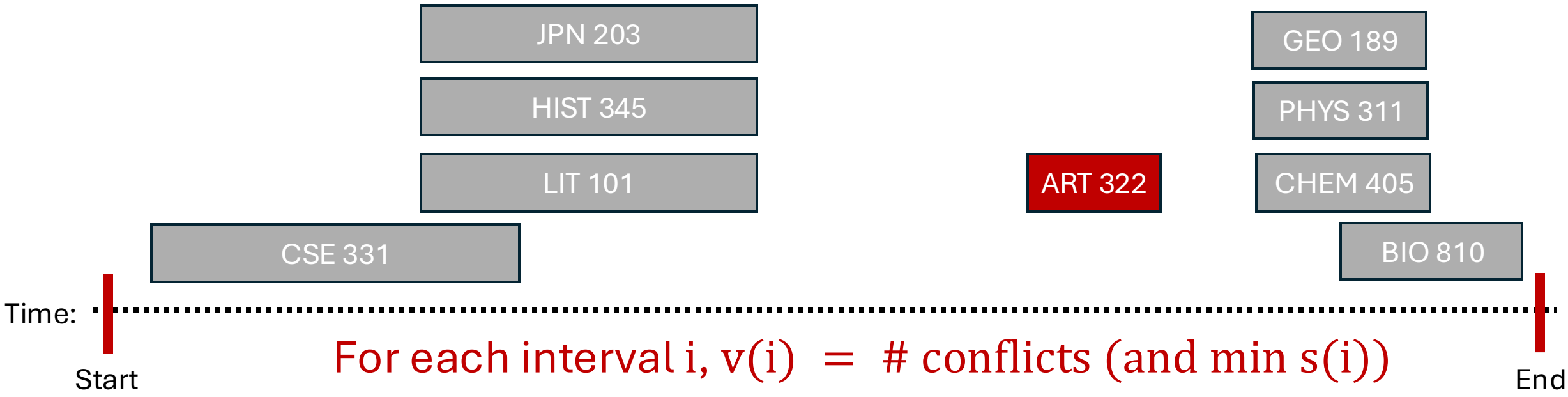

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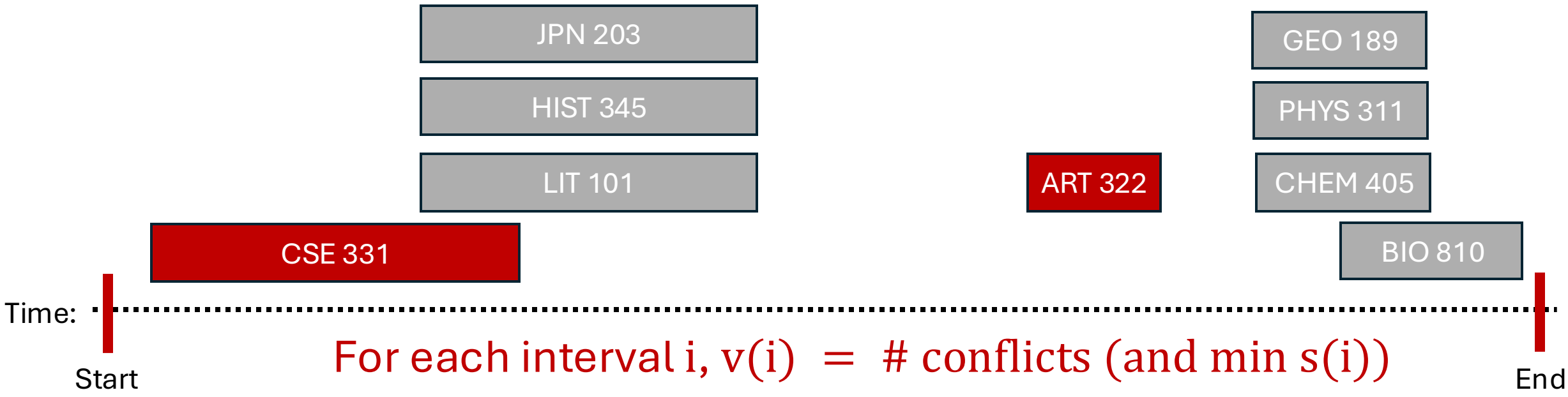

