# Binary Arithmetic

## ***Decimal Numbers -- Base 10***

$2{,}983_{10} = 2x1000 + 9x100 + 8x10 + 3x1$ or

$= 2x10^3 + 9x10^2 + 8x10^1 + 3x10^0$

Remember that $10^3$ means 10 x 10 x 10 or 10 multiplied by itself 3 times (Effectively 1 followed by 3 zeros because multiplying anything by 10 is the same as adding a 0 to the end.)

$58{,}752_{10} = 5x10{,}000 + 8x1000 + 7x100 + 5x10 + 2x1$ or

$= 5x10^4 + 8x10^3 + 7x10^2 + 5x10^1 + 2x10^0$

So, $10^4$ means 10 x 10 x 10 x 10 -- or effectively 10 times itself 4 times

1 followed by 4 zeros = 10,000

And, $10^2$ means 10 x 10 -- or effectively 10 times itself 2 times

1 followed by 2 zeros = 100

To determine the correct power, count the number of digits to the right of that number.

**REMEMBER!**

***$10^0$ = 1***, the mathematical rule states that **any number** raised to the $zero^0$ power is one

*Hence, $21^0 = 1$, $16^0 = 1$ AND* ***$2^0 = 1$***

Let's expand these:

1) $123{,}456_{10} = 1x10^5 + 2x10^4 + 3x10^3 + 4x10^2 + 5x10^1 + 6x10^0$

2) $7{,}269_{10} = 7x10^3 + 2x10^2 + 6x10^1 + 9x10^0$

3) $3{,}720{,}452 = 3x10^6 + 7x10^5 + 2x10^4 + 0x10^3 + 4x10^2 + 5x10^1 + 2x10^0$

## ***Binary Numbers—Base 2***

1) $1101_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

2) $11011_2 = 1x2^4 + 1x2^3 + 0x2^2 + 1x2^1 + 1x2^0$

3) $101011_2 = 1x2^5 + 0x2^4 + 1x2^3 + 0x2^2 + 1x2^1 + 1x2^0$

Expand these binary numbers: (for answers see end of document)

a) $11110_2 =$

b) $1010101_2 =$

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c) $1110011_2$ =

## ***Binary to Decimal Conversion***

To convert Binary numbers to their Decimal equivalent you need to be able to translate the powers of 2.

$2^0$ = 1

$2^1$ = 2

$2^2$ = 2 x 2 = 4

$2^3$ = 2 x 2 x 2 = 8

$2^4$ = 2 x 2 x 2 x 2 = 16

$2^5$ = 2 x 2 x 2 x 2 x 2 = 32

[- there are 5 twos -]

$2^6$ = 2 x 2 x 2 x 2 x 2 x 2 = 64

[- there are 6 twos -]

In general it is just easiest to remember at least the first five powers.

Working with our first expansion above:

**1)** $1101_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

(8) + (4) + (0) + (1) = $13_{10}$

So, $1101_2 = 13_{10}$

**2)** $11011_2 = 1x2^4 + 1x2^3 + 0x2^2 + 1x2^1 + 1x2^0$

(16) + (8) + (0) + (2) + (1) = $27_{10}$

So, $11011_2 = 27_{10}$

**3)** $101011_2 = 1x2^5 + 0x2^4 + 1x2^3 + 0x2^2 + 1x2^1 + 1x2^0$

(32) + (0) + (8) + (0) + (2) + (1)

So, $101011_2 = 43_{10}$

Now try these examples. Expand and convert these examples to decimal. (Answers found at end of document)

a) $1101101_2$ =

b) $11100_2$ =

c) $1010101_2$ =

d) $100001_2$ =

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## ***Decimal to Binary Conversion***

Decimal to Binary conversion is performed by a series of short division (short division is where you have a remainder)

Quotient Remainder

Divisor | Dividend

***Examples***

***1)*** ***$27_{10}$ → Binary***

2 | 27
13 R 1

Repeat until the quotient is zero (0)

2 | 27
2 | 13 R 1
2 | 6 R 1
2 | 3 R 0
2 | 1 R 1
0 R 1

Translated number is read bottom up

$27_{10}$ → Binary

$27_{10} = 1\,1\,0\,1\,1_2$

Let's check our results. Expand the binary number into powers of 2 and convert to decimal.

$1\,1\,0\,1\,1_2 = 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

$(16) + (8) + (0) + (2) + (1) = 27_{10}$

***2)*** ***$52_{10}$ → Binary***

2 | 52
2 | 26 R 0
2 | 13 R 0
2 | 6 R 1

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2 | 3 R 0

2 | 1 R 1

0 R 1

Recording from the bottom up

$52_{10} = 110100_2$

Let's check this example. Expand the binary number and convert.

