CSE443
Compilers

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Tentative short-term schedule overview

- Today: build Finite State Machine

- Monday: build parse table M

- Wednesday: example parse

- Friday: Sprint 2 discussion

- We may interchange Wednesday's and Friday's topics

Building the finite control for a bottom-up parser

- Build a finite state machine, whose states are sets of items

- Build a table (M) incorporating shift/reduce decisions

Augment grammar

Given a grammar

$G = (N,T,P,S)$

we augment to a grammar

$G' = (N \cup \{S'\}, T, P \cup \{S' \rightarrow S\}, S')$, where $S' \notin N$

G' has exactly one rule with S' on left.

CLOSURE(I)

- I is a set of items

- CLOSURE(I) fixed point construction

$CLOSURE_0(I) = I$

repeat {

$CLOSURE_{i+1}(I) =$

$CLOSURE_i(I) \cup \{ B{\rightarrow}\bullet\gamma \mid A \rightarrow \alpha\bullet B\beta \in CLOSURE_i(I) \text{ and } B \rightarrow \gamma \in P \}$

} until $CLOSURE_{i+1}(I) = CLOSURE_i(I)$

GOTO(I,X)

- GOTO(I,X) is the closure of the set of items $A \to \alpha X \bullet \beta$ s.t. $A \to \alpha \bullet X \beta \in I$

Building the LR(0) automaton

```
void items(G') {
    C = { CLOSURE( { S' -> •S } ) }
    repeat {
        for each set of items I ∈ C and
        for each grammar symbol X ∈ (NUT)
        if ( GOTO(I,X) is not empty and not already in C )
            add GOTO(I,X) to C
    } until no new sets of items are added to C
}
```

C is a set of sets of items

Terminology

- Kernel items: S' -> • S and all items with • not at left edge

- Non-kernel items: all items with • at left edge, except S' -> • S

This gives us the first state of the finite state machine, I₀

I₀

S' -> • E
———————
E -> • E + T
E -> • T
T -> • T * F
T -> • F
F -> • (E)
F -> • id

kernel item

non-kernel items are computed from CLOSURE(kernel), and therefore do not need to be explicitly stored

Next we compute GOTO(I_0,X) $\forall$ X $\in$ N $\cup$ T

N $\cup$ T = { E , T , F , + , * , (,) , id }

N.B. – augmented start symbol S' can be ignored

GOTO(I_0,E) = CLOSURE({ S' -> E $\bullet$, E -> E $\bullet$ + T })

= { S' -> E $\bullet$, E -> E $\bullet$ + T }

N.B. there is no non-terminal after the $\bullet$, so no new items are added by CLOSURE operation

I_1 | S' -> E $\bullet$
E -> E $\bullet$ + T

only kernel items

$$GOTO(I_0, T) = CLOSURE(\{ E \to T \bullet , T \to T \bullet * F \})$$

$$= \{ E \to T \bullet , T \to T \bullet * F \}$$

N.B. there is no non-terminal after the $\bullet$, so no new items are added by CLOSURE operation

I_2

E -> T $\bullet$
T -> T $\bullet$ * F

only kernel items

GOTO(I_0,F) = CLOSURE({ T -> F • })

= { T -> F • }

N.B. there is no non-terminal after the •, so no new items are added by CLOSURE operation

I_3 | T -> F • | only kernel items

GOTO(I₀, '(') = CLOSURE({ F -> (• E) })

= { F -> (• E) } ∪ { E -> • E + T , E -> • T } ∪ { T -> • T * F , T -> • F } ∪ { F -> • (E) , F -> • id }

I₄

F -> (• E)   kernel item

_____

E -> • E + T   non-kernel items
E -> • T
T -> • T * F
T -> • F
F -> • (E)
F -> • id

GOTO(I_0, id) = CLOSURE({ F -> id • })

= { F -> id • }

I_5 | F -> id • | only kernel items

N.B. there is no non-terminal after the •, so no new items are added by CLOSURE operation

GOTO(I_0 , ')') = GOTO(I_0 , +) = GOTO(I_0 , *) = GOTO(I_0 , \$) = ∅

I_0

$S' \rightarrow \bullet E$

$E \rightarrow \bullet E + T$
$E \rightarrow \bullet T$
$T \rightarrow \bullet T * F$
$T \rightarrow \bullet F$
$F \rightarrow \bullet (E)$
$F \rightarrow \bullet id$

I_1

$S' \rightarrow E \bullet$
$E \rightarrow E \bullet + T$

I_2

$E \rightarrow T \bullet$
$T \rightarrow T \bullet * F$

I_3

$T \rightarrow F \bullet$

I_5

$F \rightarrow id \bullet$

I_4

$F \rightarrow (\bullet E)$

$E \rightarrow \bullet E + T$
$E \rightarrow \bullet T$
$T \rightarrow \bullet T * F$
$T \rightarrow \bullet F$
$F \rightarrow \bullet (E)$
$F \rightarrow \bullet id$

E

T

$($

F

id

$S' \rightarrow E$
$E \rightarrow E + T$
$E \rightarrow T$
$T \rightarrow T * F$
$T \rightarrow F$
$F \rightarrow (E)$
$F \rightarrow id$
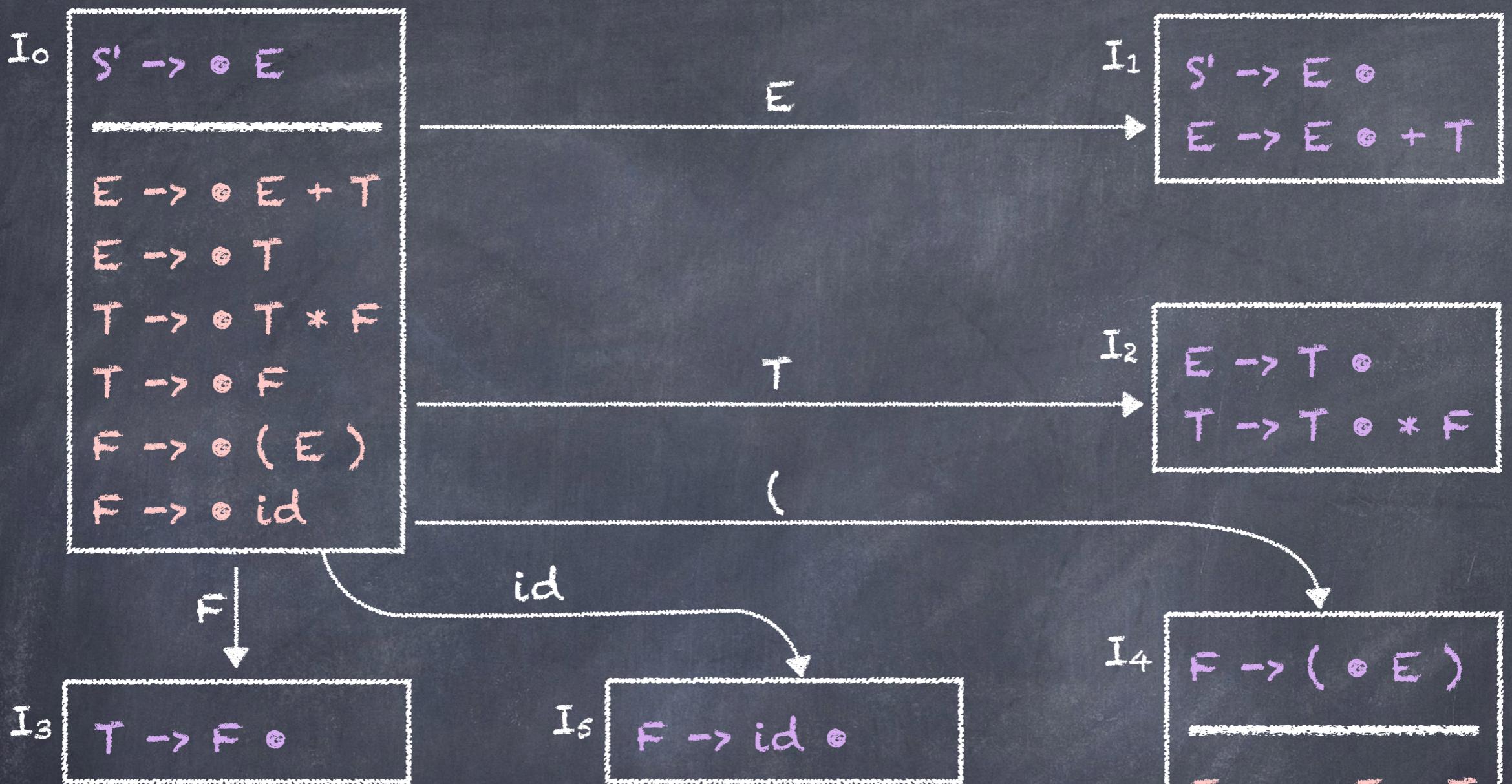
The finite state machine as at this point.

EXERCISE: complete the machine by computing $GOTO(I_k, X)$ until no new states are added.