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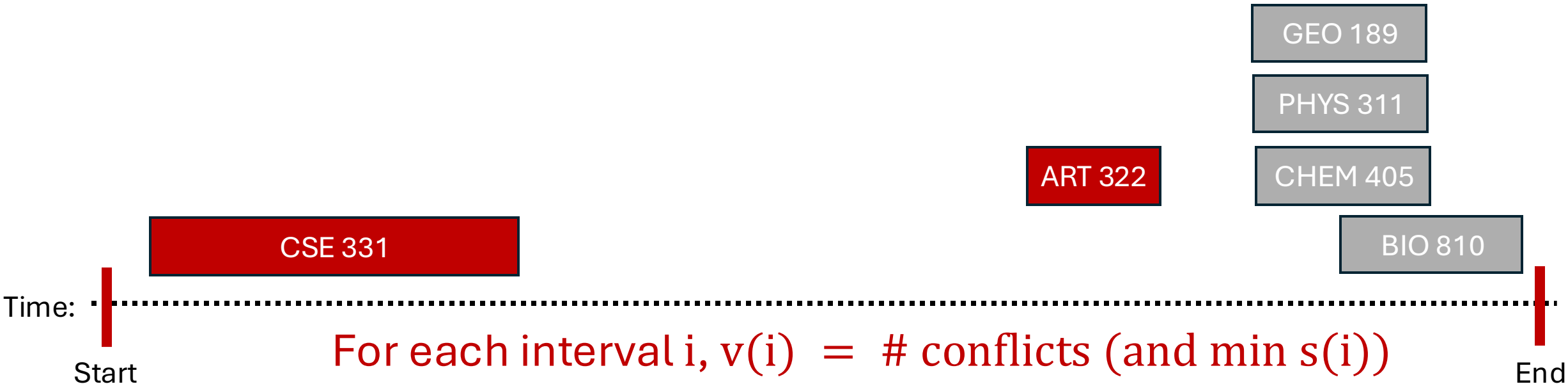

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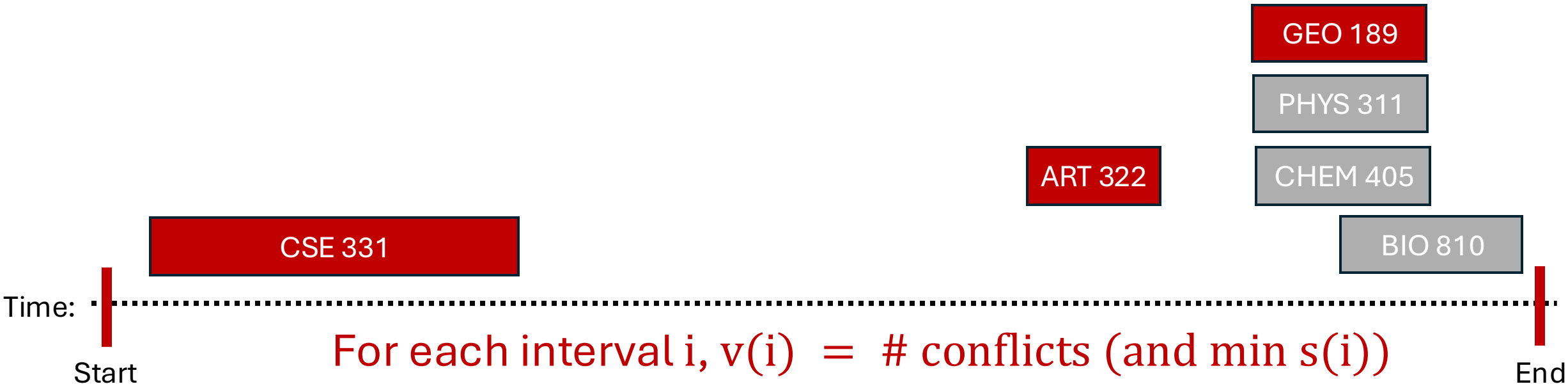