$110100_2 = 1x2^5 + 1x2^4 + 0x2^3 + 1x2^2 + 0x2^1 + 0x2^0$

(32) + (16) + (0) + (4) + (0) + (0)

$110100_2 = 52_{10}$

***3) $44_{10}$ → Binary***

2 | 44

2 | 22 R 0

2 | 11 R 0

2 | 5 R 1

2 | 2 R 1

2 | 1 R 0

0 R 1

Recording from the bottom up

$44_{10} = 101100_2$

Check this example by expanding the binary and converting.

$101100_2 = 1x2^5 + 0x2^4 + 1x2^3 + 1x2^2 + 0x2^1 + 0x2^0$

(32) + (0) + (8) + (4) + (0) + (0) = $44_{10}$

***4) $103_{10}$ → Binary***

2 | 103

2 | 51 R 1

2 | 25 R 1

2 | 12 R 1

2 | 6 R 0

2 | 3 R 0

2 | 1 R 1

0 R 1

Recording from the bottom up

$103_{10} = 1100111_2$

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Check this example by expanding the binary and converting.

$1100111_2 = 1x2^6 + 1x2^5 + 0x2^4 + 0x2^3 + 1x2^2 + 1x2^1 + 1x2^0$

$(64) + (32) + (0) + (0) + (4) + (2) + (1) = 103_{10}$

Try these examples:

a) $33_{10}$ → binary

c) $94_{10}$ → binary

b) $76_{10}$ → binary

d) $67_{10}$ → binary

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Solutions :

Expanded binary numbers

a) $11110_2 = 1x2^4 + 1x2^3 + 1x2^2 + 1x2^1 + 0x2^0$

b) $1010101_2 = 1x2^6 + 0x2^5 + 1x2^4 + 0x2^3 + 1x2^2 + 0x2^1 + 1x2^0$

c) $1110011_2 = 1x2^6 + 1x2^5 + 1x2^4 + 0x2^3 + 0x2^2 + 1x2^1 + 1x2^0$

Binary to Decimal Conversion

a) $1101101_2 = 109_{10}$

Work:

$1101101_2 = 1x2^6 + 1x2^5 + 0x2^4 + 1x2^3 + 1x2^2 + 0x2^1 + 1x2^0$

$64 + 32 + 0 + 8 + 4 + 0 + 1 = 109_{10}$

b) $11100_2 = 28_{10}$

Work:

$11100_2 = 1x2^4 + 1x2^3 + 1x2^2 + 0x2^1 + 0x2^0$

$16 + 8 + 4 + 0 + 0 = 28_{10}$

c) $1010101_2 = 85_{10}$

Work:

$1010101_2 = 1x2^6 + 0x2^5 + 1x2^4 + 0x2^3 + 1x2^2 + 0x2^1 + 1x2^0$

$64 + 0 + 16 + 0 + 4 + 0 + 1 = 85_{10}$

d) $100001_2 = 33_{10}$

Work:

$100001_2 = 1x2^5 + 0x2^4 + 0x2^3 + 0x2^2 + 0x2^1 + 1x2^0$

$32 + 0 + 0 + 0 + 0 + 1 = 33_{10}$

Decimal to Binary Conversion

a) $33_{10} \rightarrow$ binary $= 100001_2$

| | | | |
|---|---|---|---|
| 2 | 33 | | |
| 2 | 16 | R | 1 |
| 2 | 8 | R | 0 |
| 2 | 4 | R | 0 |
| 2 | 2 | R | 0 |
| 2 | 1 | R | 0 |
| | 0 | R | 1 |

↑ Record bottom up

$100001_2$

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b) $76_{10}$ → binary = $1001100_2$

2 | 76
2 | 38 R 0
2 | 19 R 0
2 | 9 R 1
2 | 4 R 1
2 | 2 R 0
2 | 1 R 0
0 R 1

Record bottom up
$1001100_2$

Let's check this answer

$1001100_2 = 1x2^6 + 0x2^5 + 0x2^4 + 1x2^3 + 1x2^2 + 0x2^1 + 0x2^0$

$64 + 0 + 0 + 8 + 4 + 0 + 0 = 76_{10}$

c) $94_{10}$ → binary = $1011110_2$

2 | 94
2 | 47 R 0
2 | 23 R 1
2 | 11 R 1
2 | 5 R 1
2 | 2 R 1
2 | 1 R 0
0 R 1

Record bottom up
$1011110_2$

Check:

$1011110_2 = + 1x2^6 + 0x2^5 + 1x2^4 + 1x2^3 + 1x2^2 + 1x2^1 + 0x2^0$

$= 64 + 0 + 16 + 8 + 4 + 2 + 0 = 94_{10}$

d) $67_{10}$ → binary = $1000011_2$

2 | 67
2 | 33 R 1
2 | 16 R 1
2 | 8 R 0
2 | 4 R 0
2 | 2 R 0
2 | 1 R 0
0 R 1

Record bottom up
$1000011_2$

Check:

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$$1000011_2 = + 1x2^6 + 0x2^5 + 0x2^4 + 0x2^3 + 0x2^2 + 1x2^1 + 1x2^0$$

$$= 64 + 0 + 0 + 0 + 0 + 2 + 1 = 67_{10}$$