

CSE 443
Compilers

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EXERCISE:
fill in the parse table

(see next slide)

Parse-table M

NON TERMINALS	id	+	*	()	\$
E						
E'						
T						
T'						
F						

$$\text{FIRST}(E) = \text{FIRST}(T) = \text{FIRST}(F) =$$

$$\{ (, id \}$$

$$\text{FIRST}(E') = \{ +, \epsilon \}$$

$$\text{FIRST}(T') = \{ *, \epsilon \}$$

$$\text{FOLLOW}(E') = \text{FOLLOW}(E) = \{), \$ \}$$

$$\text{FOLLOW}(T') = \text{FOLLOW}(T) = \{ +,), \$ \}$$

$$\text{FOLLOW}(F) = \{ +, *,), \$ \}$$

$$E \rightarrow T E'$$

$$E' \rightarrow + T E' \mid \epsilon$$

$$T \rightarrow F T'$$

$$T' \rightarrow * F T' \mid \epsilon$$

$$F \rightarrow (E) \mid id$$

For each production $A \rightarrow \alpha$ of G :

- For each terminal $a \in \text{FIRST}(\alpha)$, add $A \rightarrow \alpha$ to $M[A, a]$
- If $\epsilon \in \text{FIRST}(\alpha)$, then for each terminal b in $\text{FOLLOW}(A)$, add $A \rightarrow \alpha$ to $M[A, b]$
- If $\epsilon \in \text{FIRST}(\alpha)$ and $\$ \in \text{FOLLOW}(A)$, add $A \rightarrow \alpha$ to $M[A, \$]$

Parse-table M

NON TERMINALS	id	+	*	()	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'						
T						
T'						
F						

$FIRST(E) = FIRST(T) = FIRST(F) = \{ (, id \}$

$FIRST(E') = \{ +, \epsilon \}$

$FIRST(T') = \{ *, \epsilon \}$

$FOLLOW(E') = FOLLOW(E) = \{), \$ \}$

$FOLLOW(T') = FOLLOW(T) = \{ +,), \$ \}$

$FOLLOW(F) = \{ +, *,), \$ \}$

$E \rightarrow TE'$

$E' \rightarrow +TE' \mid \epsilon$

$T \rightarrow FT'$

$T' \rightarrow *FT' \mid \epsilon$

$F \rightarrow (E) \mid id$

For each production $A \rightarrow \alpha$ of G:

- For each terminal $a \in FIRST(\alpha)$, add $A \rightarrow \alpha$ to $M[A, a]$
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Parse-table M

NON TERMINALS	id	+	*	()	\$
E	$E \rightarrow TE'$				$E \rightarrow TE'$	
E'		$E' \rightarrow +TE'$				
T						
T'						
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$E \rightarrow TE'$

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Parse-table M

NON TERMINALS	id	+	*	()	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
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T						
T'						
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T	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'		$T' \rightarrow \epsilon$	$T' \rightarrow *FT$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
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E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'		$T' \rightarrow \epsilon$	$T' \rightarrow *FT$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F				$F \rightarrow (E)$		

$FIRST(E) = FIRST(T) = FIRST(F) =$

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NON TERMINALS	id	+	*	()	\$
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E'		$E' \rightarrow +TE'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'		$T' \rightarrow \epsilon$	$T' \rightarrow *FT$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F	$F \rightarrow id$			$F \rightarrow (E)$		

$FIRST(E) = FIRST(T) = FIRST(F) =$

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E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'		$T' \rightarrow \epsilon$	$T' \rightarrow *FT$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F	$F \rightarrow id$			$F \rightarrow (E)$		

$$\text{FIRST}(E) = \text{FIRST}(T) = \text{FIRST}(F) =$$

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$$\text{FIRST}(E') = \{ +, \epsilon \}$$

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$$E' \rightarrow +TE' \mid \epsilon$$

