

CSE 443  
Compilers

Dr. Carl Alphonse  
alphonse@buffalo.edu  
343 Davis Hall

Team  
status updates

# Items

- "How does a shift-reduce parser know when to shift and when to reduce?" [p 242]
- "...by maintaining states to keep track of where we are in a parse."
- Each state is a set of **items**.
- An **item** is a grammar rule annotated with a dot, •, somewhere on the RHS.

# Rules and items

$A \rightarrow XYZ$
$A \rightarrow \bullet XYZ$
$A \rightarrow X \bullet YZ$
$A \rightarrow XY \bullet Z$
$A \rightarrow XYZ \bullet$

$A \rightarrow \epsilon$
$A \rightarrow \bullet$

The  $\bullet$  shows where in a rule we might be during a parse.

# Building the finite control for a bottom-up parser

- Build a finite state machine, whose states are sets of items
- Build a table ( $M$ ) incorporating shift/reduce decisions

# Augment grammar

Given a grammar

$$G = (N, T, P, S)$$

we augment to a grammar

$$G' = (N \cup \{S'\}, T, P \cup \{S' \rightarrow S\}, S'), \text{ where } S' \notin N$$

$G'$  has exactly one rule with  $S'$  on left.

We need two operations to  
build our finite state machine

CLOSURE(I)

GOTO(I,X)

# CLOSURE(I)

• I is a set of items

• CLOSURE(I) fixed point construction

$$\text{CLOSURE}_0(I) = I$$

repeat {

$$\text{CLOSURE}_{i+1}(I) =$$

$$\text{CLOSURE}_i(I) \cup \{ B \rightarrow \bullet \gamma \mid A \rightarrow \alpha \bullet B \beta \in \text{CLOSURE}_i(I) \text{ and } B \rightarrow \gamma \in P \}$$

} until  $\text{CLOSURE}_{i+1}(I) = \text{CLOSURE}_i(I)$



# CLOSURE(I)

- I is a set of items
- CLOSURE(I) fixed point construction

Intuition: an item like  $A \rightarrow X \bullet Y Z$  conveys that we've already seen X, and we're expecting to see a Y followed by a Z.

The closure of this item is all the other items that are relevant at this point in the parse.

For example, if  $Y \rightarrow R S T$  is a production, then  $Y \rightarrow \bullet R S T$  is in the closure because if the next thing in the input can derive from Y, it can derive from R.

# GOTO(I, X)

• GOTO(I, X) is the closure of the set of items  $A \rightarrow \alpha X \beta$  s.t.  
 $A \rightarrow \alpha X \beta \in I$

• GOTO(I, X) construction for  $G'$  (figure 4.32):

set-of-items CLOSURE(I) {

$J = I$

repeat {

  for each item  $A \rightarrow \alpha B \beta \in J$

    for each production  $B \rightarrow \gamma \in P$

      if  $B \rightarrow \gamma$  not already in  $J$

        add  $B \rightarrow \gamma$  to  $J$

  } until no more items are added to  $J$

return  $J$

}

# Building the LR(0) automaton

```
void items(G') {  
  C = { CLOSURE( { S' → • S } ) }  
  repeat {  
    for each set of items  $I \in C$  and  
    for each grammar symbol  $X \in (NUT)$   
    if ( GOTO( $I, X$ ) is not empty and not already in C )  
      add GOTO( $I, X$ ) to C  
  } until no new sets of items are added to C  
}
```

C is a set of sets of items

# Example [p 245]

Grammar G	Augmented Grammar G'
	$S' \rightarrow E$
$E \rightarrow E + T$	$E \rightarrow E + T$
$E \rightarrow T$	$E \rightarrow T$
$T \rightarrow T * F$	$T \rightarrow T * F$
$T \rightarrow F$	$T \rightarrow F$
$F \rightarrow ( E )$	$F \rightarrow ( E )$
$F \rightarrow id$	$F \rightarrow id$