I_0

S' -> • E
_____

E -> • E + T
E -> • T
T -> • T * F
T -> • F
F -> • (E)
F -> • id

I_1

S' -> E •
E -> E • + T

E

I_2

E -> T •
T -> T • * F

T

(

id

F

I_3 T -> F •

I_5 F -> id •

I_4 F -> (• E)
_____

E -> • E + T
E -> • T
T -> • T * F
T -> • F
F -> • (E)
F -> • id
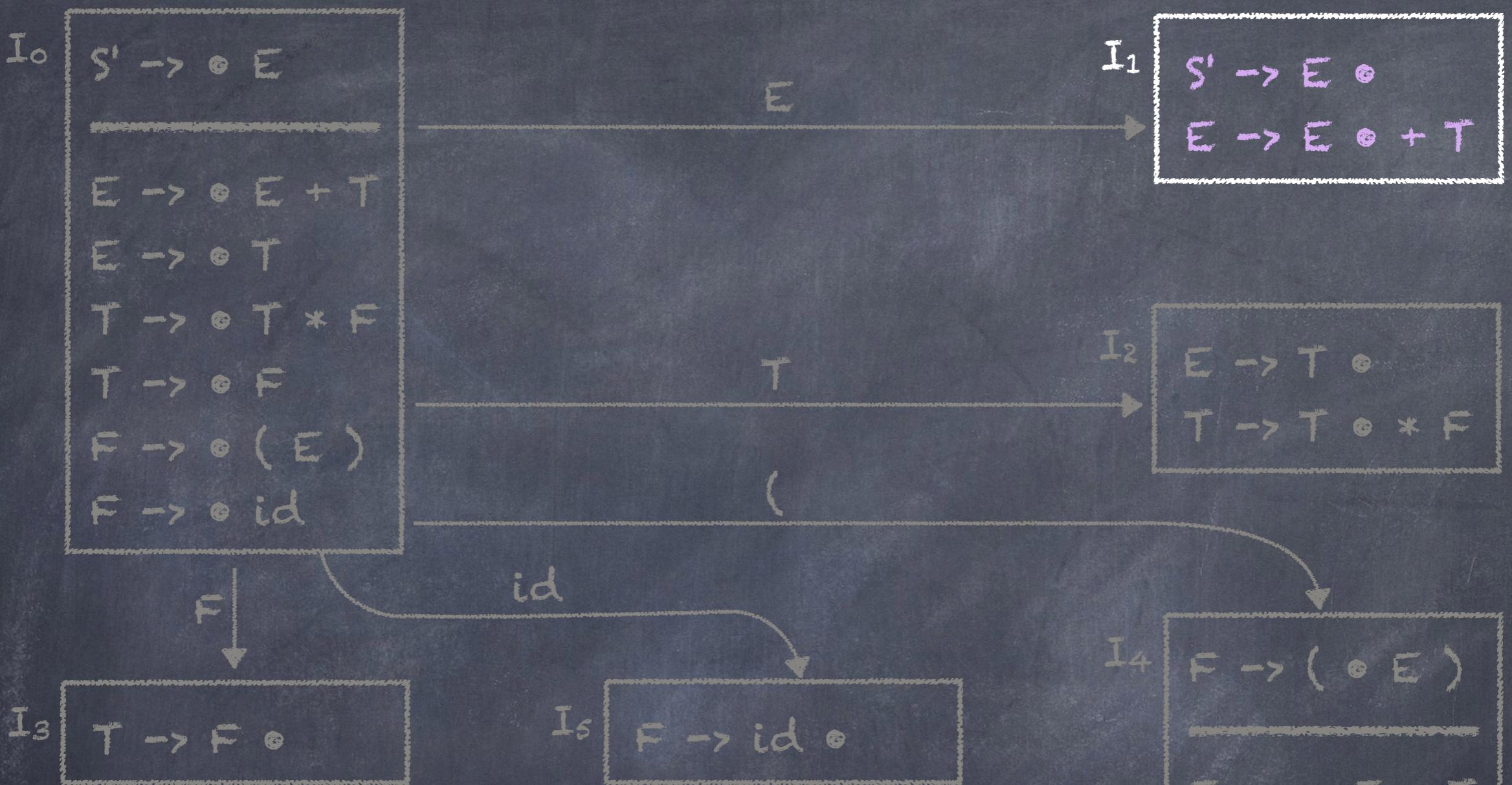
S' -> E
E -> E + T
E -> T
T -> T * F
T -> F
F -> (E)
F -> id

Compute GOTO(I_1,X) for each X
in { +, *, '(', ')', id, E, T, F, $ }
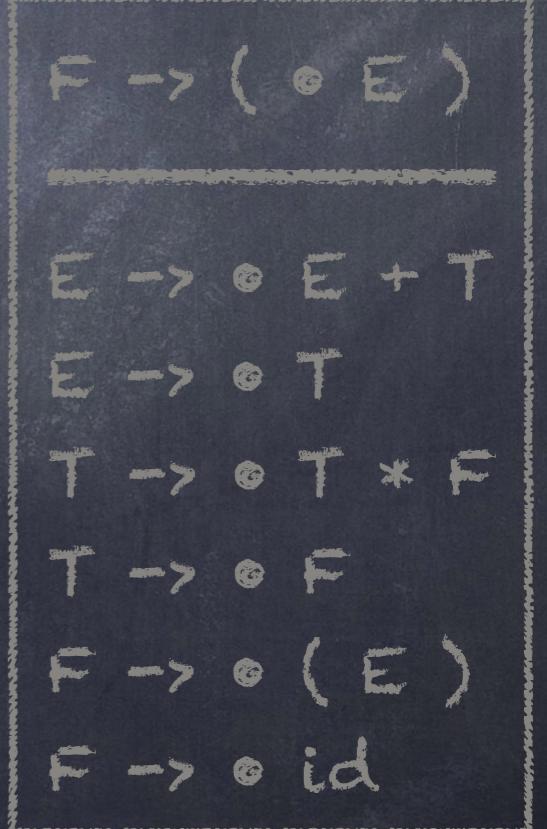
$GOTO(I_1, \$) = accept$

$GOTO(I_1, +) = CLOSURE(\{ E \rightarrow E + \bullet T \})$

$= \{ E \rightarrow E + \bullet T, T \rightarrow \bullet T * F, T \rightarrow \bullet F, F \rightarrow \bullet (E), F \rightarrow \bullet id \}$

I_6

$E \rightarrow E + \bullet T$
$T \rightarrow \bullet T * F$
$T \rightarrow \bullet F$
$F \rightarrow \bullet (E)$
$F \rightarrow \bullet id$

$GOTO(I_1, '(') = GOTO(I_1, ')') = GOTO(I_1, *) = GOTO(I_1, id) =$
$GOTO(I_1, E) = GOTO(I_1, T) = GOTO(I_1, F) = \emptyset$

$S' \rightarrow E$
$E \rightarrow E + T$
$E \rightarrow T$
$T \rightarrow T * F$
$T \rightarrow F$
$F \rightarrow (E)$
$F \rightarrow id$

I0
S' -> • E
───────────
E -> • E + T
E -> • T
T -> • T * F
T -> • F
F -> • (E)
F -> • id

I1
S' -> E •
E -> E • + T

$ → accept

I6
E -> E + • T
───────────
T -> • T * F
T -> • F
F -> • (E)
F -> • id

I2
E -> T •
T -> T • * F

I3
T -> F •

I5
F -> id •

I4
F -> (• E)
───────────
E -> • E + T
E -> • T
T -> • T * F
T -> • F
F -> • (E)
F -> • id

E
+
T
F
id
(

S' -> E
E -> E + T
E -> T
T -> T * F
T -> F
F -> (E)
F -> id
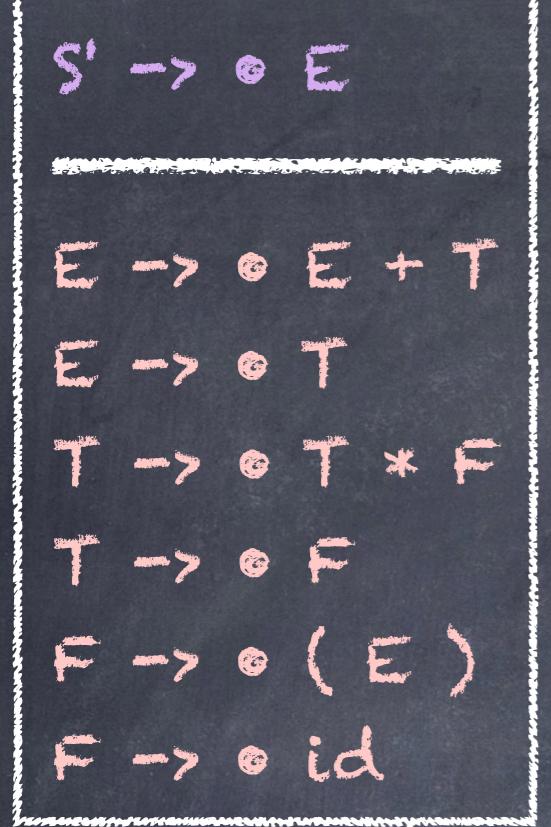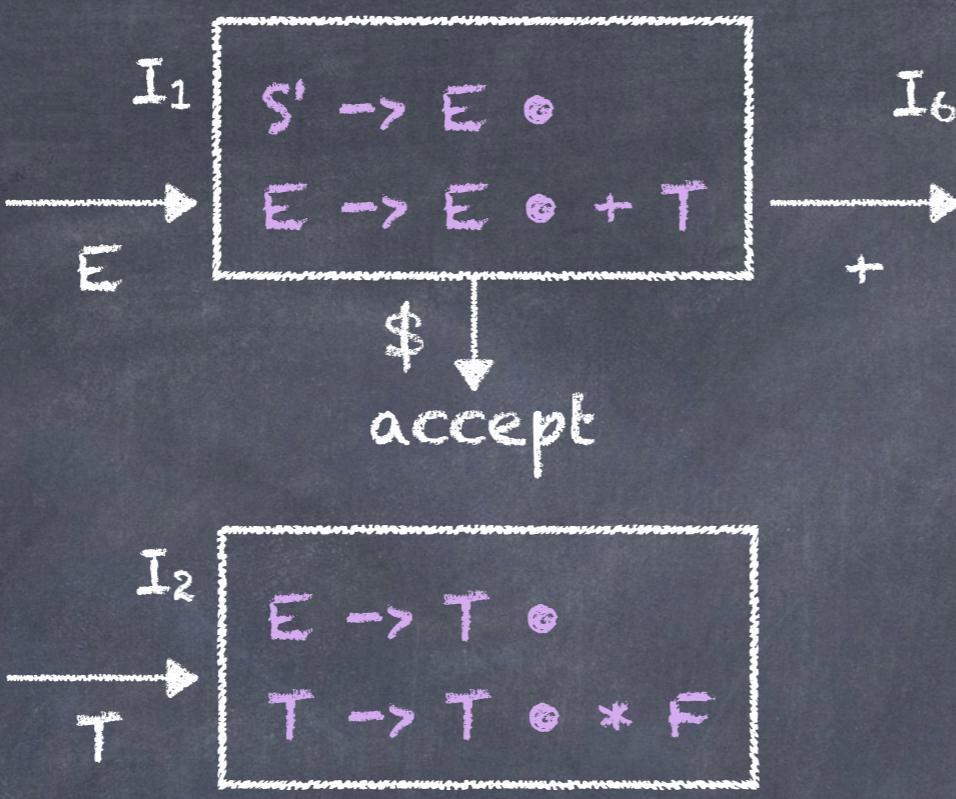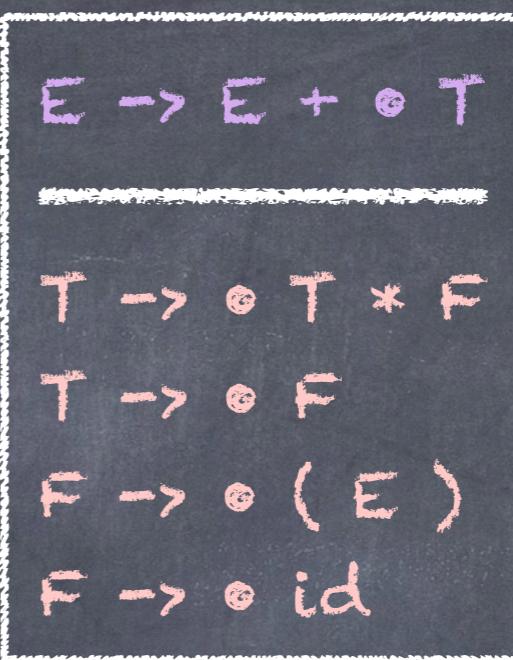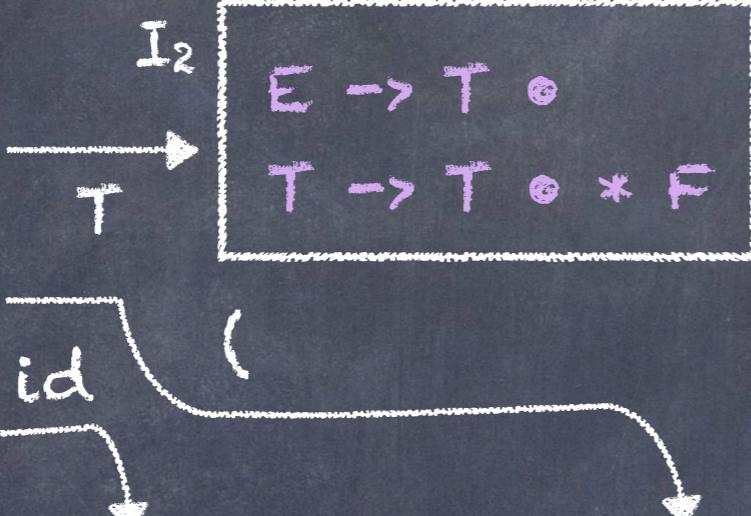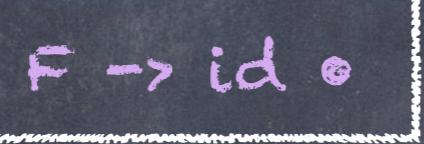
$$S' \rightarrow E$$
$$E \rightarrow E + T$$
$$E \rightarrow T$$
$$T \rightarrow T * F$$
$$T \rightarrow F$$
$$F \rightarrow (E)$$
$$F \rightarrow id$$

