

## Binary Arithmetic

### Decimal Numbers -- Base 10

$$2,983_{10} = 2 \times 1000 + 9 \times 100 + 8 \times 10 + 3 \times 1 \quad \text{or}$$

$$= 2 \times 10^3 + 9 \times 10^2 + 8 \times 10^1 + 3 \times 10^0$$

Remember that  $10^3$  means  $10 \times 10 \times 10$  or 10 multiplied by itself 3 times (Effectively 1 followed by 3 zeros because multiplying anything by 10 is the same as adding a 0 to the end.)

$$58,752_{10} = 5 \times 10,000 + 8 \times 1000 + 7 \times 100 + 5 \times 10 + 2 \times 1 \quad \text{or}$$

$$= 5 \times 10^4 + 8 \times 10^3 + 7 \times 10^2 + 5 \times 10^1 + 2 \times 10^0$$

So,  $10^4$  means  $10 \times 10 \times 10 \times 10$  -- or effectively 10 times itself 4 times

1 followed by 4 zeros = 10,000

And,  $10^2$  means  $10 \times 10$  -- or effectively 10 times itself 2 times

1 followed by 2 zeros = 100

To determine the correct power, count the number of digits to the right of that number.

### REMEMBER!

$10^0 = 1$ , the mathematical rule states that **any number** raised to the zero<sup>0</sup> power is one

Hence,  $21^0 = 1$ ,  $16^0 = 1$  AND  $2^0 = 1$

Let's expand these:

- 1)  $123,456_{10} = 1 \times 10^5 + 2 \times 10^4 + 3 \times 10^3 + 4 \times 10^2 + 5 \times 10^1 + 6 \times 10^0$
- 2)  $7,269_{10} = 7 \times 10^3 + 2 \times 10^2 + 6 \times 10^1 + 9 \times 10^0$
- 3)  $3,720,452 = 3 \times 10^6 + 7 \times 10^5 + 2 \times 10^4 + 0 \times 10^3 + 4 \times 10^2 + 5 \times 10^1 + 2 \times 10^0$

### Binary Numbers—Base 2

- 1)  $1101_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$
- 2)  $11011_2 = 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$
- 3)  $101011_2 = 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

Expand these binary numbers: (for answers see end of document)

- a)  $11110_2 =$
- b)  $1010101_2 =$
- c)  $1110011_2 =$

## Binary Arithmetic

### Binary to Decimal Conversion

To convert Binary numbers to their Decimal equivalent you need to be able to translate the powers of 2.

$$2^0 = 1$$

$$2^1 = 2$$

$$2^2 = 2 \times 2 = 4$$

$$2^3 = 2 \times 2 \times 2 = 8$$

$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$

$$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$$

[- there are 5 twos -]

$$2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$$

[- there are 6 twos -]

In general it is just easiest to remember at least the first five powers.

Working with our first expansion above:

$$1) \quad 1101_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\ (8) + (4) + (0) + (1) = 13_{10}$$

$$\text{So, } 1101_2 = 13_{10}$$

$$2) \quad 11011_2 = 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \\ (16) + (8) + (0) + (2) + (1) = 27_{10}$$

$$\text{So, } 11011_2 = 27_{10}$$

$$3) \quad 101011_2 = 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \\ (32) + (0) + (8) + (0) + (2) + (1)$$

$$\text{So, } 101011_2 = 43_{10}$$

Now try these examples. Expand and convert these examples to decimal. (Answers found at end of document)

a)  $1101101_2 =$

b)  $11100_2 =$

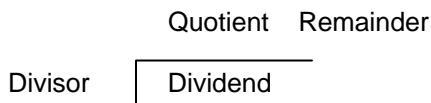
c)  $1010101_2 =$

d)  $100001_2 =$

## Binary Arithmetic

### Decimal to Binary Conversion

Decimal to Binary conversion is performed by a series of short division (short division is where you have a remainder)