# Compute items( $G'$ )

$S' \rightarrow E$   
 $E \rightarrow E + T$   
 $E \rightarrow T$   
 $T \rightarrow T * F$   
 $T \rightarrow F$   
 $F \rightarrow ( E )$   
 $F \rightarrow id$

SET OF ITEMS (I)	i	CLOSURE <sub>i</sub> (I)
$\{ S' \rightarrow \bullet E \}$	$\circ$	$\{ S' \rightarrow \bullet E \}$

# Compute items( $G'$ )

$S' \rightarrow E$   
 $E \rightarrow E + T$   
 $E \rightarrow T$   
 $T \rightarrow T * F$   
 $T \rightarrow F$   
 $F \rightarrow (E)$   
 $F \rightarrow id$

SET OF ITEMS (I)	i	CLOSURE <sub>i</sub> (I)
$\{ S' \rightarrow \bullet E \}$	0	$\{ S' \rightarrow \bullet E \}$
	1	$\text{CLOSURE}_0(I) \cup \{ E \rightarrow \bullet E + T, E \rightarrow \bullet T \}$

# Compute items( $G'$ )

$S' \rightarrow E$   
 $E \rightarrow E + T$   
 $E \rightarrow T$   
 $T \rightarrow T * F$   
 $T \rightarrow F$   
 $F \rightarrow (E)$   
 $F \rightarrow id$

SET OF ITEMS (I)	i	CLOSURE <sub>i</sub> (I)
$\{ S' \rightarrow \bullet E \}$	0	$\{ S' \rightarrow \bullet E \}$
	1	$CLOSURE_0(I) \cup \{ E \rightarrow \bullet E + T, E \rightarrow \bullet T \}$
	2	$CLOSURE_1(I) \cup \{ T \rightarrow \bullet T * F, T \rightarrow \bullet F \}$

# Compute items( $G'$ )

$S' \rightarrow E$   
 $E \rightarrow E + T$   
 $E \rightarrow T$   
 $T \rightarrow T * F$   
 $T \rightarrow F$   
 $F \rightarrow (E)$   
 $F \rightarrow id$

SET OF ITEMS (I)	i	CLOSURE <sub>i</sub> (I)
$\{ S' \rightarrow \bullet E \}$	0	$\{ S' \rightarrow \bullet E \}$
	1	$CLOSURE_0(I) \cup \{ E \rightarrow \bullet E + T, E \rightarrow \bullet T \}$
	2	$CLOSURE_1(I) \cup \{ T \rightarrow \bullet T * F, T \rightarrow \bullet F \}$
	3	$CLOSURE_2(I) \cup \{ F \rightarrow \bullet (E), F \rightarrow \bullet id \}$



# Compute items( $G'$ )

$S' \rightarrow E$   
 $E \rightarrow E + T$   
 $E \rightarrow T$   
 $T \rightarrow T * F$   
 $T \rightarrow F$   
 $F \rightarrow (E)$   
 $F \rightarrow id$

SET OF ITEMS (I)	i	CLOSURE <sub>i</sub> (I)
$\{ S' \rightarrow \bullet E \}$	0	$\{ S' \rightarrow \bullet E \}$
	1	$CLOSURE_0(I) \cup \{ E \rightarrow \bullet E + T, E \rightarrow \bullet T \}$
	2	$CLOSURE_1(I) \cup \{ T \rightarrow \bullet T * F, T \rightarrow \bullet F \}$
	3	$CLOSURE_2(I) \cup \{ F \rightarrow \bullet (E), F \rightarrow \bullet id \}$
	4	$CLOSURE_3(I) \cup \emptyset$

# Terminology

- Kernel items:  $S' \rightarrow \bullet S$  and all items with  $\bullet$  not at left edge
- Non-kernel items: all items with  $\bullet$  at left edge, except  $S' \rightarrow \bullet S$