I_2

$$E \rightarrow T \bullet$$
$$T \rightarrow T \bullet * F$$

Compute GOTO(I_2,X) for each X
in { +, *, '(', ')', id, E, T, F, $ }

$$GOTO(I_2, *) = CLOSURE(\{ T \to T * \bullet F \})$$

$$= \{ T \to T * \bullet F, F \to \bullet (E), F \to \bullet id \}$$

I_7

$$T \to T * \bullet F$$

$$F \to \bullet (E)$$
$$F \to \bullet id$$

$S' \to E$
$E \to E + T$
$E \to T$
$T \to T * F$
$T \to F$
$F \to (E)$
$F \to id$

$$GOTO(I_2, '(') = GOTO(I_2, ')') = GOTO(I_2, +) = GOTO(I_2, id) =$$
$$GOTO(I_2, E) = GOTO(I_2, T) = GOTO(I_2, F) = GOTO(I_2, \$) = \varnothing$$

I₀

S' -> ● E

E -> ● E + T
E -> ● T
T -> ● T * F
T -> ● F
F -> ● (E)
F -> ● id

I₁

S' -> E ●
E -> E ● + T

$ accept

I₆

E -> E + ● T

T -> ● T * F
T -> ● F
F -> ● (E)
F -> ● id

I₂

E -> T ●
T -> T ● * F

I₃

T -> F ●

I₅

F -> id ●

I₄

F -> (● E)

E -> ● E + T
E -> ● T
T -> ● T * F
T -> ● F
F -> ● (E)
F -> ● id

I₇

T -> T * ● F

F -> ● (E)
F -> ● id

E
T
F
id
(
*
+

S' -> E
E -> E + T
E -> T
T -> T * F
T -> F
F -> (E)
F -> id
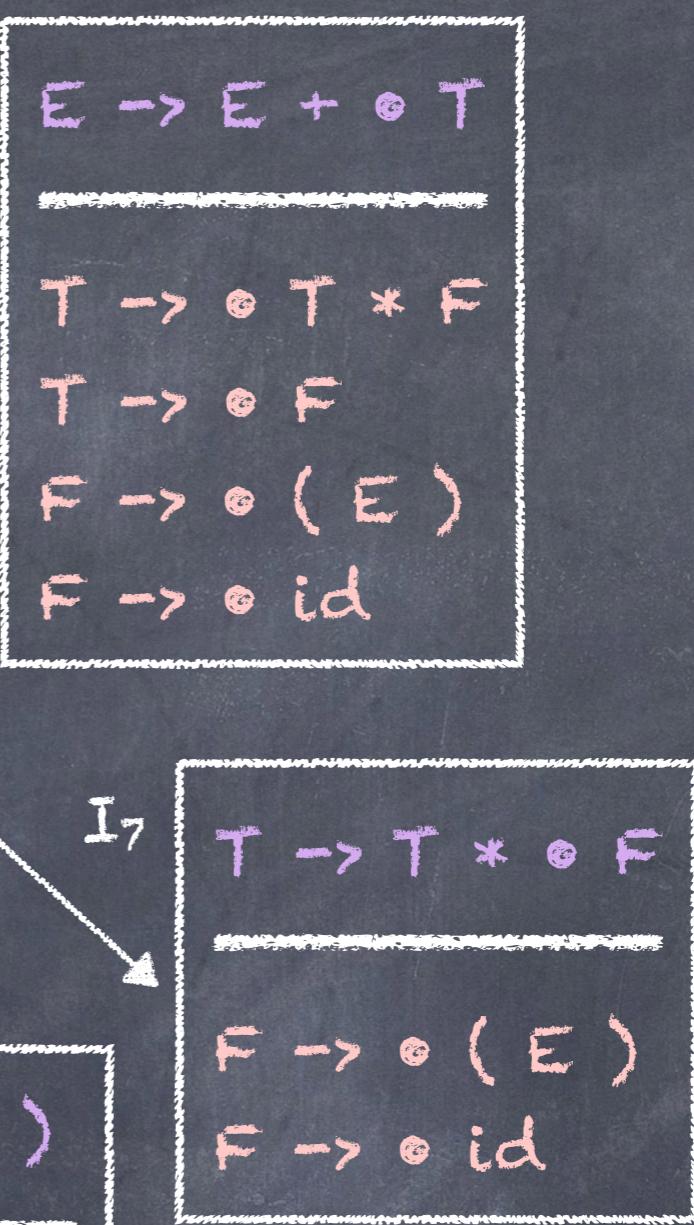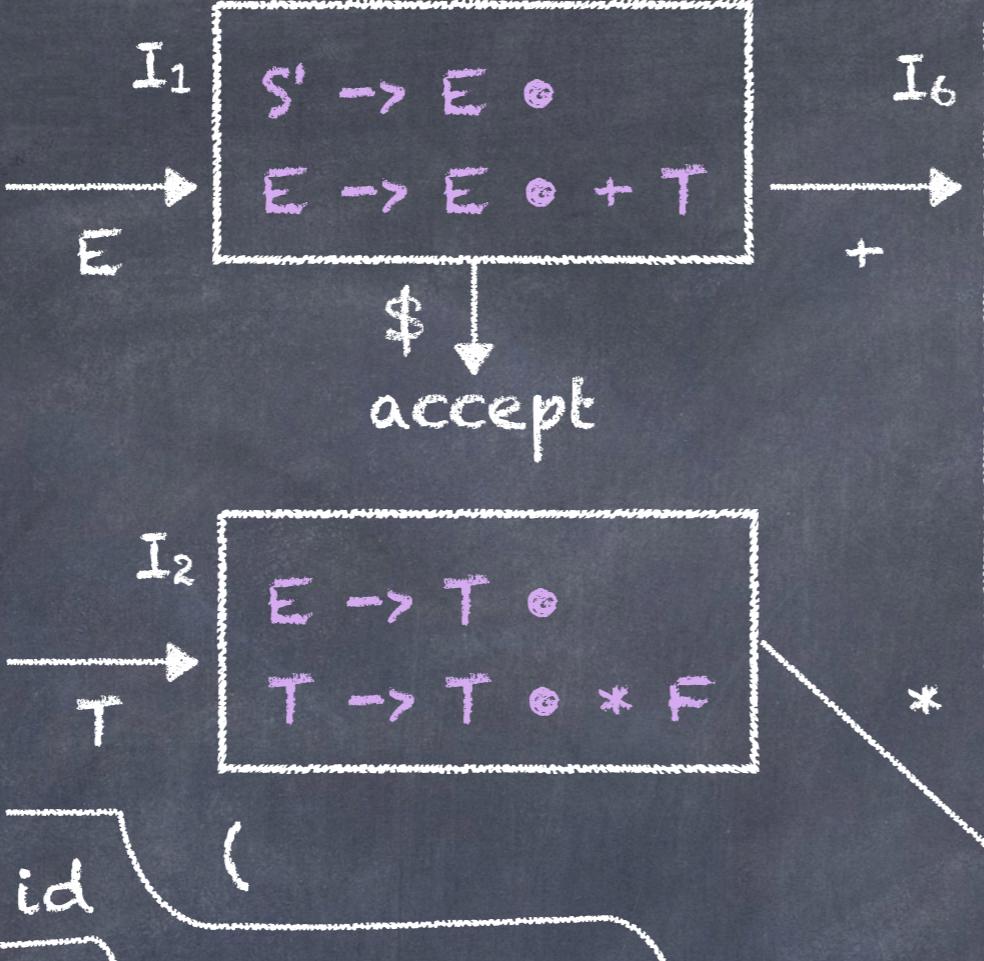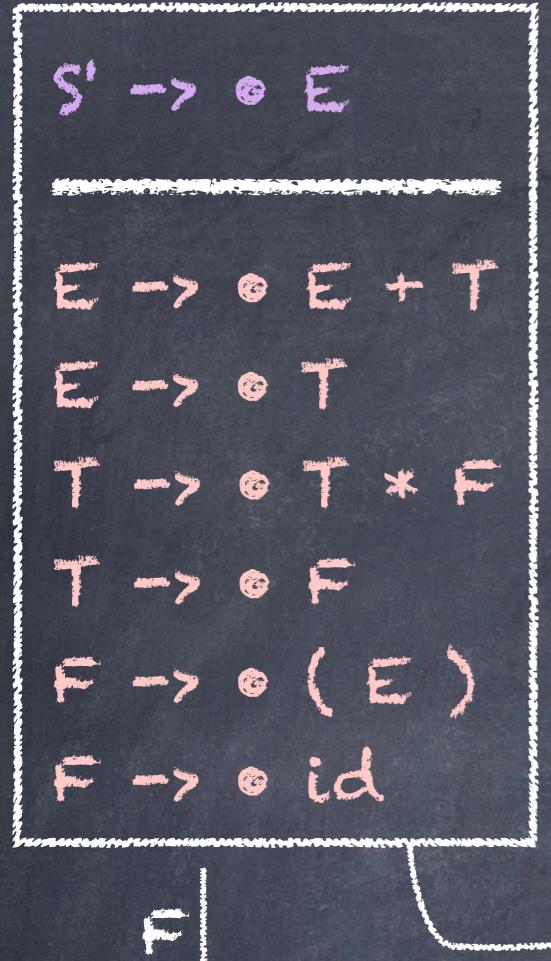
$S' \rightarrow E$
$E \rightarrow E + T$
$E \rightarrow T$
$T \rightarrow T * F$
$T \rightarrow F$
$F \rightarrow (E)$
$F \rightarrow id$