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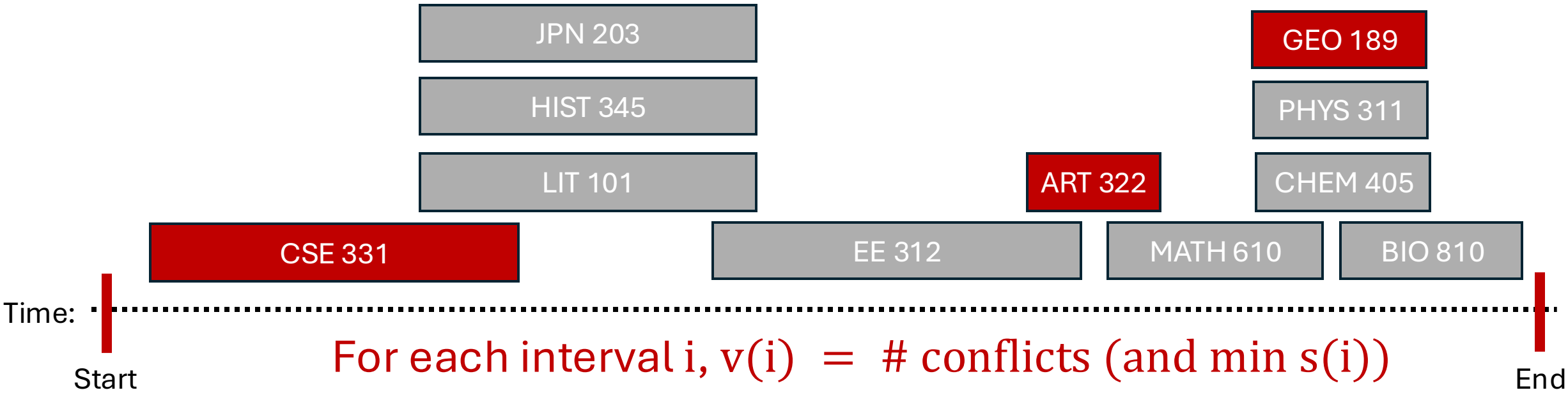