This gives us the first state of the finite state machine,  $I_0$

$I_0$

$S' \rightarrow \bullet E$

---

$E \rightarrow \bullet E + T$

$E \rightarrow \bullet T$

$T \rightarrow \bullet T * F$

$T \rightarrow \bullet F$

$F \rightarrow \bullet ( E )$

$F \rightarrow \bullet id$

kernel item

non-kernel items are computed from  $CLOSURE(\text{kernel})$ , and therefore do not need to be explicitly stored

Next we compute  $GOTO(I_0, X) \forall X \in N \cup T$

$N \cup T = \{ E, T, F, +, *, (, ), id \}$

N.B. - augmented start symbol  $S'$  can be ignored

$GOTO(I_0, E) = CLOSURE( \{ S' \rightarrow E \odot, E \rightarrow E \odot + T \} )$

$= \{ S' \rightarrow E \odot, E \rightarrow E \odot + T \}$

N.B. there is no non-terminal after the  $\odot$ , so no new items are added by CLOSURE operation

$I_1$

$S' \rightarrow E \odot$   
 $E \rightarrow E \odot + T$

only kernel items

$$\text{GOTO}(I_0, T) = \text{CLOSURE}(\{ E \rightarrow T \odot, T \rightarrow T \odot * F \})$$

$$= \{ E \rightarrow T \odot, T \rightarrow T \odot * F \}$$

N.B. there is no non-terminal after the  $\odot$ , so no new items are added by CLOSURE operation

$I_2$

$E \rightarrow T \odot$
$T \rightarrow T \odot * F$

only kernel items

$$\text{GOTO}(I_0, F) = \text{CLOSURE}(\{ T \rightarrow F \odot \})$$

$$= \{ T \rightarrow F \odot \}$$

N.B. there is no non-terminal after the  $\odot$ , so no new items are added by CLOSURE operation

$I_3$

$T \rightarrow F \odot$

only kernel items

N.B. there is a non-terminal after the  $\odot$ , so new items are added by CLOSURE operation

$$\text{GOTO}(I_0, '(') = \text{CLOSURE}(\{F \rightarrow (\odot E)\})$$

$$= \{F \rightarrow (\odot E)\} \cup \{E \rightarrow \odot E + T, E \rightarrow \odot T\} \cup \{T \rightarrow \odot T * F, T \rightarrow \odot F\} \cup \{F \rightarrow \odot (E), F \rightarrow \odot \text{id}\}$$

$I_4$   $F \rightarrow (\odot E)$

kernel item

$E \rightarrow \odot E + T$

non-kernel items

$E \rightarrow \odot T$

$T \rightarrow \odot T * F$

$T \rightarrow \odot F$

$F \rightarrow \odot (E)$

$F \rightarrow \odot \text{id}$

$$\text{GOTO}(I_0, id) = \text{CLOSURE}(\{ F \rightarrow id \bullet \})$$

$$= \{ F \rightarrow id \bullet \}$$

N.B. there is no non-terminal after the  $\bullet$ , so no new items are added by CLOSURE operation