$$T \rightarrow FT'$$

$$T' \rightarrow *FT' \mid \epsilon$$

$$F \rightarrow (E) \mid id$$

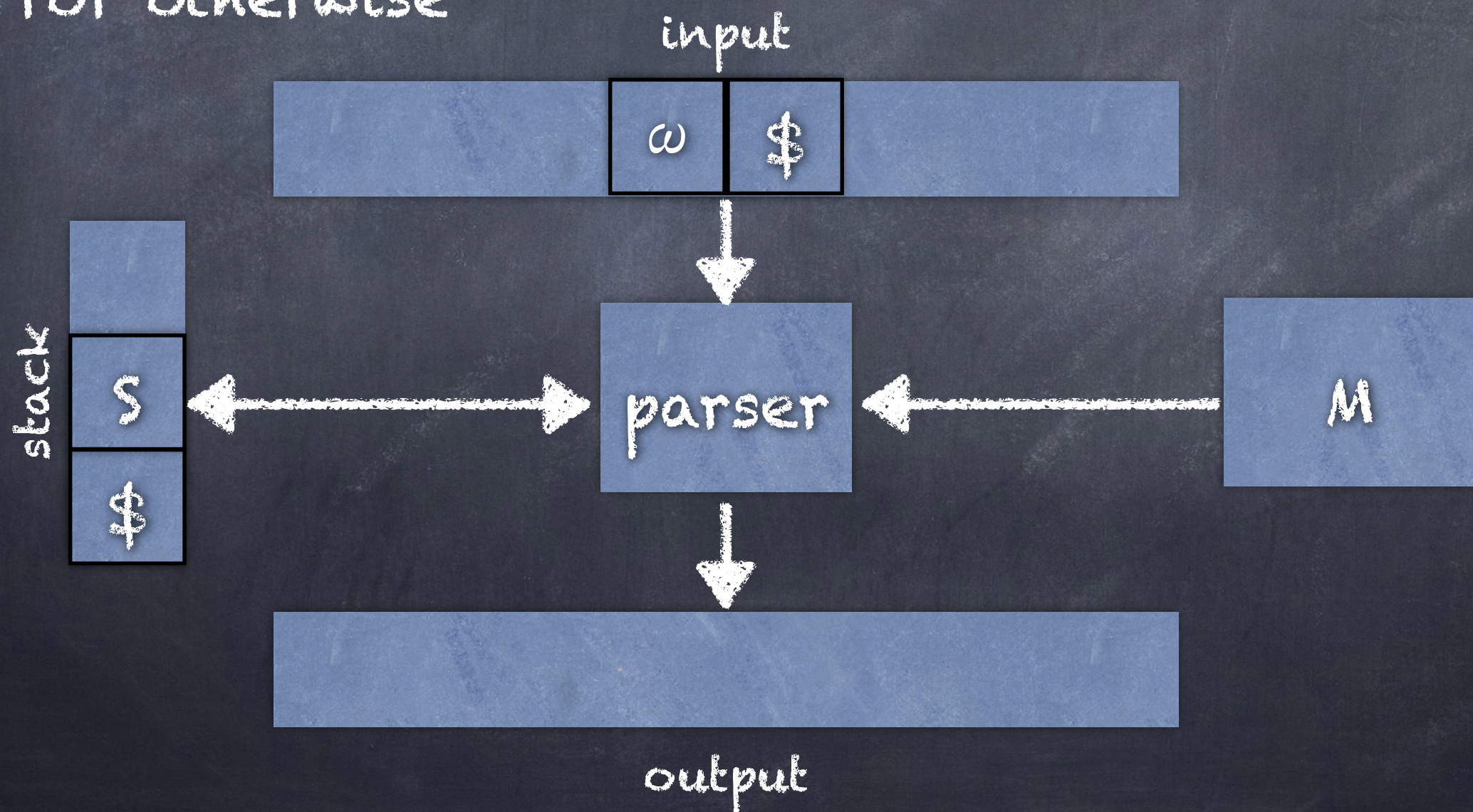
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Algorithm 4.34 [p. 226]

INPUT: A string ω and a parsing table M for a grammar $G=(N,T,P,S)$.

OUTPUT: If $\omega \in \mathcal{L}(G)$, a leftmost derivation of ω , error otherwise



Algorithm 4.34 [p. 226]

Let a be the first symbol of ω

Let X be the top stack symbol

while ($X \neq \$$) {

 if ($X == a$) { pop the stack, advance a in ω }

 else if (X is a terminal) { error }

 else if ($M[X, a]$ is blank) { error }

 else if ($M[X, a]$ is $X \rightarrow Y_1 Y_2 \dots Y_k$) {

 output $X \rightarrow Y_1 Y_2 \dots Y_k$

 pop the stack

 push $Y_k \dots Y_2 Y_1$ onto the stack

 }

 Let X be the top stack symbol

}

Accept if $a == X == \$$

INPUT (a)

id + id * id \$

STACK (X)

E \$

OUTPUT

E → T E'

if (X == a) { pop, advance a in ω }

else if (X is a terminal) { error }

else if (M[X,a] is blank) { error }

else if (M[X,a] is $X \rightarrow Y_1 Y_2 \dots Y_k$) {

 output $X \rightarrow Y_1 Y_2 \dots Y_k$

 pop

 push $Y_k \dots Y_2 Y_1$

}

Let X be the top stack symbol

	id	+	*	()	\$
E	$E \rightarrow T E'$			$E \rightarrow T E'$		
E'		$E' \rightarrow + T E'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow F T'$			$T \rightarrow F T'$		
T'		$T' \rightarrow \epsilon$	$T' \rightarrow * F T'$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F	$F \rightarrow id$			$F \rightarrow (E)$		

INPUT (a)

id + id * id \$

id + id * id \$

STACK (X)

E \$

T E' \$

OUTPUT

E → T E'