#### **Examples**

1)  $27_{10} \rightarrow \text{Binary}$

$$\begin{array}{r} 2 \mid 27 \\ 13 \qquad R \ 1 \end{array}$$

Repeat until the quotient is zero (0)

$$\begin{array}{r} 2 \mid 27 \\ 2 \mid 13 \qquad R \ 1 \\ 2 \mid 6 \qquad R \ 1 \\ 2 \mid 3 \qquad R \ 0 \\ 1 \qquad R \ 1 \end{array}$$

Translated number is read bottom up  
Recording the bits from left to right.

$$\begin{array}{l} 27_{10} \rightarrow \text{Binary} \\ \qquad\qquad\qquad \longrightarrow \\ 27_{10} = 11011_2 \end{array}$$

Let's check our results. Expand the binary number into powers of 2 and convert to decimal.

$$\begin{aligned} 11011_2 &= 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \\ (16) &+ (8) + (0) + (2) + (1) = 27_{10} \end{aligned}$$

2)  $52_{10} \rightarrow \text{Binary}$

$$\begin{array}{r} 2 \mid 52 \\ 2 \mid 26 \qquad R \ 0 \\ 2 \mid 13 \qquad R \ 0 \\ 2 \mid 6 \qquad R \ 1 \\ 2 \mid 3 \qquad R \ 0 \\ 1 \qquad R \ 1 \end{array}$$

Recording from the bottom up  
Recording the bits from left to right.

$$\longrightarrow$$

$$52_{10} = 110100_2$$

## Binary Arithmetic

Let's check this example. Expand the binary number and convert.

$$\begin{aligned}110100_2 &= 1 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 0 \times 2^0 \\(32) &+ (16) + (0) + (4) + (0) + (0)\end{aligned}$$

$$110100_2 = 52_{10}$$

**3)  $44_{10} \rightarrow \text{Binary}$**

$$\begin{array}{r} 2 \mid 44 \\ 2 \mid 22 \quad R \ 0 \\ 2 \mid 11 \quad R \ 0 \\ 2 \mid 5 \quad R \ 1 \\ 2 \mid 2 \quad R \ 1 \\ 1 \quad R \ 0 \end{array}$$

Recording from the bottom up

Recording the bits from left to right.



$$44_{10} = 101100_2$$

Check this example by expanding the binary and converting.

$$\begin{aligned}101100_2 &= 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 0 \times 2^0 \\(32) &+ (0) + (8) + (4) + (0) + (0) = 44_{10}\end{aligned}$$

**4)  $103_{10} \rightarrow \text{Binary}$**

$$\begin{array}{r} 2 \mid 103 \\ 2 \mid 51 \quad R \ 1 \\ 2 \mid 25 \quad R \ 1 \\ 2 \mid 12 \quad R \ 1 \\ 2 \mid 6 \quad R \ 0 \\ 2 \mid 3 \quad R \ 0 \\ 1 \quad R \ 1 \end{array}$$

Recording from the bottom up

Recording the bits from left to right.



$$103_{10} = 1100111_2$$

Check this example by expanding the binary and converting.

$$\begin{aligned}1100111_2 &= 1 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \\(64) &+ (32) + (0) + (0) + (4) + (2) + (1) = 103_{10}\end{aligned}$$

## Binary Arithmetic

Try these examples:

a)  $33_{10} \rightarrow$  binary

c)  $94_{10} \rightarrow$  binary

b)  $76_{10} \rightarrow$  binary

d)  $67_{10} \rightarrow$  binary

## Binary Arithmetic

### Solutions :

Expanded binary numbers

- a)  $11110_2 = 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$
- b)  $1010101_2 = 1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$
- c)  $1110011_2 = 1 \times 2^6 + 1 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

Binary to Decimal Conversion

a)  $1101101_2 = 109_{10}$

Work:

$$1101101_2 = 1 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\ 64 + 32 + 0 + 8 + 4 + 0 + 1 = 109_{10}$$

b)  $11100_2 = 28_{10}$

Work:

$$11100_2 = 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 0 \times 2^0 \\ 16 + 8 + 4 + 0 + 0 = 28_{10}$$

c)  $1010101_2 = 85_{10}$

Work:

$$1010101_2 = 1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\ 64 + 0 + 16 + 0 + 4 + 0 + 1 = 85_{10}$$