I_3 | $T \rightarrow F \bullet$

Compute GOTO(I_3,X) for each X
in { +, *, '(', ')', id, E, T, F, $ }

$GOTO(I_3,E) = GOTO(I_3,T) = GOTO(I_3,F) = GOTO(I_3,+) = GOTO(I_3,*) = GOTO(I_3,'(') = GOTO(I_3,')') = GOTO(I_3,id) = GOTO(I_3,\$) = \varnothing$

I₀

S' -> • E
───────────
E -> • E + T
E -> • T
T -> • T * F
T -> • F
F -> • (E)
F -> • id

I₁

S' -> E •
E -> E • + T
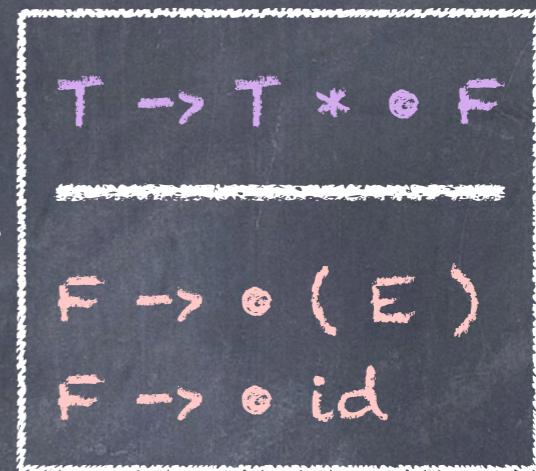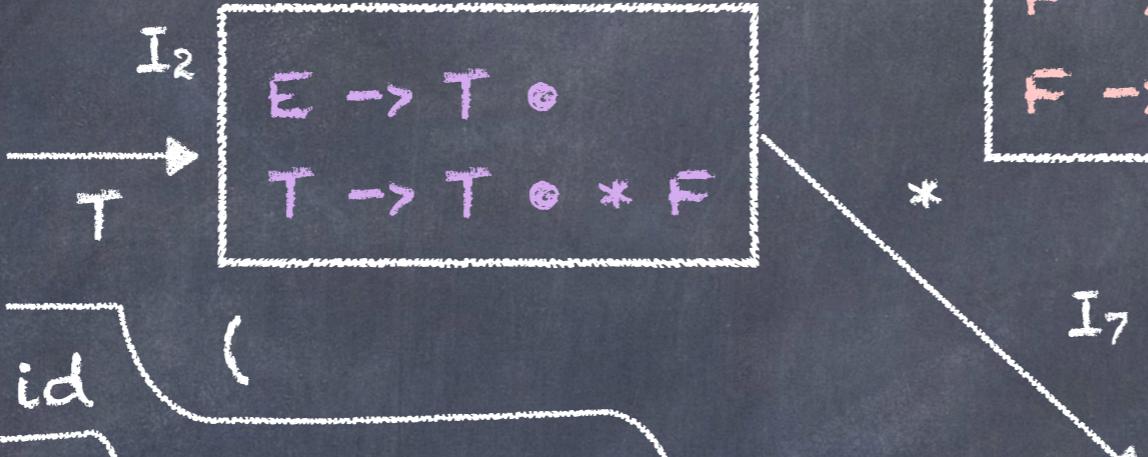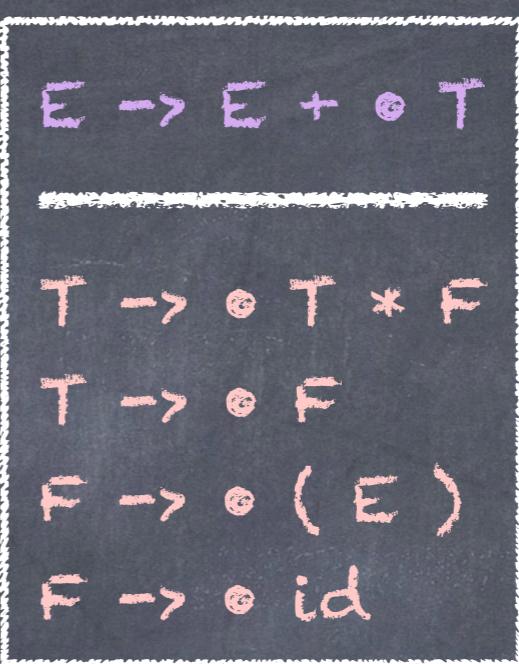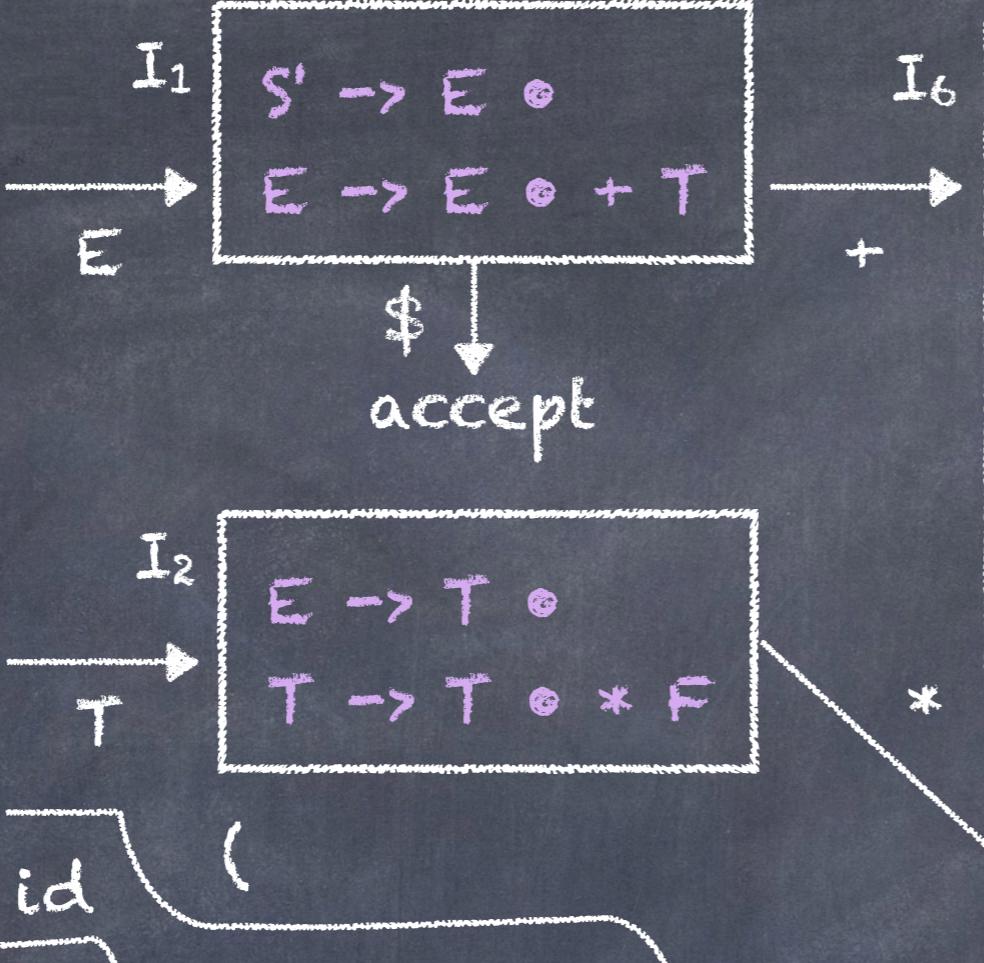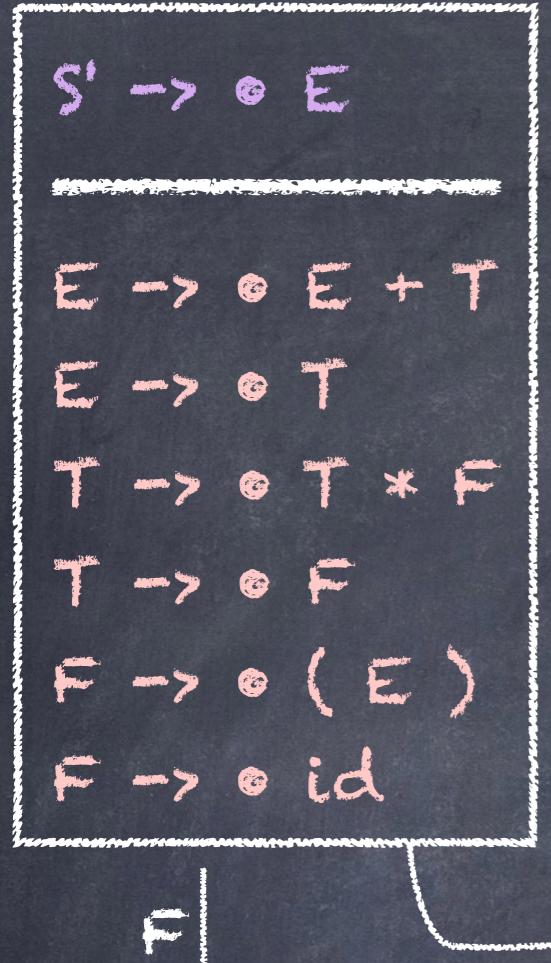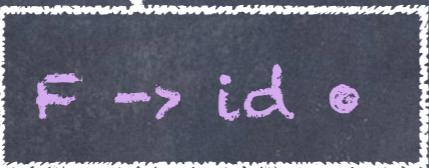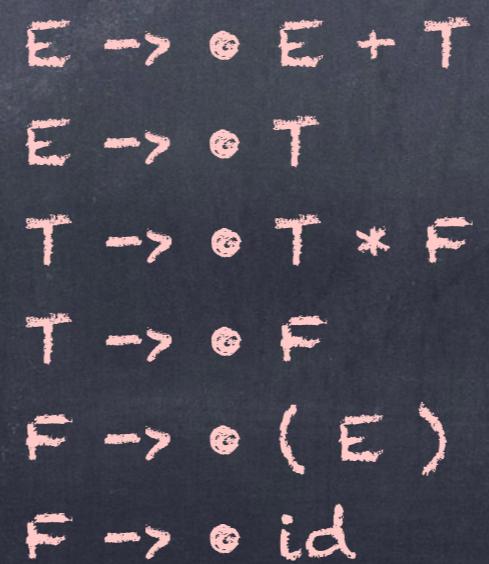
$\$$ → accept