T → F T'

```

if (X == a) { pop, advance a in ω }
else if (X is a terminal) { error }
else if (M[X,a] is blank) { error }
else if (M[X,a] is X → Y1 Y2 ... Yk) {
  output X → Y1 Y2 ... Yk
  pop
  push Yk ... Y2 Y1
}

```

Let X be the top stack symbol

	id	+	*	()	\$
E	E → T E'			E → T E'		
E'		E' → + T E'			E' → ε	E' → ε
T	T → F T'			T → F T'		
T'		T' → ε	T' → * F T'		T' → ε	T' → ε
F	F → id			F → (E)		

INPUT (a)

id + id * id \$

id + id * id \$

id + id * id \$

STACK (X)

E \$

T E' \$

F T' E' \$

OUTPUT

E → T E'

T → F T'

F → id

```

if (X == a) { pop, advance a in ω }
else if (X is a terminal) { error }
else if (M[X,a] is blank) { error }
else if (M[X,a] is X → Y1 Y2 ... Yk) {
  output X → Y1 Y2 ... Yk
  pop
  push Yk ... Y2 Y1
}

```

Let X be the top stack symbol

	id	+	*	()	\$
E	E → T E'			E → T E'		
E'		E' → + T E'			E' → ε	E' → ε
T	T → F T'			T → F T'		
T'		T' → ε	T' → * F T'		T' → ε	T' → ε
F	F → id			F → (E)		

INPUT (a)

id + id * id \$

id + id * id \$

id + id * id \$

id + id * id \$

STACK (X)

E \$

T E' \$

F T' E' \$

id T' E' \$

OUTPUT

E → T E'

T → F T'

F → id

```

if (X == a) { pop, advance a in ω }
else if (X is a terminal) { error }
else if (M[X,a] is blank) { error }
else if (M[X,a] is X → Y1 Y2 ... Yk) {
  output X → Y1 Y2 ... Yk
  pop
  push Yk ... Y2 Y1
}
  
```

Let X be the top stack symbol

	id	+	*	()	\$
E	E → T E'			E → T E'		
E'		E' → + T E'			E' → ε	E' → ε
T	T → F T'			T → F T'		
T'		T' → ε	T' → * F T'		T' → ε	T' → ε
F	F → id			F → (E)		

INPUT (a)

id + id * id \$

id + id * id \$

id + id * id \$

id + id * id \$

id + id * id \$

STACK (X)

E \$

T E' \$

F T' E' \$

id T' E' \$

T' E' \$

OUTPUT

$E \rightarrow T E'$

$T \rightarrow F T'$

$F \rightarrow id$

$T' \rightarrow \epsilon$

```

if (X == a) { pop, advance a in w }
else if (X is a terminal) { error }
else if (M[X,a] is blank) { error }
else if (M[X,a] is  $X \rightarrow Y_1 Y_2 \dots Y_k$ ) {
    output  $X \rightarrow Y_1 Y_2 \dots Y_k$ 
    pop
    push  $Y_k \dots Y_2 Y_1$ 
}

```

Let X be the top stack symbol

	id	+	*	()	\$
E	$E \rightarrow T E'$			$E \rightarrow T E'$		
E'		$E' \rightarrow + T E'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow F T'$			$T \rightarrow F T'$		
T'		$T' \rightarrow \epsilon$	$T' \rightarrow * F T'$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F	$F \rightarrow id$			$F \rightarrow (E)$		

INPUT (a)

id + id * id \$

id + id * id \$

id + id * id \$

id + id * id \$

id + id * id \$

id + id * id \$

STACK (X)

E \$

T E' \$

F T' E' \$

id T' E' \$

T' E' \$

E' \$

OUTPUT

$E \rightarrow T E'$

$T \rightarrow F T'$

$F \rightarrow id$

$T' \rightarrow \epsilon$

$E' \rightarrow + T E'$

```

if (X == a) { pop, advance a in w }
else if (X is a terminal) { error }
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else if (M[X,a] is  $X \rightarrow Y_1 Y_2 \dots Y_k$ ) {
  output  $X \rightarrow Y_1 Y_2 \dots Y_k$ 
  pop
  push  $Y_k \dots Y_2 Y_1$ 
}
  
```

Let X be the top stack symbol

	id	+	*	()	\$
E	$E \rightarrow T E'$			$E \rightarrow T E'$		
E'		$E' \rightarrow + T E'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow F T'$			$T \rightarrow F T'$		
T'		$T' \rightarrow \epsilon$	$T' \rightarrow * F T'$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F	$F \rightarrow id$			$F \rightarrow (E)$		