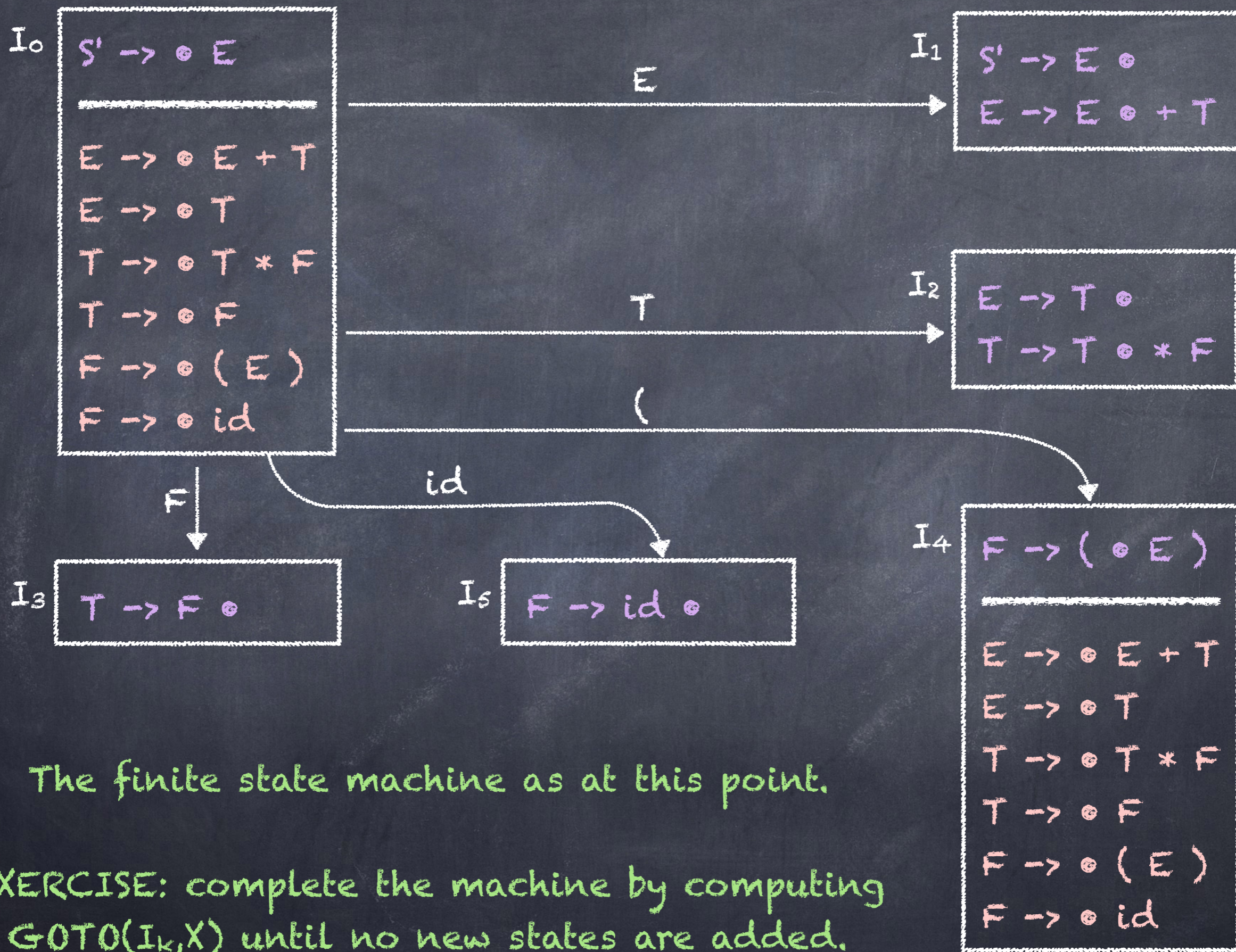
$I_0$

$F \rightarrow id \bullet$

only kernel items

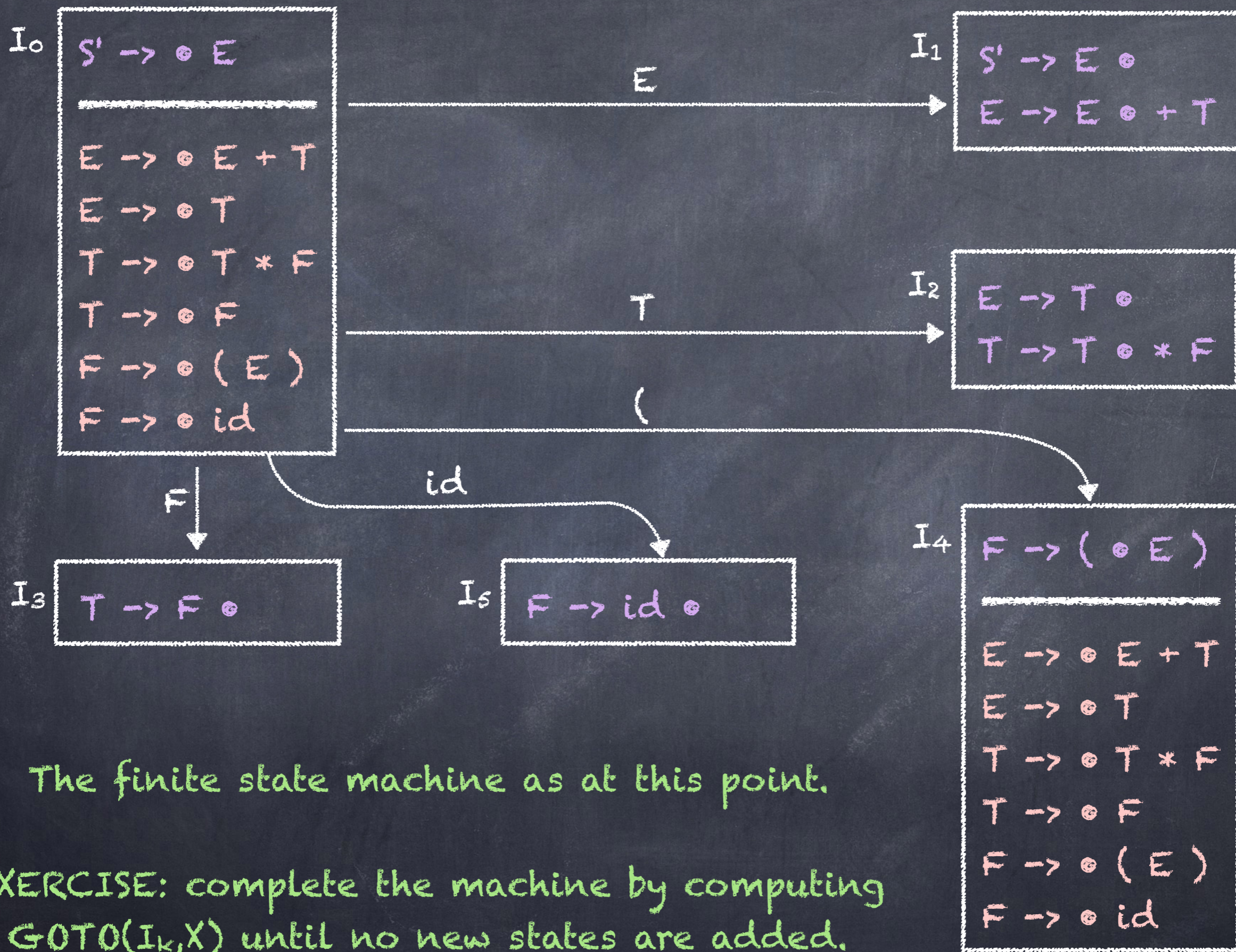
$$\text{GOTO}(I_0, ')') = \text{GOTO}(I_0, +) = \text{GOTO}(I_0, *) = \text{GOTO}(I_0, \$) = \emptyset$$