I₆

E -> E + • T
───────────
T -> • T * F
T -> • F
F -> • (E)
F -> • id

I₂

E -> T •
T -> T • * F

I₇

T -> T * • F
───────────
F -> • (E)
F -> • id

I₃ T -> F •

I₅ F -> id •

I₄

F -> (• E)
───────────
E -> • E + T
E -> • T
T -> • T * F
T -> • F
F -> • (E)
F -> • id

E (arrow to I₁)
T (arrow to I₂)
F (arrow to I₃)
+ (arrow to I₆)
* (arrow to I₇)
id (arrow to I₅)
((arrow to I₄)

S' -> E
E -> E + T
E -> T
T -> T * F
T -> F
F -> (E)
F -> id

Compute GOTO(I_4,X) for each X
in { +, *, '(', ')', id, E, T, F, \$ }

$S' \rightarrow E$
$E \rightarrow E + T$
$E \rightarrow T$
$T \rightarrow T * F$
$T \rightarrow F$
$F \rightarrow (E)$
$F \rightarrow id$

I_4

$F \rightarrow (\bullet E)$

$E \rightarrow \bullet E + T$
$E \rightarrow \bullet T$
$T \rightarrow \bullet T * F$
$T \rightarrow \bullet F$
$F \rightarrow \bullet (E)$
$F \rightarrow \bullet id$

$GOTO(I4,E) =$
$CLOSURE(\{ F \rightarrow (E \bullet), E \rightarrow E \bullet + T \}) =$
$\{ F \rightarrow (E \bullet), E \rightarrow E \bullet + T \}$

I_8
$E \rightarrow E \bullet + T$
$F \rightarrow (E \bullet)$

$GOTO(I_4,T) = CLOSURE(\{ E \rightarrow T \bullet, T \rightarrow T \bullet * F \}) = \{ E \rightarrow T \bullet,$
$T \rightarrow T \bullet * F \} = I_2$

$GOTO(I_4,F) = CLOSURE(\{ T \rightarrow F \bullet \}) = I_3$

$GOTO(I_4,'(') = CLOSURE(\{ F \rightarrow (\bullet E) \}) = I_4$

$GOTO(I_4,id) = CLOSURE(\{ F \rightarrow id \bullet \}) = I_5$

$GOTO(I_4 , ')') = GOTO(I_4 , +) = GOTO(I_4 , *) = GOTO(I_4 , \$) = \varnothing$

I₀
$$S' \rightarrow \bullet E$$
$$E \rightarrow \bullet E + T$$
$$E \rightarrow \bullet T$$
$$T \rightarrow \bullet T * F$$
$$T \rightarrow \bullet F$$
$$F \rightarrow \bullet (E)$$
$$F \rightarrow \bullet \, id$$

I₁
$$S' \rightarrow E \bullet$$
$$E \rightarrow E \bullet + T$$

$$\$$$
accept

I₆
$$E \rightarrow E + \bullet T$$
$$T \rightarrow \bullet T * F$$
$$T \rightarrow \bullet F$$
$$F \rightarrow \bullet (E)$$
$$F \rightarrow \bullet \, id$$

I₂
$$E \rightarrow T \bullet$$
$$T \rightarrow T \bullet * F$$

I₇
$$T \rightarrow T * \bullet F$$
$$F \rightarrow \bullet (E)$$
$$F \rightarrow \bullet \, id$$

I₃
$$T \rightarrow F \bullet$$

I₅
$$F \rightarrow id \bullet$$

I₄
$$F \rightarrow (\bullet E)$$
$$E \rightarrow \bullet E + T$$
$$E \rightarrow \bullet T$$
$$T \rightarrow \bullet T * F$$
$$T \rightarrow \bullet F$$
$$F \rightarrow \bullet (E)$$
$$F \rightarrow \bullet \, id$$

I₈
$$E \rightarrow E \bullet + T$$
$$F \rightarrow (E \bullet)$$

Edges: E, +, T, *, F, id, (, $

$$S' \rightarrow E$$
$$E \rightarrow E + T$$
$$E \rightarrow T$$
$$T \rightarrow T * F$$
$$T \rightarrow F$$
$$F \rightarrow (E)$$
$$F \rightarrow id$$

$S' \rightarrow E$
$E \rightarrow E + T$
$E \rightarrow T$
$T \rightarrow T * F$
$T \rightarrow F$
$F \rightarrow (E)$
$F \rightarrow id$

I_5 | $F \rightarrow id \bullet$

Compute GOTO(I_5,X) for each X
in { +, *, '(', ')', id, E, T, F, $ }

$GOTO(I_5,E) = GOTO(I_5,T) = GOTO(I_5,F) = GOTO(I_5,+) =$
$GOTO(I_5,*) = GOTO(I_5,'(') = GOTO(I_5,')') = GOTO(I_5,id) =$
$GOTO(I_5,\$) = \varnothing$

I_0

$S' \rightarrow \bullet E$

$E \rightarrow \bullet E + T$
$E \rightarrow \bullet T$
$T \rightarrow \bullet T * F$
$T \rightarrow \bullet F$
$F \rightarrow \bullet (E)$
$F \rightarrow \bullet id$

I_1

$S' \rightarrow E \bullet$
$E \rightarrow E \bullet + T$

I_6

$E \rightarrow E + \bullet T$

$T \rightarrow \bullet T * F$
$T \rightarrow \bullet F$
$F \rightarrow \bullet (E)$
$F \rightarrow \bullet id$

$\$$ accept

I_2

$E \rightarrow T \bullet$
$T \rightarrow T \bullet * F$

I_7

$T \rightarrow T * \bullet F$

$F \rightarrow \bullet (E)$
$F \rightarrow \bullet id$

I_3 $T \rightarrow F \bullet$

I_5 $F \rightarrow id \bullet$

I_4

$F \rightarrow (\bullet E)$

$E \rightarrow \bullet E + T$
$E \rightarrow \bullet T$
$T \rightarrow \bullet T * F$
$T \rightarrow \bullet F$
$F \rightarrow \bullet (E)$
$F \rightarrow \bullet id$

I_8

$E \rightarrow E \bullet + T$
$F \rightarrow (E \bullet)$

E
T
F
$+$
$*$
id
$($

$S' \rightarrow E$
$E \rightarrow E + T$
$E \rightarrow T$
$T \rightarrow T * F$
$T \rightarrow F$
$F \rightarrow (E)$
$F \rightarrow id$
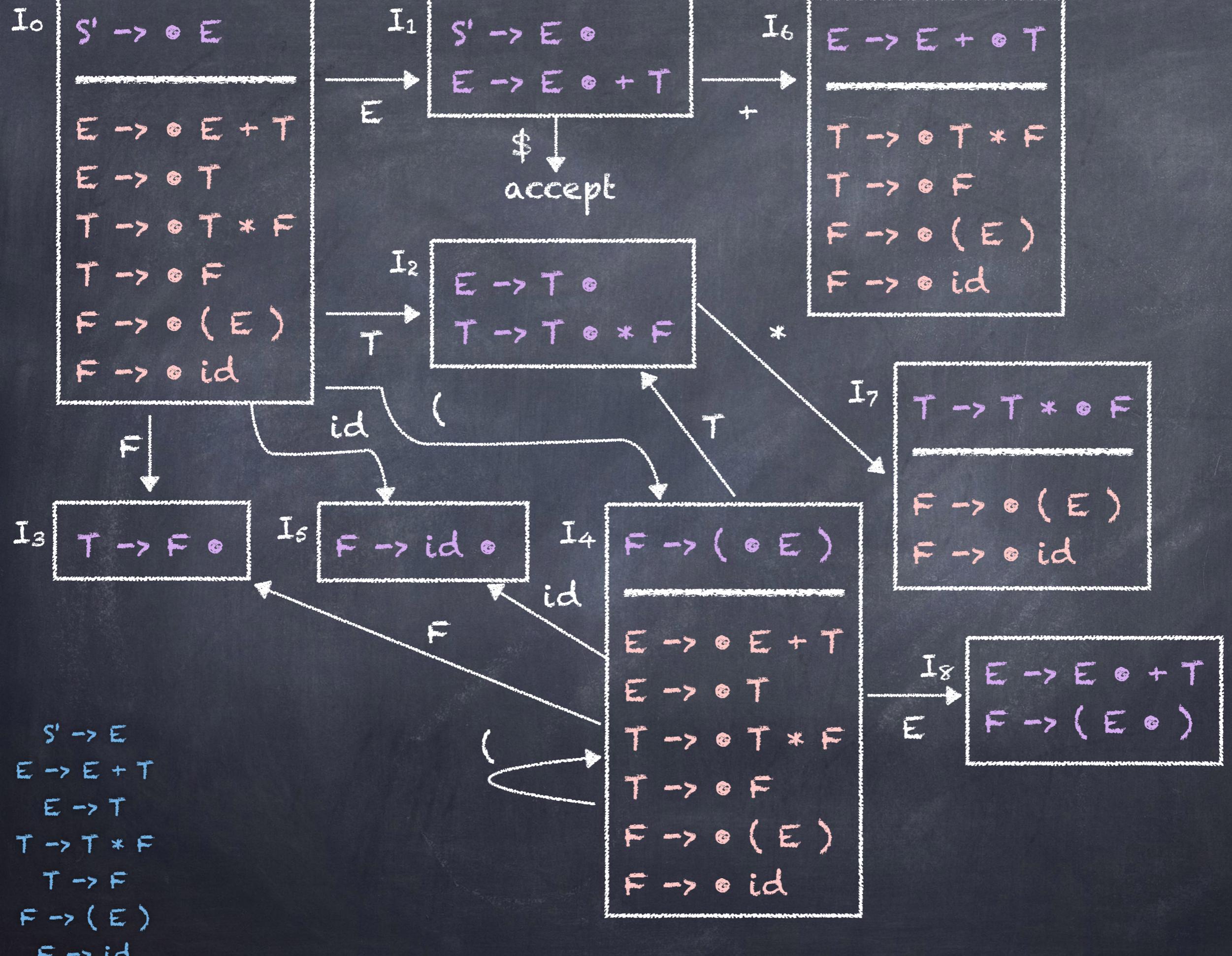
I_6

$$E \rightarrow E + \bullet T$$

$$T \rightarrow \bullet T * F$$
$$T \rightarrow \bullet F$$
$$F \rightarrow \bullet (E)$$
$$F \rightarrow \bullet id$$

$$S' \rightarrow E$$
$$E \rightarrow E + T$$
$$E \rightarrow T$$
$$T \rightarrow T * F$$
$$T \rightarrow F$$
$$F \rightarrow (E)$$
$$F \rightarrow id$$