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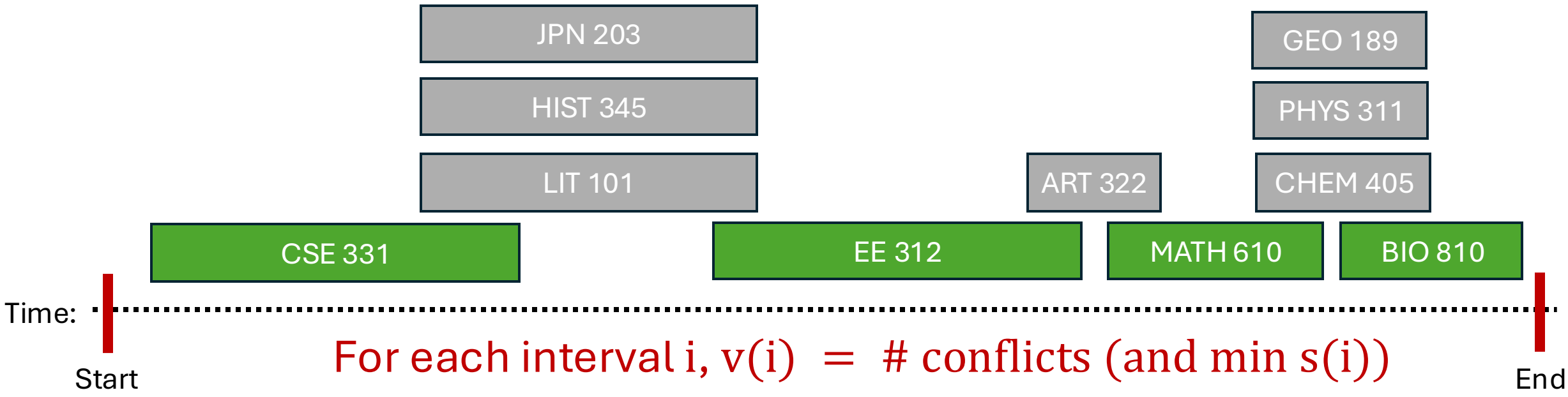
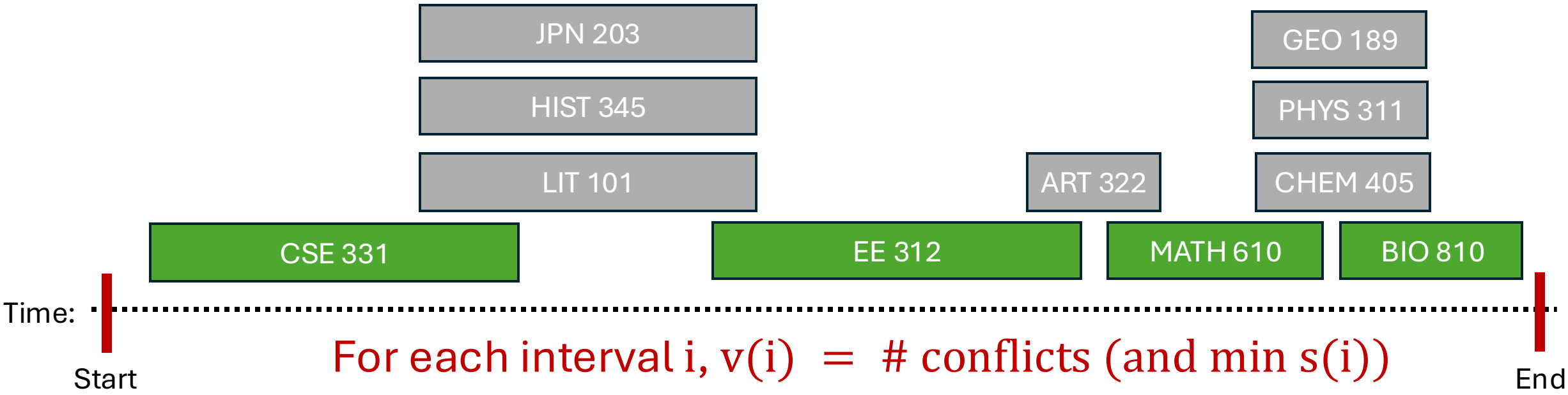


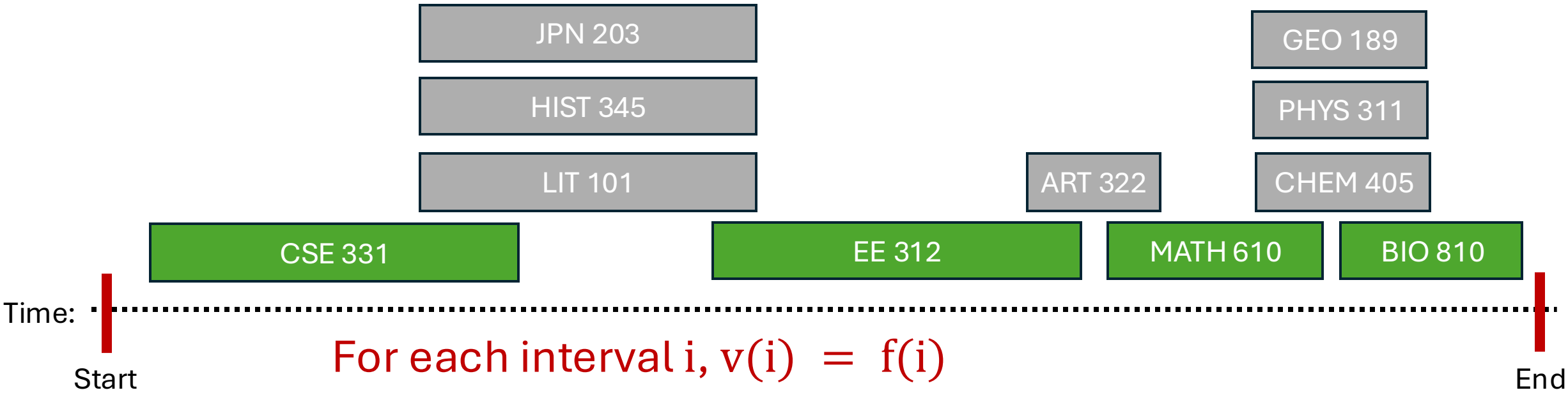
For each interval i, v(i) = # conflicts (and min s(i))

I GIVE UP!!!!



