Logic Gates: AND, OR, NOT (Inverter)

Using AND

A: John Kerry ran for President in 2004.
B: George Bush ran for President in 2004.

The letters A along with the letter B are called **variables**. They represent or hold the place of other things.

John Kerry **AND** George Bush ran for President in 2004.

This sentence is True only if both parts of the sentence are True.

An AND gate can have more than two inputs

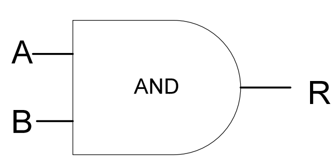
- If at least one input is a 0, then the output is a 0.
- When all inputs are 1, the output is a 1.

All of these symbols are equivalent:
A AND B ➔ A Λ B ➔ A⋅ B ➔ AB

Truth Table (AND)

A	B	A⋅B
0	0	0
0	1	0
1	0	0
1	1	1

A	B	R
0	0	0
0	1	0
1	0	0
1	1	1



Using OR

A: John Kerry
B: George Bush

John Kerry **OR** George Bush won the Presidency in 2004.

This sentence is True when either part of the sentence is True

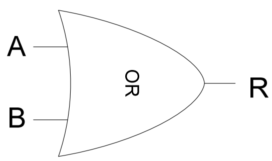
An OR gate can have more than two inputs

- If at least one input is a 1, then the output is a 1.
- If **all** the inputs are 0, the output is a 0.

All of these symbols are equivalent
A OR B ➔ A V B ➔ A+ B
Truth Table (OR)

A	B	A+B
0	0	0
0	1	1
1	0	1
1	1	1

A	B	R
0	0	0
0	1	1
1	0	1
1	1	1



Using NOT (Invert)

C: John Edwards is running for President.

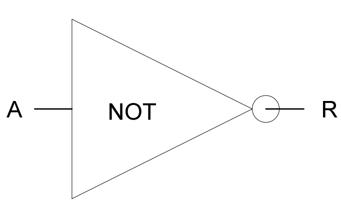
The action of NOT flips or inverts the statement between True and False

~C = NOT(John Edwards is running for President.)
In other words: John Edwards is NOT running for President

All of these symbols are equivalent

NOT ➔ ~ ➔ $\overline{}$
Truth Table (NOT)

C	~C = R
0	1
1	0



Let's look at the following information:

A new building is creating a security system with three alarm types. A flood alarm, a fire alarm and an intrusion alarm. If the intrusion or fire alarm goes off, the police department is called. If the fire or flood alarm goes off the fire department is called. If the flood alarm goes off don't call the police department. Similarly, if the intrusion alarm goes off don't call the fire department.

A = intrusion alarm
B = flood alarm
C = fire alarm

If intrusion OR fire alarm → call police
If fire OR flood alarm → call fire department
If flood → NOT police
If intrusion → NOT fire

S0:
A+C = Police
B+C = Fire Department

Could include
~A = Fire department
~B = Police department

Truth Tables are an "easy" way to determine the outcome of a logic equation.

A+C = P(olice)
B+C = F(ire)

Truth Table:
The statements above can be taken independently.

This is a two(2) variable truth table. It has four rows.

A	C	A+C=P
0	0	0
0	1	1
1	0	1
1	1	1

A similar Truth Table would be created for the other statement

B	C	B+C=F
0	0	0
0	1	1
1	0	1
1	1	1

These statements could also have been solved in a single Truth Table

This is a truth table that starts with 3 columns and 8 rows.

A = intrusion alarm
B = flood alarm
C = fire alarm

			or	or	(any row with a 1→ 1)
A	B	C	A+C	B+C	
A	B	C	P	F	
0	0	0			
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

A = intrusion alarm
B = flood alarm
C = fire alarm

A+C = P(olice)

			or	or	(any row with a 1→ 1)
A	B	C	A+C	B+C	
A	B	C	P	F	
0	0	**0**	0		
0	0	**1**	1		
0	1	**0**	0		
0	1	**1**	1		
1	0	**0**	1		
1	0	**1**	1		
1	1	**0**	1		
1	1	**1**	1		

B+C = F(ire)

			or	or	(any row with a 1→ 1)
A	B	C	A+C	B+C	
A	B	C	P	F	
0	**0**	**0**	0	0	
0	**0**	**1**	1	0	
0	**1**	**0**	0	1	
0	**1**	**1**	1	1	
1	**0**	**0**	1	0	
1	**0**	**1**	1	1	
1	**1**	**0**	1	1	
1	1	1	1	1	

Now we can ask some questions:

1) If a robber enters the building and sets a fire thinking to cover his tracks which alarm goes off? In other words A = 1, C = 1, which alarm goes off. (Ans. Both)

