

## Binary Arithmetic

### Binary Addition

$$\begin{array}{r}
 & 1 \\
 0 & 1 \\
 + & 0 & 1 \\
 \hline
 1 & 0
 \end{array}$$

(1+1 = 0 carry 1 to the next column, so 1+1=10)

$$\begin{array}{r}
 1 & 0 \\
 + & 0 & 1 \\
 \hline
 1 & 1
 \end{array}$$

(no carry)

But,

$$\begin{array}{r}
 & 1 \\
 1 & 1 \\
 + & 0 & 1 \\
 \hline
 1 & 0 & 0
 \end{array}$$

(1 + 1 = 0 carry 1 to next column, then again 1 + 1 is 0  
carry 1 to next column)

There are only two **carry** combinations

$$\begin{array}{l}
 1 + 1 = 10 \\
 1 + 1 + 1 = 11
 \end{array}$$

$$\begin{array}{r}
 & 1 \\
 1 & 0 & 1 \\
 + & 0 & 0 & 1 \\
 \hline
 1 & 1 & 0
 \end{array}$$

(1 + 1 = 0 carry 1 to next column)

Now try this:

$$\begin{array}{r}
 & 1 & 1 \\
 1 & 0 & 1 \\
 + & 0 & 1 & 1 \\
 \hline
 1 & 0 & 0 & 0
 \end{array}$$

(all columns involve carry)

And this,

$$\begin{array}{r}
 & 1 & 1 \\
 1 & 1 & 1 \\
 + & 0 & 1 & 1 \\
 \hline
 1 & 0 & 1 & 0
 \end{array}$$

(all columns involve carry, remember 1+1+1 = 1 carry 1 to next column)

Try these examples:

$$\begin{array}{r}
 1 & 0 & 1 & 1 & 0 & 1 & 0 \\
 + & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
 \hline$$

$$\begin{array}{r}
 1 & 1 & 0 & 0 & 0 & 1 & 1 \\
 + & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\
 \hline$$

$$\begin{array}{r}
 1 & 1 & 0 & 1 & 0 & 1 & 1 \\
 + & 1 & 1 & 1 & 0 & 1 & 1 \\
 \hline$$

## Binary Arithmetic

$$\begin{array}{r}
 \textcolor{red}{1} \textcolor{red}{1} \\
 1 0 1 1 0 1 0 \\
 + 0 1 1 0 0 0 0 \\
 \hline
 1 0 0 0 \textcolor{red}{1} 0 1 0
 \end{array}$$

$$\begin{array}{r}
 \textcolor{red}{1} \\
 1 1 0 0 0 1 1 \\
 + 0 1 1 1 1 0 0 \\
 \hline
 1 0 0 \textcolor{red}{1} 1 1 1 1
 \end{array}$$

$$\begin{array}{r}
 \textcolor{red}{1} \textcolor{red}{1} \textcolor{red}{1} \textcolor{red}{1} \\
 1 1 0 1 0 1 1 \\
 + 1 1 1 0 1 1 \\
 \hline
 1 0 1 0 0 \textcolor{red}{1} 1 0
 \end{array}$$

It doesn't matter how many digits (bits) there are in the binary number the rules are the same.  
Remember:

$$\begin{aligned}
 1 + 0 &= 1 \\
 1 + 1 &= 10 \\
 1 + 1 + 1 &= 11
 \end{aligned}$$

Another, longer example:

$$\begin{array}{r}
 \textcolor{red}{1} \textcolor{red}{1} \textcolor{red}{1} \textcolor{red}{1} \textcolor{red}{1} \textcolor{red}{1} \\
 \textcolor{blue}{1} \textcolor{blue}{1} \textcolor{blue}{1} \textcolor{blue}{0} 0 1 \textcolor{red}{1} \textcolor{red}{0} 1 0 \textcolor{red}{1} \textcolor{red}{1} \\
 + \textcolor{blue}{1} \textcolor{blue}{1} \textcolor{blue}{1} 0 \textcolor{blue}{0} 1 \textcolor{blue}{1} \textcolor{blue}{1} \textcolor{blue}{1} \textcolor{blue}{0} 0 0 0 \textcolor{red}{1} \\
 \hline
 \textcolor{blue}{1} \textcolor{blue}{0} 0 \textcolor{blue}{1} \textcolor{blue}{0} 0 0 \textcolor{blue}{0} \textcolor{blue}{0} 0 1 \textcolor{blue}{1} \textcolor{blue}{0} 1 1 \textcolor{blue}{0} 1 0 0
 \end{array}$$

Now try these examples: (results on the next page)

a)  $\begin{array}{r}
 1011 \textcolor{blue}{0001} \\
 + 111 \textcolor{blue}{1100} \\
 \hline
 \end{array}$

b)  $\begin{array}{r}
 1010 \textcolor{blue}{1100} \textcolor{blue}{1111} \\
 + 1 \textcolor{blue}{0000} 0101 \\
 \hline
 \end{array}$

c)  $\begin{array}{r}
 1100 \textcolor{blue}{1100} \textcolor{blue}{1100} \\
 + 11 \textcolor{blue}{1010} \textcolor{blue}{0100} \\
 \hline
 \end{array}$

d)  $\begin{array}{r}
 1111 \textcolor{blue}{0000} \textcolor{blue}{1111} \\
 + 10 \textcolor{blue}{1010} \textcolor{blue}{0101} \\
 \hline
 \end{array}$

**Binary Arithmetic**

- a)      1011 0001 + 111 1100 = 1 0010 1101
- b)      1010 1100 1111 + 1 0000 0101 = 1011 1101 0100
- c)      1100 1100 1100 + 11 1010 0100 = 1 0000 0111 0000
- d)      1111 0000 1111 + 10 1010 0101 = 1 0001 1011 0100