Def (code): A code \( C \) of block length \( n \) over an alphabet \( \Sigma \) is a subset of \( \Sigma^n \).

Example: repetition/parity code \( \Sigma = \{0, 1\} \) alphabet size \( q = 1 \leq 1 \) alphabet size

Alternate view: \( C \subseteq \Sigma^n \), \( |C| = M \)

\( C : [M] \rightarrow \Sigma^n \)

Def: \( \dim(C) = \log_q |C| \) typically \( k = \dim(C) \) \( k \leq n \)

Ex: \( \Sigma = \{0, 1\} \) (binary code) \( q = 2 \)
Parity code: \( |C_0| = 2^4 \Rightarrow \dim(C_0) = 4 \), \( n = 5 \)

\( C_0 : \{0, 1\}^4 \rightarrow \{0, 1\}^5 \)

\( C_0 \oplus (x_1, x_2, x_3, x_4) = (x_1, x_2, x_3, x_4, x_1 \oplus x_2 \oplus x_3 \oplus x_4) \)

Repetition: \( C_{3, \text{rep}} \) each bit is repeated 3 times

\( k = 4 \), \( n = 3 \times 4 = 12 \)

Redundancy: \( \text{Def: } n - k \)

Def (rate): Rate of a code of block length \( n \) & \( \dim k \) is \( R(C) = \frac{k}{n} \)

Q: \( R(C_0) = \frac{4}{5} \)
\( R(C_{3, \text{rep}}) = \frac{1}{3} \)

\( h = 102, k = 100 \)
\( h = 4, k = 2 \)

\( h \leq R \rightarrow 0 \) less redundancy

1 \( \leq R \rightarrow 0 \) more redundancy