INPUT (a)	STACK (X)	OUTPUT
id + id * id \$	E \$	$E \rightarrow T E'$
id + id * id \$	T E' \$	$T \rightarrow F T'$
id + id * id \$	F T' E' \$	$F \rightarrow id$
id + id * id \$	id T' E' \$	
id + id * id \$	T' E' \$	$T' \rightarrow \epsilon$
id + id * id \$	E' \$	$E' \rightarrow + T E'$
id + id * id \$	+ T E' \$	

```

if (X == a) { pop, advance a in w }
else if (X is a terminal) { error }
else if (M[X,a] is blank) { error }
else if (M[X,a] is  $X \rightarrow Y_1 Y_2 \dots Y_k$ ) {
  output  $X \rightarrow Y_1 Y_2 \dots Y_k$ 
  pop
  push  $Y_k \dots Y_2 Y_1$ 
}

```

Let X be the top stack symbol

	id	+	*	()	\$
E	$E \rightarrow T E'$			$E \rightarrow T E'$		
E'		$E' \rightarrow + T E'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow F T'$			$T \rightarrow F T'$		
T'		$T' \rightarrow \epsilon$	$T' \rightarrow * F T'$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F	$F \rightarrow id$			$F \rightarrow (E)$		

INPUT (a)	STACK (X)	OUTPUT
id + id * id \$	E \$	$E \rightarrow T E'$
id + id * id \$	T E' \$	$T \rightarrow F T'$
id + id * id \$	F T' E' \$	$F \rightarrow id$
id + id * id \$	id T' E' \$	
id + id * id \$	T' E' \$	$T' \rightarrow \epsilon$
id + id * id \$	E' \$	$E' \rightarrow + T E'$
id + id * id \$	+ T E' \$	
id + id * id \$	T E' \$	$T \rightarrow F T'$

```

if (X == a) { pop, advance a in w }
else if (X is a terminal) { error }
else if (M[X,a] is blank) { error }
else if (M[X,a] is  $X \rightarrow Y_1 Y_2 \dots Y_k$ ) {
  output  $X \rightarrow Y_1 Y_2 \dots Y_k$ 
  pop
  push  $Y_k \dots Y_2 Y_1$ 
}

```

Let X be the top stack symbol

	id	+	*	()	\$
E	$E \rightarrow T E'$			$E \rightarrow T E'$		
E'		$E' \rightarrow + T E'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow F T'$			$T \rightarrow F T'$		
T'		$T' \rightarrow \epsilon$	$T' \rightarrow * F T'$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F	$F \rightarrow id$			$F \rightarrow (E)$		

INPUT (a)

id + id * id \$

id + id * id \$

id + id * id \$

id + id * id \$

id + id * id \$

id + id * id \$

id + id * id \$

id + id * id \$

id + id * id \$

STACK (X)

E \$

T E' \$

F T' E' \$

id T' E' \$

T' E' \$

E' \$

+ T E' \$

T E' \$

F T' E' \$

OUTPUT

$E \rightarrow T E'$

$T \rightarrow F T'$

$F \rightarrow id$

$T' \rightarrow \epsilon$

$E' \rightarrow + T E'$

$T \rightarrow F T'$

$F \rightarrow id$

if $(X == a)$ { pop, advance a in ω }
 else if (X is a terminal) { error }
 else if ($M[X,a]$ is blank) { error }
 else if ($M[X,a]$ is $X \rightarrow Y_1 Y_2 \dots Y_k$) {
 output $X \rightarrow Y_1 Y_2 \dots Y_k$
 pop
 push $Y_k \dots Y_2 Y_1$
 }

Let X be the top stack symbol

	id	+	*	()	\$
E	$E \rightarrow T E'$			$E \rightarrow T E'$		
E'		$E' \rightarrow + T E'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow F T'$			$T \rightarrow F T'$		
T'		$T' \rightarrow \epsilon$	$T' \rightarrow * F T'$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F	$F \rightarrow id$			$F \rightarrow (E)$		

INPUT (a)

id + id * id \$

id + id * id \$

id + id * id \$

id + id * id \$

id + id * id \$

id + id * id \$

id + id * id \$

id + id * id \$

id + id * id \$

id + id * id \$

STACK (X)

E \$

T E' \$

F T' E' \$

id T' E' \$

T' E' \$

E' \$

+ T E' \$

T E' \$

F T' E' \$

id T' E' \$

OUTPUT

$E \rightarrow T E'$

$T \rightarrow F T'$

$F \rightarrow id$

$T' \rightarrow \epsilon$

$E' \rightarrow + T E'$

$T \rightarrow F T'$

$F \rightarrow id$

```

if (X == a) { pop, advance a in w }
else if (X is a terminal) { error }
else if (M[X,a] is blank) { error }
else if (M[X,a] is  $X \rightarrow Y_1 Y_2 \dots Y_k$ ) {
  output  $X \rightarrow Y_1 Y_2 \dots Y_k$ 
  pop
  push  $Y_k \dots Y_2 Y_1$ 
}
  