For each interval i, v(i) = # conflicts (and min s(i))

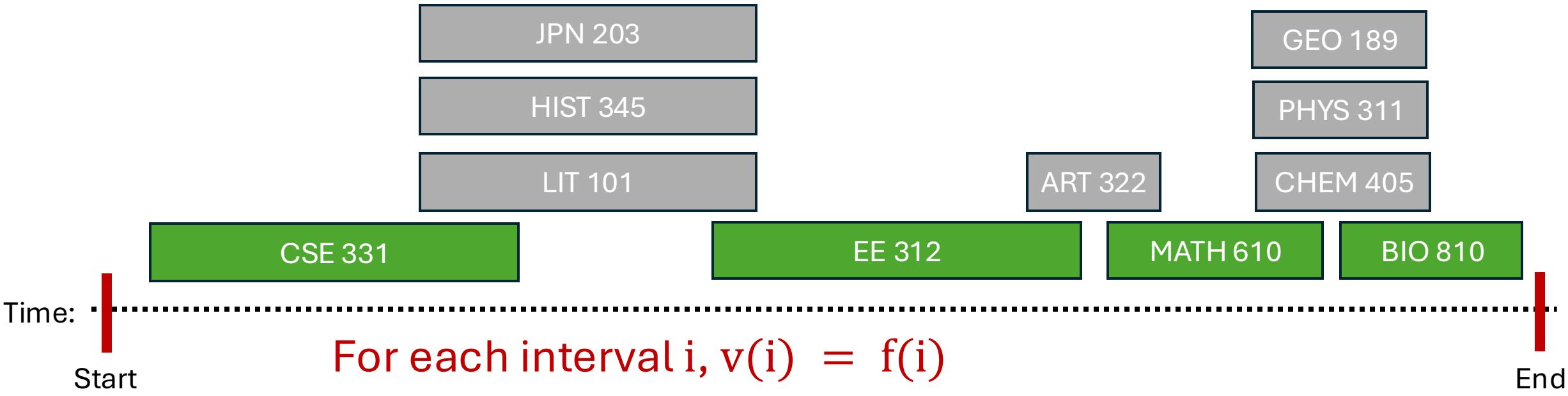
Attempt IV: Finish Time (Okay)



For each interval i, $v(i) = f(i)$

Attempt IV: Finish Time (….)

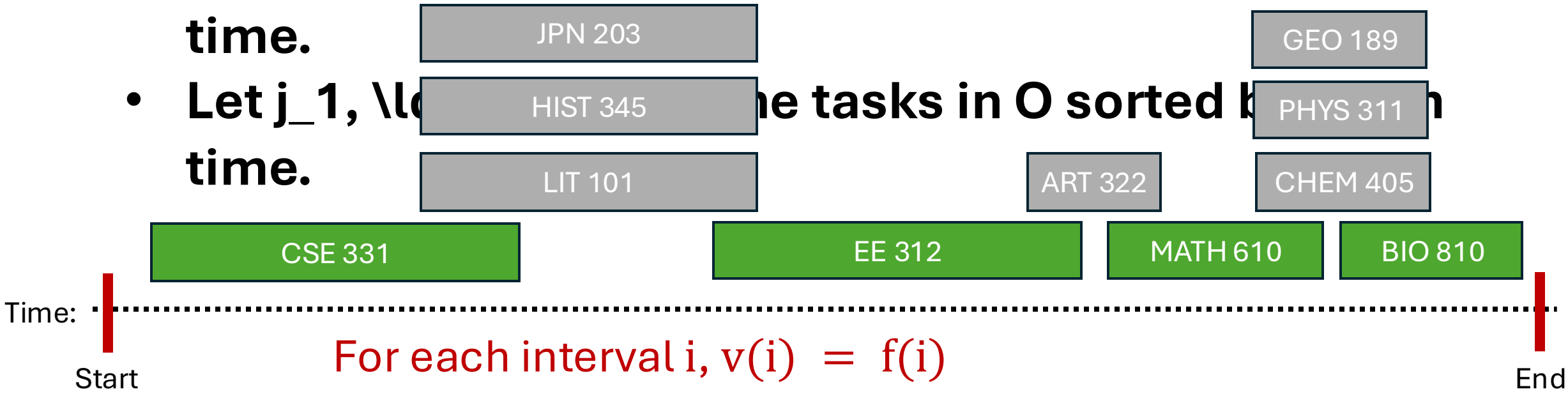
Wait… does that actually work?