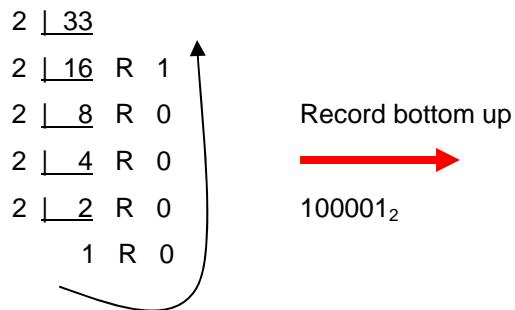
d)  $100001_2 = 33_{10}$

Work:

$$100001_2 = 1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\ 32 + 0 + 0 + 0 + 0 + 1 = 33_{10}$$

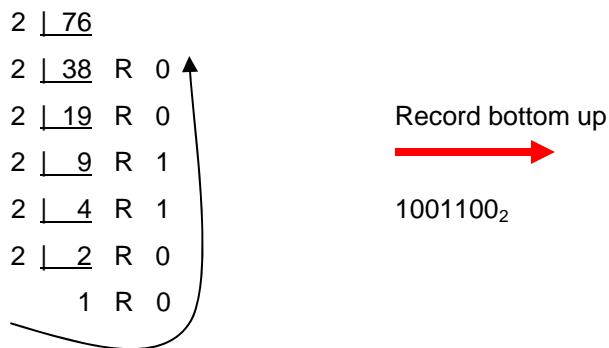
Decimal to Binary Conversion

a)  $33_{10} \rightarrow \text{binary} = 100001_2$



## Binary Arithmetic

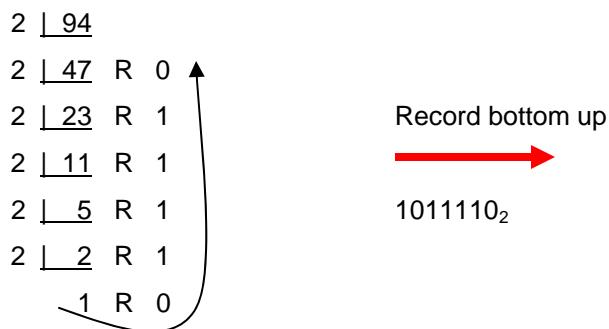
b)  $76_{10} \rightarrow \text{binary} = 1001100_2$



Let's check this answer

$$1001100_2 = 1x2^6 + 0x2^5 + 0x2^4 + 1x2^3 + 1x2^2 + 0x2^1 + 0x2^0 \\ 64 + 0 + 0 + 8 + 4 + 0 + 0 = 76_{10}$$

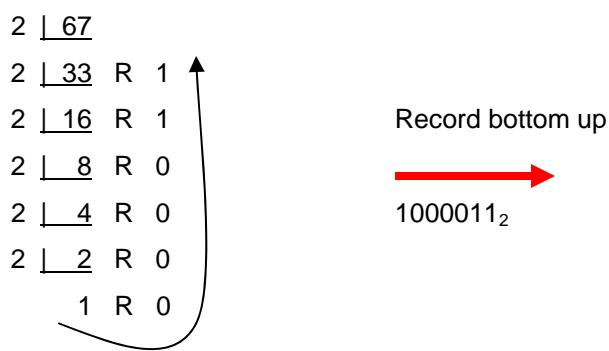
c)  $94_{10} \rightarrow \text{binary} = 1011110_2$



Check:

$$1011110_2 = + 1x2^6 + 0x2^5 + 1x2^4 + 1x2^3 + 1x2^2 + 1x2^1 + 0x2^0 \\ = 64 + 0 + 16 + 8 + 4 + 2 + 0 = 94_{10}$$

d)  $67_{10} \rightarrow \text{binary} = 1000011_2$



Check:

$$1000011_2 = + 1x2^6 + 0x2^5 + 0x2^4 + 0x2^3 + 0x2^2 + 1x2^1 + 1x2^0 \\ = 64 + 0 + 0 + 0 + 0 + 2 + 1 = 67_{10}$$