```

Let X be the top stack symbol

	id	+	*	()	\$
E	$E \rightarrow T E'$			$E \rightarrow T E'$		
E'		$E' \rightarrow + T E'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow F T'$			$T \rightarrow F T'$		
T'		$T' \rightarrow \epsilon$	$T' \rightarrow * F T'$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F	$F \rightarrow id$			$F \rightarrow (E)$		

INPUT (a)

id + id * id \$

id + id * id \$

id + id * id \$

id + id * id \$

id + id * id \$

id + id * id \$

id + id * id \$

id + id * id \$

id + id * id \$

id + id * id \$

id + id * id \$

STACK (X)

E \$

T E' \$

F T' E' \$

id T' E' \$

T' E' \$

E' \$

+ T E' \$

T E' \$

F T' E' \$

id T' E' \$

T' E' \$

OUTPUT

E → T E'

T → F T'

F → id

T' → ε

E' → + T E'

T → F T'

F → id

T' → * F T'

```

if (X == a) { pop, advance a in ω }
else if (X is a terminal) { error }
else if (M[X,a] is blank) { error }
else if (M[X,a] is X → Y1 Y2 ... Yk) {
  output X → Y1 Y2 ... Yk
  pop
  push Yk ... Y2 Y1
}
  
```

Let X be the top stack symbol

	id	+	*	()	\$
E	E → T E'			E → T E'		
E'		E' → + T E'			E' → ε	E' → ε
T	T → F T'			T → F T'		
T'		T' → ε	T' → * F T'		T' → ε	T' → ε
F	F → id			F → (E)		

INPUT (a)

STACK (X)

OUTPUT

```

if (X == a) { pop, advance a in ω }
else if (X is a terminal) { error }
else if (M[X,a] is blank) { error }
else if (M[X,a] is X → Y1 Y2 ... Yk) {
  output X → Y1 Y2 ... Yk
  pop
  push Yk ... Y2 Y1
}
  
```

	id	+	*	()	\$
E	E → T E'			E → T E'		
E'		E' → + T E'			E' → ε	E' → ε
T	T → F T'			T → F T'		
T'		T' → ε	T' → * F T'		T' → ε	T' → ε
F	F → id			F → (E)		

Let X be the top stack symbol

id + id * id \$

+ T E' \$

id + id * id \$

T E' \$

T → F T'

id + id * id \$

F T' E' \$

F → id

id + id * id \$

id T' E' \$

id + id * id \$

T' E' \$

T' → * F T'

id + id * id \$

* F T' E' \$

INPUT (a)

STACK (X)

OUTPUT

```

if (X == a) { pop, advance a in w }
else if (X is a terminal) { error }
else if (M[X,a] is blank) { error }
else if (M[X,a] is  $X \rightarrow Y_1 Y_2 \dots Y_k$ ) {
  output  $X \rightarrow Y_1 Y_2 \dots Y_k$ 
  pop
  push  $Y_k \dots Y_2 Y_1$ 
}
  
```

Let X be the top stack symbol

	id	+	*	()	\$
E	$E \rightarrow T E'$			$E \rightarrow T E'$		
E'		$E' \rightarrow + T E'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow F T'$			$T \rightarrow F T'$		
T'		$T' \rightarrow \epsilon$	$T' \rightarrow * F T'$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F	$F \rightarrow id$			$F \rightarrow (E)$		

id + id * id \$

+ T E' \$

id + id * id \$

T E' \$

$T \rightarrow F T'$

id + id * id \$

F T' E' \$

$F \rightarrow id$

id + id * id \$

id T' E' \$

id + id * id \$

T' E' \$

$T' \rightarrow * F T'$

id + id * id \$

* F T' E' \$

id + id * id \$

F T' E' \$

$F \rightarrow id$

INPUT (a)

STACK (X)

OUTPUT

```

if (X == a) { pop, advance a in w }
else if (X is a terminal) { error }
else if (M[X,a] is blank) { error }
else if (M[X,a] is X -> Y1 Y2 ... Yk) {
  output X -> Y1 Y2 ... Yk
  pop
  push Yk ... Y2 Y1
}
  
```

Let X be the top stack symbol

	id	+	*	()	\$
E	$E \rightarrow T E'$			$E \rightarrow T E'$		
E'		$E' \rightarrow + T E'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow F T'$			$T \rightarrow F T'$		
T'		$T' \rightarrow \epsilon$	$T' \rightarrow * F T'$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F	$F \rightarrow id$			$F \rightarrow (E)$		

id + id * id \$

+ T E' \$

id + id * id \$

T E' \$

$T \rightarrow F T'$

id + id * id \$

F T' E' \$

$F \rightarrow id$

id + id * id \$

id T' E' \$

id + id * id \$

T' E' \$

$T' \rightarrow * F T'$

id + id * id \$

* F T' E' \$

id + id * id \$

F T' E' \$

$F \rightarrow id$

id + id * id \$

id T' E' \$

INPUT (a)