Compute GOTO(I_6,X) for each X
in { +, *, '(', ')', id, E, T, F, $ }

$GOTO(I_6,T) = CLOSURE(\{ E \to E + T \bullet, T \to T \bullet * F \}) =$
$\{ E \to E + T \bullet, T \to T \bullet * F \}$

$GOTO(I_6,F) = CLOSURE(\{ T \to F \bullet \}) = I_3$

$GOTO(I_6,'(') = CLOSURE(\{ F \to (\bullet E) \}) = I_4$

$GOTO(I_6,id) = CLOSURE(\{ F \to id \bullet \}) = I_5$

$GOTO(I_6 , E) = GOTO(I_6 , ')') = GOTO(I_6 , +) = GOTO(I_6 , *) =$
$GOTO(I_6 , \$) = \varnothing$

I0
$S' \rightarrow \bullet E$

$E \rightarrow \bullet E + T$
$E \rightarrow \bullet T$
$T \rightarrow \bullet T * F$
$T \rightarrow \bullet F$
$F \rightarrow \bullet (E)$
$F \rightarrow \bullet id$

I1
$S' \rightarrow E \bullet$
$E \rightarrow E \bullet + T$
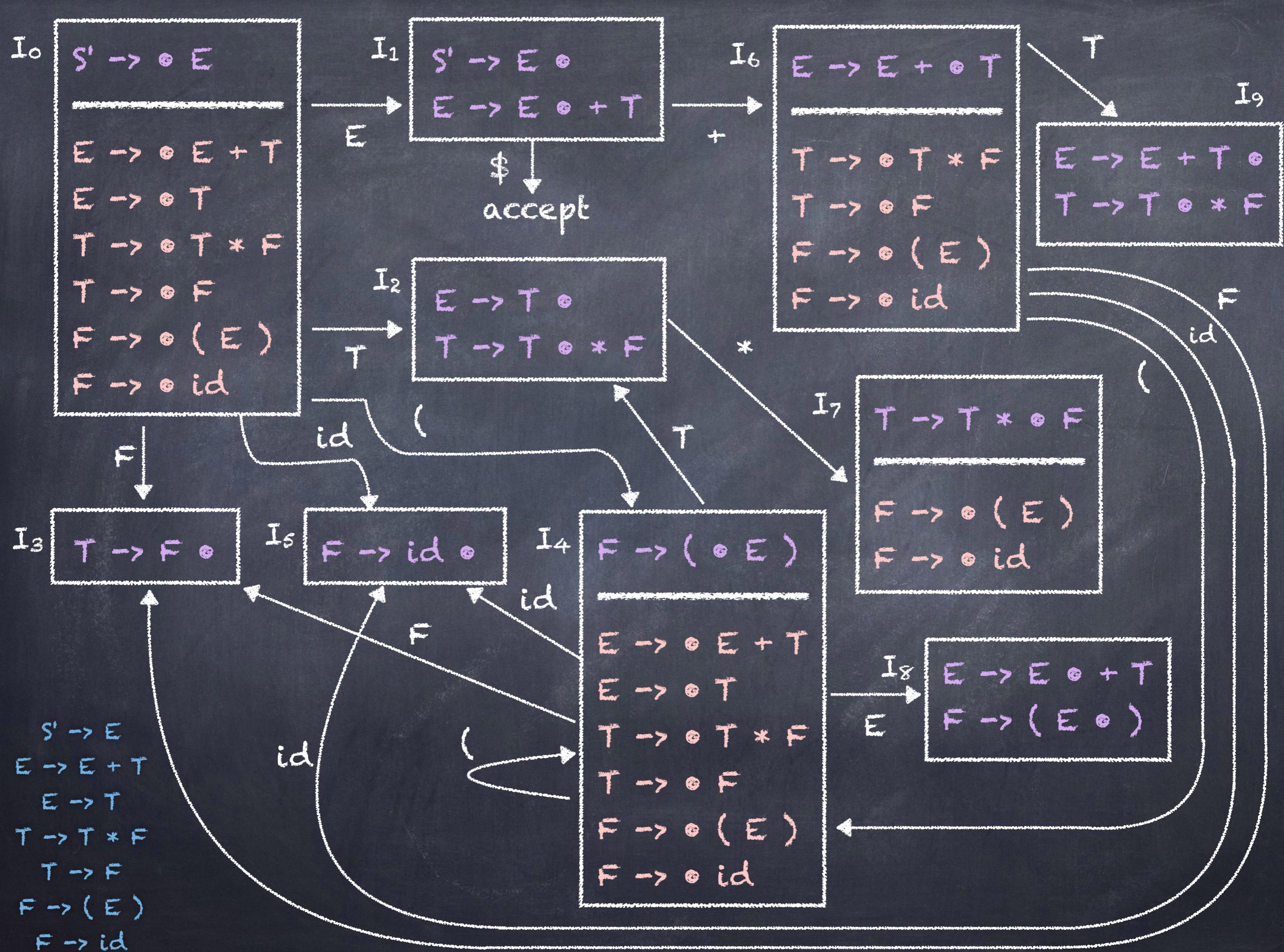
$\$$

accept

I6
$E \rightarrow E + \bullet T$

$T \rightarrow \bullet T * F$
$T \rightarrow \bullet F$
$F \rightarrow \bullet (E)$
$F \rightarrow \bullet id$

I9
$E \rightarrow E + T \bullet$
$T \rightarrow T \bullet * F$

I2
$E \rightarrow T \bullet$
$T \rightarrow T \bullet * F$

I7
$T \rightarrow T * \bullet F$

$F \rightarrow \bullet (E)$
$F \rightarrow \bullet id$

I3
$T \rightarrow F \bullet$

I5
$F \rightarrow id \bullet$

I4
$F \rightarrow (\bullet E)$

$E \rightarrow \bullet E + T$
$E \rightarrow \bullet T$
$T \rightarrow \bullet T * F$
$T \rightarrow \bullet F$
$F \rightarrow \bullet (E)$
$F \rightarrow \bullet id$

I8
$E \rightarrow E \bullet + T$
$F \rightarrow (E \bullet)$

$S' \rightarrow E$
$E \rightarrow E + T$
$E \rightarrow T$
$T \rightarrow T * F$
$T \rightarrow F$
$F \rightarrow (E)$
$F \rightarrow id$

E
T
F
id
(
+
*

$$S' \rightarrow E$$
$$E \rightarrow E + T$$
$$E \rightarrow T$$
$$T \rightarrow T * F$$
$$T \rightarrow F$$
$$F \rightarrow (E)$$
$$F \rightarrow id$$

I_7

$T \rightarrow T * \bullet F$
$F \rightarrow \bullet (E)$
$F \rightarrow \bullet id$

Compute GOTO(I_7,X) for each X
in { +, *, '(', ')', id, E, T, F, \$ }

$GOTO(I_7,F) = CLOSURE(\{T \rightarrow T * F \bullet \}) = \{T \rightarrow T * F \bullet \}$

$GOTO(I_7,'(') = CLOSURE(\{F \rightarrow (\bullet E)\}) = I_4$

$GOTO(I_7,id) = CLOSURE(\{F \rightarrow id \bullet \}) = I_5$

I_{10}

$$T \rightarrow T * F \bullet$$

$GOTO(I_7, E) = GOTO(I_7, T) = GOTO(I_7, ')') = GOTO(I_7, +) = GOTO(I_7, *) = GOTO(I_7, \$) = \emptyset$

I₀
$S' \rightarrow \bullet E$
———
$E \rightarrow \bullet E + T$
$E \rightarrow \bullet T$
$T \rightarrow \bullet T * F$
$T \rightarrow \bullet F$
$F \rightarrow \bullet (E)$
$F \rightarrow \bullet id$

I₁
$S' \rightarrow E \bullet$
$E \rightarrow E \bullet + T$

accept ($\$$)

I₆
$E \rightarrow E + \bullet T$
———
$T \rightarrow \bullet T * F$
$T \rightarrow \bullet F$
$F \rightarrow \bullet (E)$
$F \rightarrow \bullet id$

I₉
$E \rightarrow E + T \bullet$
$T \rightarrow T \bullet * F$

I₂
$E \rightarrow T \bullet$
$T \rightarrow T \bullet * F$

I₇
$T \rightarrow T * \bullet F$
———
$F \rightarrow \bullet (E)$
$F \rightarrow \bullet id$

I₃
$T \rightarrow F \bullet$

I₅
$F \rightarrow id \bullet$

I₄
$F \rightarrow (\bullet E)$
———
$E \rightarrow \bullet E + T$
$E \rightarrow \bullet T$
$T \rightarrow \bullet T * F$
$T \rightarrow \bullet F$
$F \rightarrow \bullet (E)$
$F \rightarrow \bullet id$

I₁₀
$T \rightarrow T * F \bullet$

I₈
$E \rightarrow E \bullet + T$
$F \rightarrow (E \bullet)$

$S' \rightarrow E$
$E \rightarrow E + T$
$E \rightarrow T$
$T \rightarrow T * F$
$T \rightarrow F$
$F \rightarrow (E)$
$F \rightarrow id$

$$S' \to E$$
$$E \to E + T$$
$$E \to T$$
$$T \to T * F$$
$$T \to F$$
$$F \to (E)$$
$$F \to id$$

