Homework 4 due November 17

1. This problem demonstrates that definitions of connectivity beyond the usual 4- and 8-connectivity can be applied when computing the Euler Number of a scene.

Submit an m-file EN12.m which determines the Euler number of a binary image using *12-connectivity*. Two pixels p1 and p2 are defined to be 12-adjacent if the city-block distance between p1 and p2 is less than or equal to two. The first line of EN12.m should be

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function en = EN12(InIm)
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where InIm is a uint8 binary image with foreground pixels = 0 and background pixels = 1, and en is the Euler number of the entire image using 12-connectivity. So for instance if there are two 12-connected blobs in InIm, and one of them has three 12-connected holes, then en = 2-3 = -1. Note that the blobs B1={(1,1),(1,2)} and B2={(3,2),(3,3)} are actually a single 12-connected blob with 4 pixels even though there is white space between them.

Problem 1 is to be submitted electronically, the remaining three problems are to be handed in as hardcopy at the beginning of lecture on Wednesday Nov 17.

2. Let all 3 classes in a 2-D recognition problem be Gaussian with mean vectors

class 1: $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$, class 2: $\begin{bmatrix} 4 \\ -4 \end{bmatrix}$, class 3: $\begin{bmatrix} -2 \\ -2 \end{bmatrix}$

and have equal prior probabilities.

(a) Assume the three classes have the same covariance matrix, which is the 2x2 identity matrix. Find the minimum error classifier. Sketch, and give formulas for each decision boundary.

(b) Repeat if the covariance matrix for class 3 is changed to

Class 3: $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

while the other two remain the same (identity matrices).

3. Problem 2 concerns minimum error classification. For this problem, find the minimum loss classifier for this same 3-class problem if there is no loss for correct classification, ie $\lambda(\omega r|\omega s)=0$ if $\omega r=\omega s$, $\lambda(\omega r|\omega s)=1$ for misclassifying data which is truly from class 1, ie for $\omega r=2$ or 3 when $\omega s=1$, and the loss is otherwise $\lambda(\omega r|\omega s)=2$. In other words, your loss if you misclassify class 1 data is only half as big as your loss with all other errors. For this problem assume all three covariance matrices are 2x2 identity matrices as in 2(a) above, and all three classes have equal prior probabilities. Sketch, and give formulas for each decision boundary.

4. Define a *nibble* as a binary string 4 bits long. Let L be the set of all binary strings 3 nibbles long that have the property that at least 2 of the 3 nibbles have even parity. So for instance if nibbles 1 has odd parity (an odd number of 1's), then nibbles 2 and 3 must have even parity for this string to be in L. Specific examples: 011000011111 is in L, since the first and third nibbles have even parity, while 010010111001 is not in L, since only one nibble has even parity.

(a) Sketch a finite state automaton which will produce L. Identify the start state and the terminal state, and show what symbol is written as each edge is traversed.

(b) Specify a grammar G which produces L. In other words, specify the 4-tuple (V_n,V_t,P,S).