Claim: The Finish First Algorithm is Optimal

Proof Ideas:

- **Let A be the set returned by the algorithm and O be the optimal list.**
- **Let i_1, \ldots, i_k be the tasks in A sorted by add time.**
- **Let j_1, \l[illegible]e tasks in O sorted b[illegible]n time.**



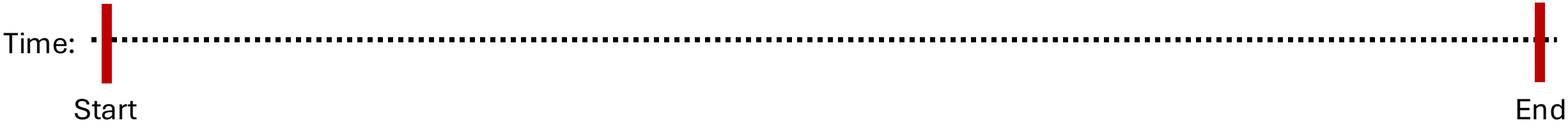
For each interval i, $v(i) = f(i)$

Claim: The Finish First Algorithm is Optimal

Proof Ideas:

- Let $i_1, \ldots, i_k$ be the tasks returned by algorithm (sorted by finish/add time).
- Let $j_1, \ldots, j_m$ be the tasks in optimal solution (sorted by finish time)
- We want to show k = m

Q: What can we say about the first job in each list?



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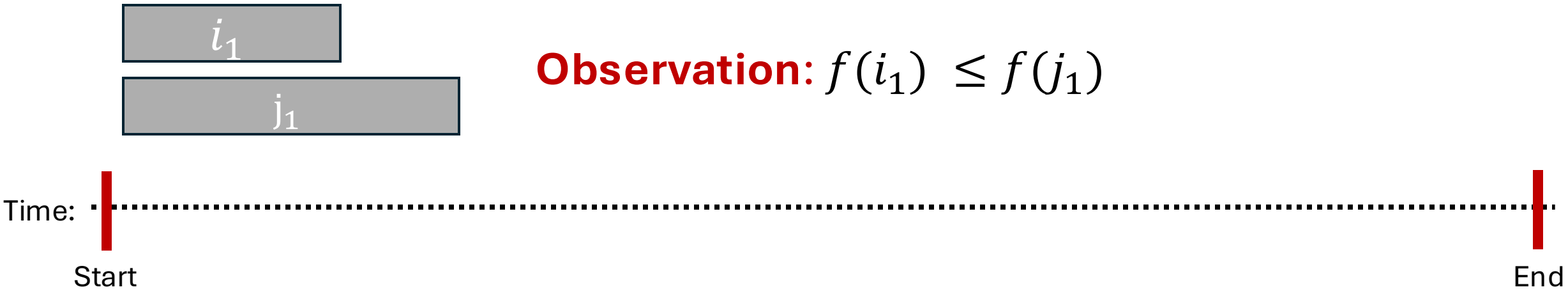
Proof Ideas:

- Let $i_1, \ldots, i_k$ be the tasks returned by algorithm (sorted by finish/add time).
- Let $j_1, \ldots, j_m$ be the tasks in optimal solution (sorted by finish time)