STACK (X)

OUTPUT

```

if (X == a) { pop, advance a in w }
else if (X is a terminal) { error }
else if (M[X,a] is blank) { error }
else if (M[X,a] is  $X \rightarrow Y_1 Y_2 \dots Y_k$ ) {
  output  $X \rightarrow Y_1 Y_2 \dots Y_k$ 
  pop
  push  $Y_k \dots Y_2 Y_1$ 
}
  
```

Let X be the top stack symbol

	id	+	*	()	\$
E	$E \rightarrow T E'$			$E \rightarrow T E'$		
E'		$E' \rightarrow + T E'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow F T'$			$T \rightarrow F T'$		
T'		$T' \rightarrow \epsilon$	$T' \rightarrow * F T'$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F	$F \rightarrow id$			$F \rightarrow (E)$		

id + id * id \$

+ T E' \$

id + id * id \$

T E' \$

$T \rightarrow F T'$

id + id * id \$

F T' E' \$

$F \rightarrow id$

id + id * id \$

id T' E' \$

id + id * id \$

T' E' \$

$T' \rightarrow * F T'$

id + id * id \$

* F T' E' \$

id + id * id \$

F T' E' \$

$F \rightarrow id$

id + id * id \$

id T' E' \$

id + id * id \$

T' E' \$

$T' \rightarrow \epsilon$

INPUT (a)

STACK (X)

OUTPUT

if (X == a) { pop, advance a in ω }
 else if (X is a terminal) { error }
 else if (M[X,a] is blank) { error }
 else if (M[X,a] is $X \rightarrow Y_1 Y_2 \dots Y_k$) {
 output $X \rightarrow Y_1 Y_2 \dots Y_k$
 pop
 push $Y_k \dots Y_2 Y_1$
 }

Let X be the top stack symbol

	id	+	*	()	\$
E	$E \rightarrow T E'$			$E \rightarrow T E'$		
E'		$E' \rightarrow + T E'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow F T'$			$T \rightarrow F T'$		
T'		$T' \rightarrow \epsilon$	$T' \rightarrow * F T'$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F	$F \rightarrow id$			$F \rightarrow (E)$		

id + id * id \$

+ T E' \$

id + id * id \$

T E' \$

T \rightarrow F T'

id + id * id \$

F T' E' \$

F \rightarrow id

id + id * id \$

id T' E' \$

id + id * id \$

T' E' \$

T' \rightarrow * F T'

id + id * id \$

* F T' E' \$

id + id * id \$

F T' E' \$

F \rightarrow id

id + id * id \$

id T' E' \$

id + id * id \$

T' E' \$

T' \rightarrow ϵ

id + id * id \$

E' \$

E' \rightarrow ϵ

INPUT (a)

STACK (X)

OUTPUT

```

while (X ≠ $) {
  if (X == a) { pop the stack, advance a in ω }
  else if (X is a terminal) { error }
  else if (M[X,a] is blank) { error }
  else if (M[X,a] is X → Y1 Y2 ... Yk) {
    output X → Y1 Y2 ... Yk
    pop the stack
    push Yk ... Y2 Y1 onto the stack
  }
  Let X be the top stack symbol
}

```

Accept if a == X == \$

id + id * id \$

id T' E' \$

id + id * id \$

T' E' \$

T' → * F T'

id + id * id \$

* F T' E' \$

id + id * id \$

F T' E' \$

F → id

id + id * id \$

id T' E' \$

id + id * id \$

T' E' \$

T' → ε

id + id * id \$

E' \$

E' → ε

id + id * id \$

\$

Since both a and X are \$, accept

INPUT (a)	STACK (X)	OUTPUT
id + id * id \$	E \$	$E \rightarrow T E'$
id + id * id \$	T E' \$	$T \rightarrow F T'$
id + id * id \$	F T' E' \$	$F \rightarrow id$
id + id * id \$	id T' E' \$	
id + id * id \$	T' E' \$	$T' \rightarrow \epsilon$
id + id * id \$	E' \$	$E' \rightarrow + T E'$
id + id * id \$	+ T E' \$	
id + id * id \$	T E' \$	$T \rightarrow F T'$
id + id * id \$	F T' E' \$	$F \rightarrow id$
id + id * id \$	id T' E' \$	
id + id * id \$	T' E' \$	$T' \rightarrow * F T'$
id + id * id \$	* F T' E' \$	
id + id * id \$	F T' E' \$	$F \rightarrow id$
id + id * id \$	id T' E' \$	
id + id * id \$	T' E' \$	$T' \rightarrow \epsilon$
id + id * id \$	E' \$	$E' \rightarrow \epsilon$
id + id * id \$	\$	

Since both a and X are \$, accept

Error recovery in predictive parsing

Basic idea: if input is unexpected given the current state of the parse, try to "synchronize" so that parse can continue (even though no machine code generation will occur once an error has been detected)

Strategies for error recovery

Skip input symbols until a synchronizing token appears.