I_8

$$E \to E \bullet + T$$
$$F \to (E \bullet)$$
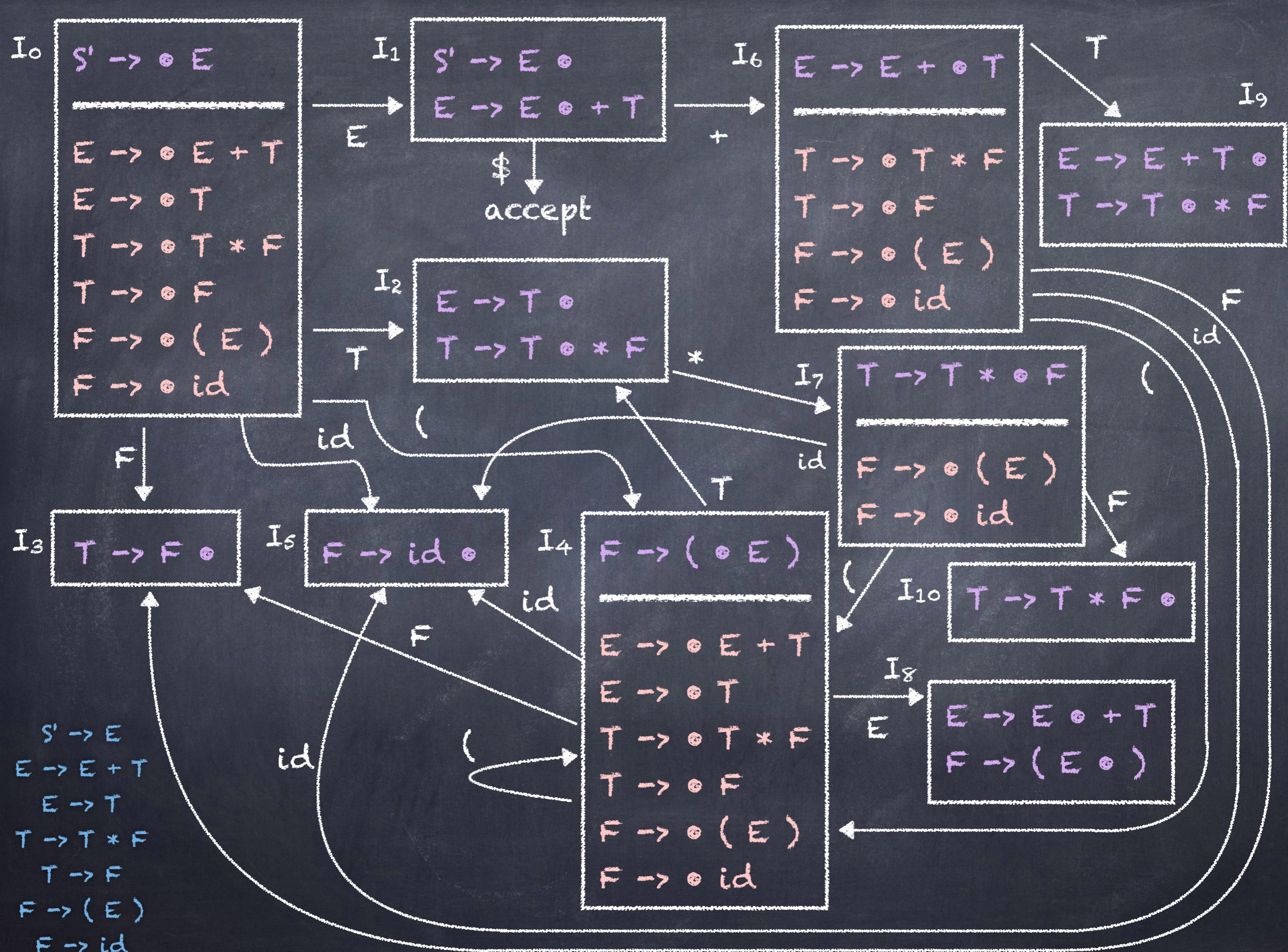
Compute GOTO(I_8,X) for each X
in { +, *, '(', ')', id, E, T, F, $ }

$S' \rightarrow E$
$E \rightarrow E + T$
$E \rightarrow T$
$T \rightarrow T * F$
$T \rightarrow F$
$F \rightarrow (E)$
$F \rightarrow id$

$GOTO(I_8, ')') = CLOSURE(\{ F \rightarrow (E) \bullet \}) = \{ F \rightarrow (E) \bullet \}$

$GOTO(I_8, +) = CLOSURE(\{ E \rightarrow E + \bullet T\}) = I_6$

I_{11}

$$F \rightarrow (E) \bullet$$

$GOTO(I_8, '(') = GOTO(I_8, *) = GOTO(I_8, E) = GOTO(I_8, T) = GOTO(I_8, F) = GOTO(I_8, \$) = GOTO(I_8, id) = \emptyset$

I₀
$S' \rightarrow \bullet E$
───────────
$E \rightarrow \bullet E + T$
$E \rightarrow \bullet T$
$T \rightarrow \bullet T * F$
$T \rightarrow \bullet F$
$F \rightarrow \bullet (E)$
$F \rightarrow \bullet id$

I₁
$S' \rightarrow E \bullet$
$E \rightarrow E \bullet + T$

$ \$ $
accept

I₆
$E \rightarrow E + \bullet T$
───────────
$T \rightarrow \bullet T * F$
$T \rightarrow \bullet F$
$F \rightarrow \bullet (E)$
$F \rightarrow \bullet id$

I₉
$E \rightarrow E + T \bullet$
$T \rightarrow T \bullet * F$

I₂
$E \rightarrow T \bullet$
$T \rightarrow T \bullet * F$

I₇
$T \rightarrow T * \bullet F$
───────────
$F \rightarrow \bullet (E)$
$F \rightarrow \bullet id$

I₃
$T \rightarrow F \bullet$

I₅
$F \rightarrow id \bullet$

I₄
$F \rightarrow (\bullet E)$
───────────
$E \rightarrow \bullet E + T$
$E \rightarrow \bullet T$
$T \rightarrow \bullet T * F$
$T \rightarrow \bullet F$
$F \rightarrow \bullet (E)$
$F \rightarrow \bullet id$

I₁₀
$T \rightarrow T * F \bullet$

I₁₁
$F \rightarrow (E) \bullet$

I₈
$E \rightarrow E \bullet + T$
$F \rightarrow (E \bullet)$

$S' \rightarrow E$
$E \rightarrow E + T$
$E \rightarrow T$
$T \rightarrow T * F$
$T \rightarrow F$
$F \rightarrow (E)$
$F \rightarrow id$
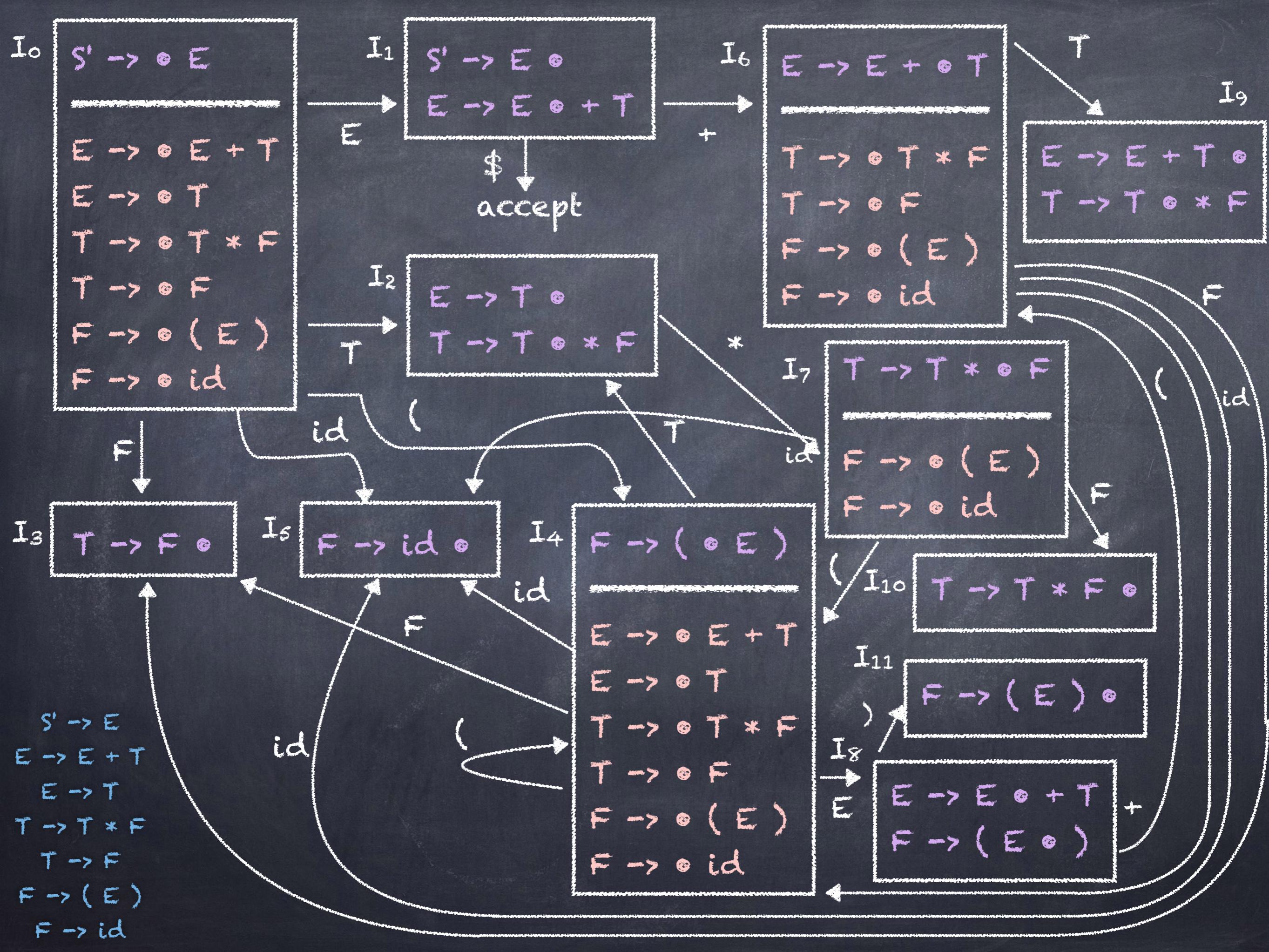
S' -> E
E -> E + T
E -> T
T -> T * F
T -> F
F -> (E)
F -> id

I_9

E -> E + T ⊙
T -> T ⊙ * F

Compute GOTO(I₉,X) for each X
in { +, *, '(', ')', id, E, T, F, $ }

$S' \rightarrow E$
$E \rightarrow E + T$
$E \rightarrow T$
$T \rightarrow T * F$
$T \rightarrow F$
$F \rightarrow (E)$
$F \rightarrow id$