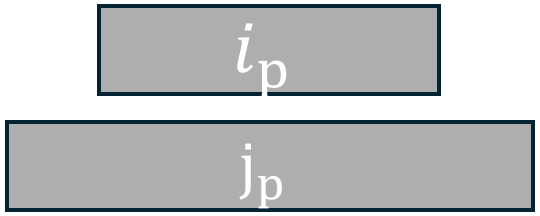
Observation: $f(i_1) \leq f(j_1)$

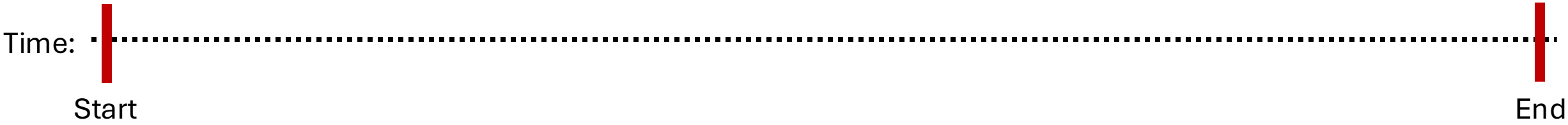
Claim: The Finish First Algorithm is Optimal

Proof Ideas:

- Assume now that $f(i_p) \leq f(j_p)$ for some p.
  - That is, assume the pth in the algorithms list ends before the pth job in the optimal list.

Q: What can we say about the $p + 1$ job in each list?



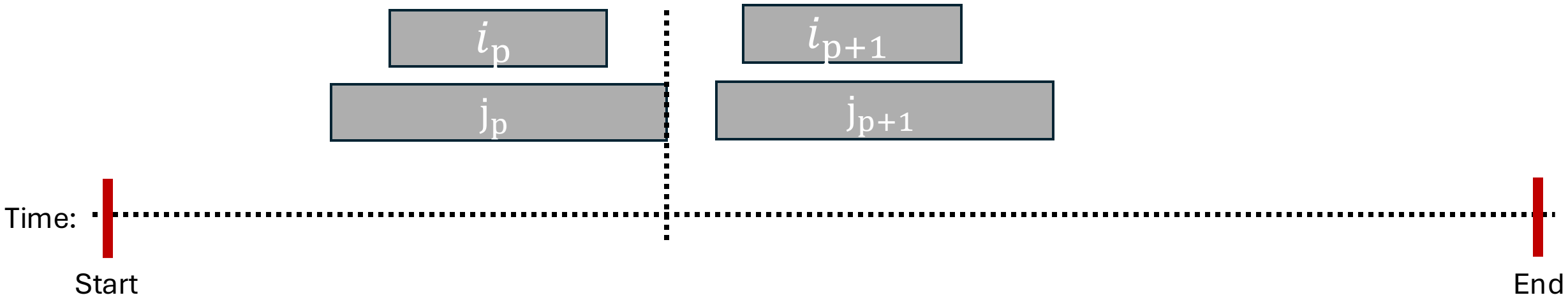


Claim: The Finish First Algorithm is Optimal

Proof Ideas:

- Assume now that $f(i_p) \leq f(j_p)$ for some p.
  - That is, assume the pth in the algorithms list ends before the pth job in the optimal list.

Observation: The algorithm could have added j_{p+1}!