1. $\text{synch}(A)$ includes $\text{FOLLOW}(A)$
2. ... $\langle \text{expr} \rangle ;$ $\text{FOLLOW}(\langle \text{expr} \rangle)$ is $\{';'\}$, but if $';$ is missing, then include all keyword-beginning statement starts in $\text{synch}(A)$
3. add contents of $\text{FIRST}(A)$ to $\text{synch}(A)$
4. if $A \rightarrow \epsilon$ use this "absorb" A
5. if terminal a on the stack cannot be matched, pop it and issue a message " a was inserted". (Synch is set to all other tokens.)

synch is added to each blank $M[X,a]$ when $a \in \text{FOLLOW}(X)$

$$\text{FOLLOW}(E) = \text{FOLLOW}(E') = \{ \text{) , } \$ \}$$

NON TERMINALS	id	+	*	()	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$	<i>synch</i>	<i>synch</i>
E'		$E' \rightarrow +TE'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'		$T' \rightarrow \epsilon$	$T' \rightarrow *FT$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F	$F \rightarrow \text{id}$			$F \rightarrow (E)$		

synch is added to each blank $M[X,a]$ when $a \in \text{FOLLOW}(X)$

$$\text{FOLLOW}(T) = \text{FOLLOW}(T') = \{ +,), \$ \}$$

NON TERMINALS	id	+	*	()	\$
E	$E \rightarrow T E'$			$E \rightarrow T E'$	<i>synch</i>	<i>synch</i>
E'		$E' \rightarrow + T E'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow F T'$	<i>synch</i>		$T \rightarrow F T'$	<i>synch</i>	<i>synch</i>
T'		$T' \rightarrow \epsilon$	$T' \rightarrow * F T$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F	$F \rightarrow \text{id}$			$F \rightarrow (E)$		

synch is added to each blank $M[X,a]$ when $a \in \text{FOLLOW}(X)$

$$\text{FOLLOW}(F) = \{ +, *,), \$ \}$$

NON TERMINALS	id	+	*	()	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$	<i>synch</i>	<i>synch</i>
E'		$E' \rightarrow +TE'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow FT'$	<i>synch</i>		$T \rightarrow FT'$	<i>synch</i>	<i>synch</i>
T'		$T' \rightarrow \epsilon$	$T' \rightarrow *FT$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F	$F \rightarrow \text{id}$	<i>synch</i>	<i>synch</i>	$F \rightarrow (E)$	<i>synch</i>	<i>synch</i>

Modified parse-table M

NON TERMINALS	id	+	*	()	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$	synch	synch
E'		$E' \rightarrow +TE'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow FT'$	synch		$T \rightarrow FT'$	synch	synch
T'		$T' \rightarrow \epsilon$	$T' \rightarrow *FT$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F	$F \rightarrow id$	synch	synch	$F \rightarrow (E)$	synch	synch

synch is added to each blank $M[X,a]$ when $a \in FOLLOW(X)$

- $FOLLOW(E) = FOLLOW(E') = \{), \$ \}$
- $FOLLOW(T) = FOLLOW(T') = \{ +,), \$ \}$
- $FOLLOW(F) = \{ +, *,), \$ \}$

Algorithm 4.34 (modified w/error handling)

Let a be the first symbol of ω

Let X be the top stack symbol

while ($X \neq \$$) {

 if ($x == a$) { pop the stack, advance a in ω }

 else if (X is a terminal) { pop stack }

 else if ($M[X,a]$ is blank) { advance a in ω }

 else if ($M[X,a]$ is synch) { pop stack }

 else if ($M[X,a]$ is $X \rightarrow Y_1 Y_2 \dots Y_k$) {

 output $X \rightarrow Y_1 Y_2 \dots Y_k$

 pop the stack

 push $Y_k \dots Y_2 Y_1$ onto the stack

 }

 Let X be the top stack symbol

}

Accept if $a == X == \$$

INPUT (a)

+ id * + id \$

STACK (X)

E \$

OUTPUT

???

if (x == a) { pop, advance a in ω }

else if (X is a terminal) { pop }

else if (M[X,a] is blank) { advance a in ω }

else if (M[X,a] is synch) { pop }

else if (M[X,a] is $X \rightarrow Y_1 Y_2 \dots Y_k$) {
 output $X \rightarrow Y_1 Y_2 \dots Y_k$
 pop
 push $Y_k \dots Y_2 Y_1$
}

Let X be the top stack symbol

	id	+	*	()	\$
E	$E \rightarrow T E'$			$E \rightarrow T E'$	synch	synch
E'		$E' \rightarrow + T E'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow F T'$	synch		$T \rightarrow F T'$	synch	synch
T'		$T' \rightarrow \epsilon$	$T' \rightarrow * F T$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F	$F \rightarrow id$	synch	synch	$F \rightarrow (E)$	synch	synch