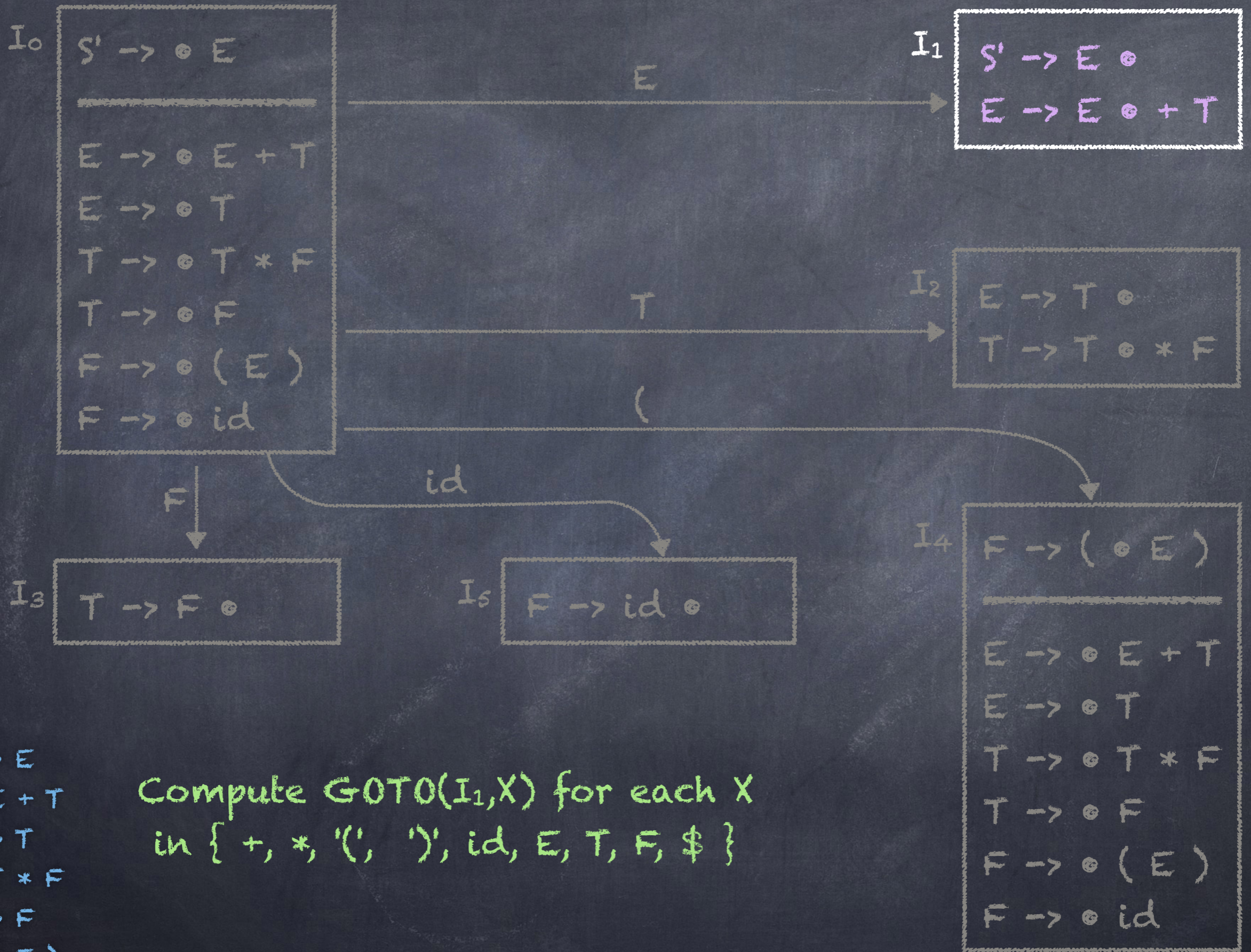
The finite state machine as at this point.

EXERCISE: complete the machine by computing  $GOTO(I_k, X)$  until no new states are added.



The finite state machine as at this point.

EXERCISE: complete the machine by computing  $GOTO(I_k, X)$  until no new states are added.



Compute  $GOTO(I_1, X)$  for each  $X$  in  $\{ +, *, '(', ')', id, E, T, F, \$ \}$

$S' \rightarrow E$   
 $E \rightarrow E + T$   
 $E \rightarrow T$   
 $T \rightarrow T * F$   
 $T \rightarrow F$   
 $F \rightarrow ( E )$   
 $F \rightarrow id$