2) If a hurricane crashes into the building with water that breaks doors and windows, which alarm goes off? In this case A = 1, B = 1 (Ans. Both)

			or	or	(any row with a 1→ 1)
A	B	C	A+C	B+C	
A	B	C	P	F	
0	0	0	0	0	
0	0	1	1	0	
0	1	0	0	1	
0	1	1	1	1	
1	0	0	1	0	
1	0	1	1	1	Ans. 1
1	1	0	1	1	Ans. 2
1	1	1	1	1	

Logic problems can be solved even if we do not know the set of sentences being solved. Computers are designed to help solve problems without knowing in advance what problems they will be solving. That is why they are general purpose computers.

Let's examine some Logic problems.

The number of initial columns is the same as the number of variables.

1) R = AB + B
    2 Variables (A, B)
    2 initial columns
    4 rows to include all possibilities

		AND	OR
A	B	A·B	AB+B
0	0	0	0
0	1	0	1
1	0	0	0
1	1	1	1

Question: When A = 0, B = 1 what is R

R = 1

2) R = ~AC + B
3 Variables (A, B, C)
3 initial columns
8 rows to include all possibilities

Question:
If A= 0, B= 1, C=1 What is R?
R = 1

A	B	C	~A
0	0	1	1
0	0	0	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

A	B	C	~A	~A·C
0	0	0	1	0
0	0	1	1	1
0	1	0	1	0
0	1	1	1	1
1	0	0	0	0
1	0	1	0	0
1	1	0	0	0
1	1	1	0	0

A	B	C	~A	~A·C	~AC+B
0	0	1	0	0	0
0	0	1	1	1	1
0	1	0	1	0	1
0	1	1	1	1	1
1	0	0	0	0	0
1	0	1	0	0	0
1	1	0	0	0	1
1	1	1	0	0	1

Try these example: Note the dot (·) will only be included if critical
Two letters next to each other AB = A· B

Note: Order of operations matters
The order is: Parenthesis ()
NOT~
AND · ex: AB or A·B
OR + ex: A+B

1) R = ~A + ~BC If A = 0, B = 1, C = 1, then what is R?

2) R = ~A · ~B + A If A = 1, B = 0, then what is R?

3) R = ~A · (~B + A) If A = 1, B = 0, then what is R?
Be careful question 2 and 3 are not the same. Order of operations matters.

4) R= (~B + C)A If A = 0, B = 1, C = 1 then what is R?
Order of operations matters here.

5) R = ((A+~B)(B+C)) If A = 1, B = 1, C = 0 then what is R?
Order of operations matters here.

Solutions:

1) R = ~A + ~BC **If A = 0, B = 1, C = 1, then what is R?**
R = 1

A	B	C	~A	~B	~BC	~A + ~BC
0	0	0	1	1	0	1
0	0	1	1	1	1	1
0	1	0	1	0	0	1
0	**1**	**1**	**1**	**0**	**0**	**1**
1	0	0	0	1	0	0
1	0	1	0	1	1	1
1	1	0	0	0	0	0
1	1	1	0	0	0	0

2) R = ~A · ~B + A **If A = 1, B = 0, then what is R?**
R = 1

A	B	~A	~B	~A · ~B	~A · ~B + A
0	0	1	1	1	1
0	1	1	0	0	0
1	**0**	**0**	**1**	**0**	**1**
1	1	0	0	0	1

3) R = ~A · (~B + A) **If A = 1, B = 0, then what is R?**
R = 0

A	B	~A	~B	(~B + A)	~A · (~B + A)
0	0	1	1	1	1
0	1	1	0	0	0
1	**0**	**0**	**1**	**1**	**0**
1	1	0	0	1	0

4) R= (~B + C)A **If A = 0, B = 1, C = 1 then what is R?**
R = 0

A	B	C	~B	(~B+C)	(~B + C)A
0	0	0	1	1	0
0	0	1	1	1	0
0	1	0	0	0	0
0	**1**	**1**	**0**	**1**	**0**
1	0	0	1	1	1
1	0	1	1	1	1
1	1	0	0	0	0
1	1	1	0	1	1

5) R = ((A+~B)(B+C)) If A = 1, B = 1, C = 0 then what is R?
R = 1

A	B	C	~B	(A+ ~B)	(B+C)	((A+ ~B)(B+C))
0	0	0	1	1	0	0
0	0	1	1	1	1	1
0	1	0	0	0	1	0
0	1	1	0	0	1	0
1	0	0	1	1	0	0
1	0	1	1	1	1	1
1	**1**	**0**	**0**	**1**	**1**	**1**
1	1	1	0	1	1	1