$$GOTO(I_9, *) = CLOSURE(\{ T \rightarrow T * \bullet F \}) = I_7$$

$$GOTO(I_9, \text{'('}) = GOTO(I_9, \text{')'}) = GOTO(I_9, +) = GOTO(I_9, id) = GOTO(I_9, \$) =$$
$$GOTO(I_9, E) = GOTO(I_9, T) = GOTO(I_9, F) = \emptyset$$
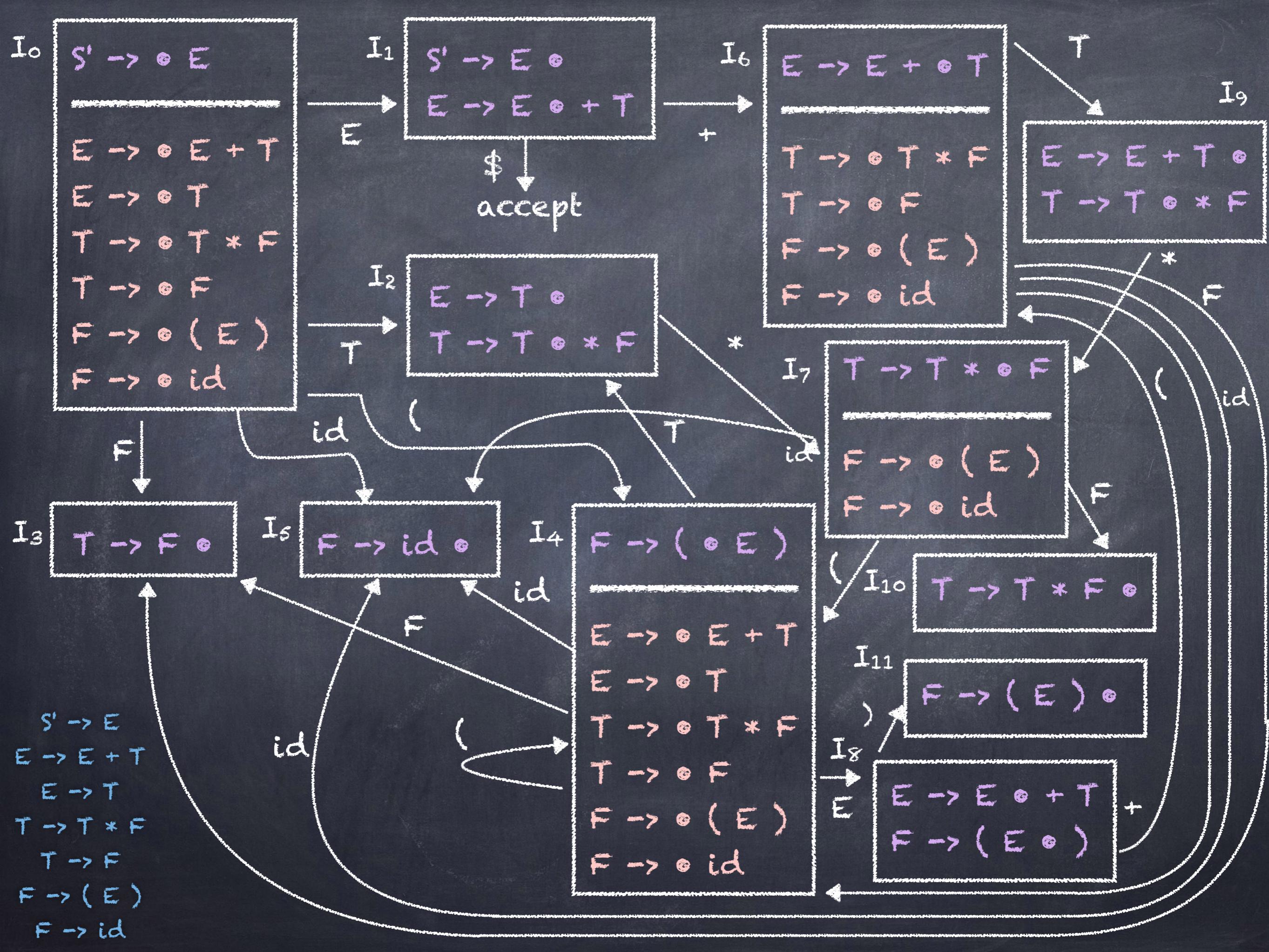
$$S' \rightarrow E$$
$$E \rightarrow E + T$$
$$E \rightarrow T$$
$$T \rightarrow T * F$$
$$T \rightarrow F$$
$$F \rightarrow (E)$$
$$F \rightarrow id$$

$$I_{10} \quad \boxed{T \rightarrow T * F \bullet}$$

Compute $GOTO(I_{10}, X)$ for each X
in $\{ +, *, '(', ')', id, E, T, F, \$ \}$

$$GOTO(I_{10},E) = GOTO(I_{10},T) = GOTO(I_{10},F) = GOTO(I_{10},+) =$$
$$GOTO(I_{10},*) = GOTO(I_{10},'(') = GOTO(I_{10},')') = GOTO(I_{10},id) =$$
$$GOTO(I_{10},\$) = \varnothing$$

I0
S' → • E

E → • E + T
E → • T
T → • T * F
T → • F
F → • (E)
F → • id

I1
S' → E •
E → E • + T

accept

$

I6
E → E + • T

T → • T * F
T → • F
F → • (E)
F → • id

I9
E → E + T •
T → T • * F

I2
E → T •
T → T • * F

I7
T → T * • F

F → • (E)
F → • id

I3
T → F •

I5
F → id •

I4
F → (• E)

E → • E + T
E → • T
T → • T * F
T → • F
F → • (E)
F → • id

I10
T → T * F •

I11
F → (E) •

I8
E → E • + T
F → (E •)

S' → E
E → E + T
E → T
T → T * F
T → F
F → (E)
F → id
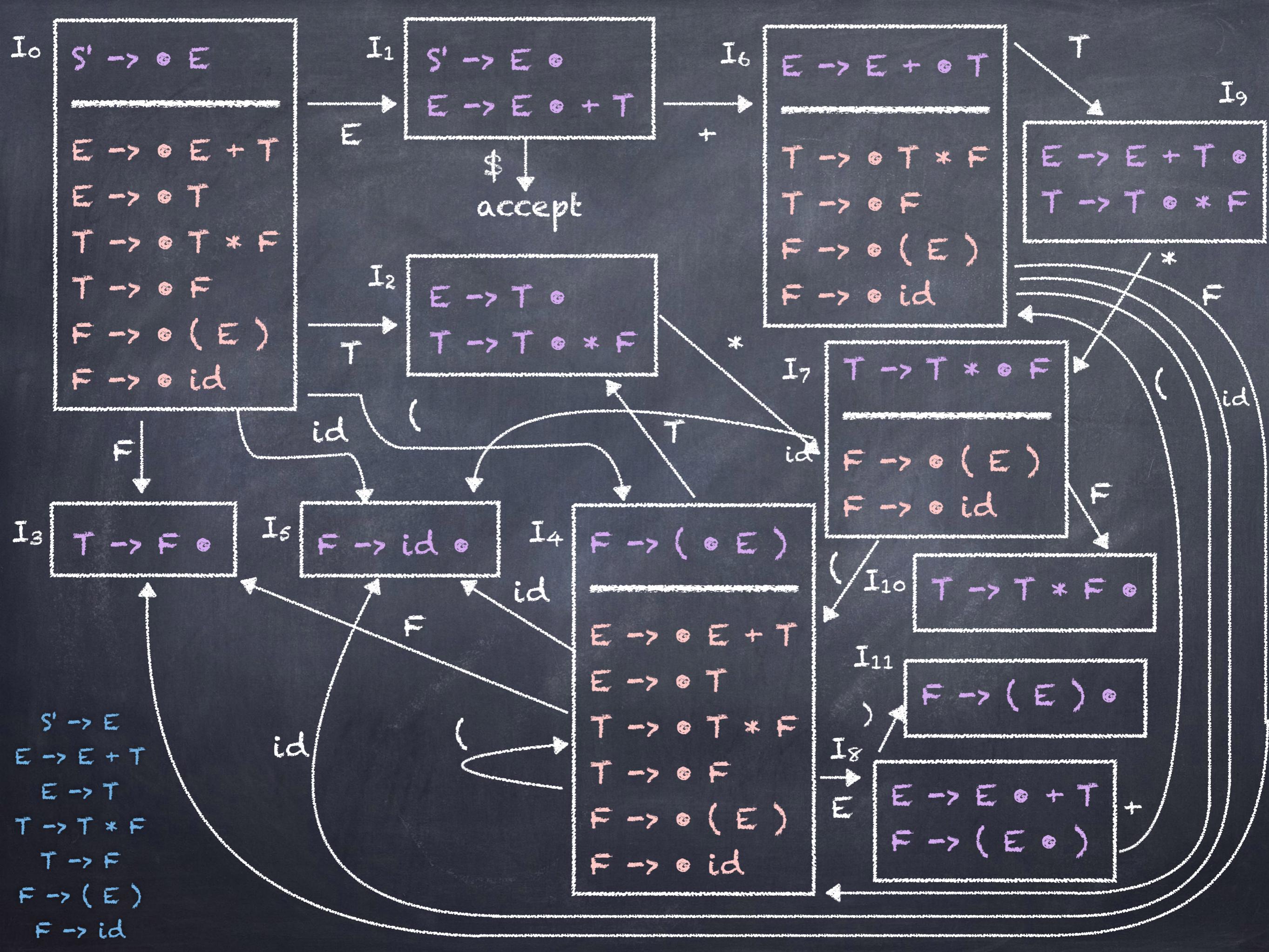
$S' \rightarrow E$
$E \rightarrow E + T$
$E \rightarrow T$
$T \rightarrow T * F$
$T \rightarrow F$
$F \rightarrow (E)$
$F \rightarrow id$

I_{11}

$F \rightarrow (E) \bullet$

Compute GOTO(I_{11},X) for each X
in { +, *, '(', ')', id, E, T, F, $ }

$GOTO(I_{11},E) = GOTO(I_{11},T) = GOTO(I_{11},F) = GOTO(I_{11},+) = GOTO(I_{11},*) = GOTO(I_{11},'(') = GOTO(I_{11},')') = GOTO(I_{11},id) = GOTO(I_{11},\$) = \varnothing$

I_0

$S' \rightarrow \bullet E$

$E \rightarrow \bullet E + T$
$E \rightarrow \bullet T$
$T \rightarrow \bullet T * F$
$T \rightarrow \bullet F$
$F \rightarrow \bullet (E)$
$F \rightarrow \bullet id$

I_1

$S' \rightarrow E \bullet$
$E \rightarrow E \bullet + T$

E

$\$$

accept

I_6

$E \rightarrow E + \bullet T$

$T \rightarrow \bullet T * F$
$T \rightarrow \bullet F$
$F \rightarrow \bullet (E)$
$F \rightarrow \bullet id$

T

I_9

$E \rightarrow E + T \bullet$
$T \rightarrow T \bullet * F$

$+$

I_2

$E \rightarrow T \bullet$
$T \rightarrow T \bullet * F$

T

$*$

I_7

$T \rightarrow T * \bullet F$

$F \rightarrow \bullet (E)$
$F \rightarrow \bullet id$

F

I_3

$T \rightarrow F \bullet$

I_5

$F \rightarrow id \bullet$

I_4

$F \rightarrow (\bullet E)$

$E \rightarrow \bullet E + T$
$E \rightarrow \bullet T$
$T \rightarrow \bullet T * F$
$T \rightarrow \bullet F$
$F \rightarrow \bullet (E)$
$F \rightarrow \bullet id$

I_{10}

$T \rightarrow T * F \bullet$

I_{11}

$F \rightarrow (E) \bullet$

I_8

$E \rightarrow E \bullet + T$
$F \rightarrow (E \bullet)$

E

$+$

id

(

id

(

F

T

id

id

*

F

id

(

)

$S' \rightarrow E$
$E \rightarrow E + T$
$E \rightarrow T$
$T \rightarrow T * F$
$T \rightarrow F$
$F \rightarrow (E)$
$F \rightarrow id$