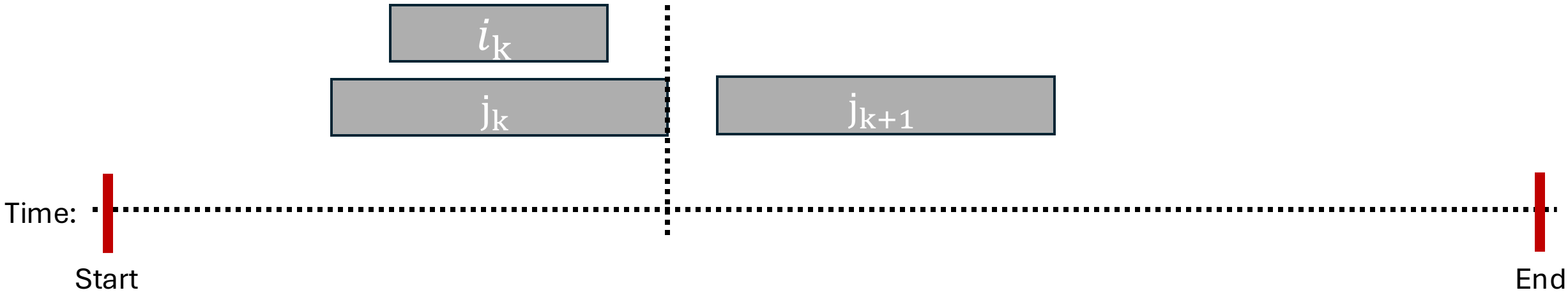


Claim: The Finish First Algorithm is Optimal

Proof Ideas:

- Assume now that $f(i_k) \leq f(j_k)$ and m > k.

Q: Why is this a problem?



Claim: The Finish First Algorithm is Optimal

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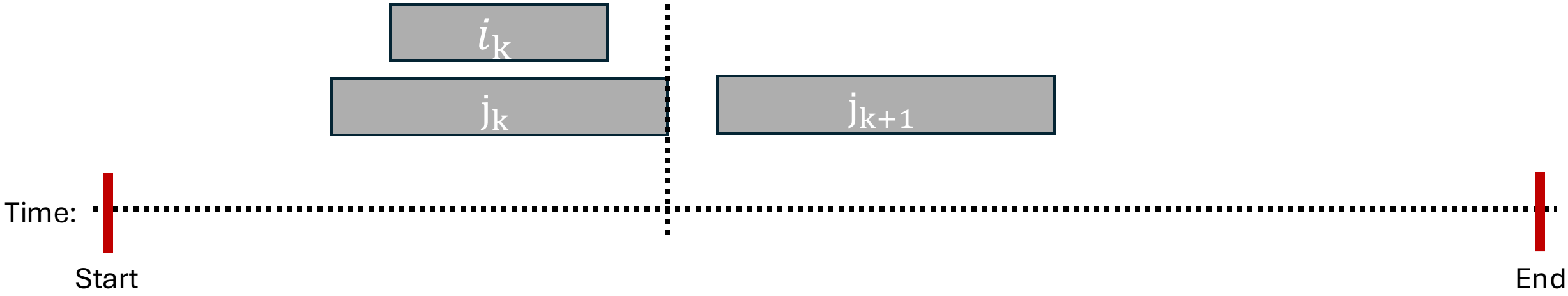

Runtime Analysis

- Basic Algorithm Outline:
  - S is empty
  - While R is not empty:
    - Pick i in R
    - Add i to S
    - Remove all tasks that conflict with i from R

Runtime Analysis

- **Input**: List of tasks R
  - S is empty
  - While R is not empty:
    - Find i in R with earliest finish time
    - Add i to S
    - Remove all tasks that conflict with i from R
  - Return S

Runtime Analysis

- **Input**: List of tasks R
  - S is empty list **<- O(1) time**
  - While R is not empty: **<- O(n) iterations**
    - Find i in R with earliest finish time **<- O(n) time to search**
    - Add i to S **<- O(1) time**
    - Remove all tasks that conflict with i from R **<- O(n) time**
  - Return S **<- O(1) time**

Runtime Analysis: O(n^2)

- **Input**: List of tasks R
  - S is empty list **<- O(1) time**
  - While R is not empty: **<- O(n) iterations**
    - Find i in R with earliest finish time **<- O(n) time to search**
    - Add i to S **<- O(1) time**
    - Remove all tasks that conflict with i from R **<- O(n) time**
  - Return S **<- O(1) time**

Q: Can we do this faster?

Runtime Analysis: O(nlog(n))

- **Input**: List of tasks R
  - Sort R by finish time **<- O(nlog(n)) time**
  - S is empty list **<- O(1) time**
  - Set last_job_fin to be 0 **<- O(1) time**
  - For j = 1 up to n: **<- O(n) iterations**
    - if s(j) > last_job_fin: **<- O(1) time**
      - Add i to S **<- O(1) time**
      - Set last_job_fin to be f(i) **<- O(1) time**
  - Return S **<- O(1) time**