INPUT (a)

+ id * + id \$

STACK (X)

E \$

OUTPUT

error, skip +

if (x == a) { pop, advance a in ω }

else if (X is a terminal) { pop }

else if (M[X,a] is blank) { advance a in ω }

else if (M[X,a] is synch) { pop }

else if (M[X,a] is $X \rightarrow Y_1 Y_2 \dots Y_k$) {
 output X \rightarrow $Y_1 Y_2 \dots Y_k$
 pop
 push $Y_k \dots Y_2 Y_1$
}

Let X be the top stack symbol

	id	+	*	()	\$
E	$E \rightarrow T E'$			$E \rightarrow T E'$	synch	synch
E'		$E' \rightarrow + T E'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow F T'$	synch		$T \rightarrow F T'$	synch	synch
T'		$T' \rightarrow \epsilon$	$T' \rightarrow * F T$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F	$F \rightarrow id$	synch	synch	$F \rightarrow (E)$	synch	synch

INPUT (a)

STACK (X)

OUTPUT

+ id * + id \$

E \$

error, skip +

+ id * + id \$

E \$

$E \rightarrow T E'$

+ id * + id \$

T E' \$

$T \rightarrow F T'$

+ id * + id \$

F T' E' \$

$F \rightarrow id$

+ id * + id \$

id T' E' \$

+ id * + id \$

T' E' \$

$T' \rightarrow * F T'$

+ id * + id \$

* F T' E' \$

+ id * + id \$

F T' E' \$

???

if (x == a) { pop, advance a in ω }
 else if (X is a terminal) { pop }
 else if (M[X,a] is blank) { advance a in ω }
 else if (M[X,a] is synch) { pop }
 else if (M[X,a] is $X \rightarrow Y_1 Y_2 \dots Y_k$) {
 output $X \rightarrow Y_1 Y_2 \dots Y_k$
 pop
 push $Y_k \dots Y_2 Y_1$
 }
 Let X be the top stack symbol

	id	+	*	()	\$
E	$E \rightarrow T E'$			$E \rightarrow T E'$	synch	synch
E'		$E' \rightarrow + T E'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow F T'$	synch		$T \rightarrow F T'$	synch	synch
T'		$T' \rightarrow \epsilon$	$T' \rightarrow * F T$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F	$F \rightarrow id$	synch	synch	$F \rightarrow (E)$	synch	synch

INPUT (a)

+ id * + id \$
 + id * + id \$
 + id * + id \$
 + id * + id \$
 + id * + id \$
 + id * + id \$
 + id * + id \$
 + id * + id \$

STACK (X)

E \$
 E \$
 T E' \$
 F T' E' \$
 id T' E' \$
 T' E' \$
 * F T' E' \$
 F T' E' \$

OUTPUT

error, skip +
 E → T E'
 T → F T'
 F → id
 T' → * F T'
 error, M[F, +] = synch, pop F

if (x == a) { pop, advance a in ω }
 else if (X is a terminal) { pop }
 else if (M[X,a] is blank) { advance a in ω }
 else if (M[X,a] is synch) { pop }
 else if (M[X,a] is X → Y1 Y2 ... Yk) {
 output X → Y1 Y2 ... Yk
 pop
 push Yk ... Y2 Y1
 }
 Let X be the top stack symbol

	id	+	*	()	\$
E	E → T E'			E → T E'	synch	synch
E'		E' → + T E'			E' → ε	E' → ε
T	T → F T'	synch		T → F T'	synch	synch
T'		T' → ε	T' → * F T		T' → ε	T' → ε
F	F → id	synch	synch	F → (E)	synch	synch

INPUT (a)	STACK (X)	OUTPUT
+ id * + id \$	E \$	error, skip +
+ id * + id \$	E \$	$E \rightarrow T E'$
+ id * + id \$	T E' \$	$T \rightarrow F T'$
+ id * + id \$	F T' E' \$	$F \rightarrow id$
+ id * + id \$	id T' E' \$	
+ id * + id \$	T' E' \$	$T' \rightarrow * F T'$
+ id * + id \$	* F T' E' \$	
+ id * + id \$	F T' E' \$	error, $M[F, +] = \text{synch}$, pop F
+ id * + id \$	T' E' \$	$T' \rightarrow \epsilon$
+ id * + id \$	E' \$	$E' \rightarrow + T E'$
+ id * + id \$	+ T E' \$	
+ id * + id \$	T E' \$	$T \rightarrow F T'$
+ id * + id \$	F T' E' \$	$F \rightarrow id$
+ id * + id \$	id T' E' \$	
+ id * + id \$	T' E' \$	$T' \rightarrow \epsilon$
+ id * + id \$	E' \$	$E' \rightarrow \epsilon$
+ id * + id \$	\$	

Since there were errors, do not accept.