

## Binary Arithmetic

### Decimal Numbers -- Base 10

$$2,983_{10} = 2 \times 1000 + 9 \times 100 + 8 \times 10 + 3 \times 1 \quad \text{or}$$

$$= 2 \times 10^3 + 9 \times 10^2 + 8 \times 10^1 + 3 \times 10^0$$

Remember that  $10^3$  means  $10 \times 10 \times 10$  or 10 multiplied by itself 3 times (Effectively 1 followed by 3 zeros because multiplying anything by 10 is the same as adding a 0 to the end.)

$$58,752_{10} = 5 \times 10,000 + 8 \times 1000 + 7 \times 100 + 5 \times 10 + 2 \times 1 \quad \text{or}$$

$$= 5 \times 10^4 + 8 \times 10^3 + 7 \times 10^2 + 5 \times 10^1 + 2 \times 10^0$$

So,  $10^4$  means  $10 \times 10 \times 10 \times 10$  -- or effectively 10 times itself 4 times  
1 followed by 4 zeros = 10,000

And,  $10^2$  means  $10 \times 10$  -- or effectively 10 times itself 2 times  
1 followed by 2 zeros = 100

To determine the correct power, count the number of digits to the right of that number.

### **REMEMBER!**

$10^0 = 1$ , the mathematical rule states that **any number** raised to the zero<sup>0</sup> power is one

Hence,  $21^0 = 1$ ,  $16^0 = 1$  AND  $2^0 = 1$

Let's expand these:

$$1) \quad 123,456_{10} = 1 \times 10^5 + 2 \times 10^4 + 3 \times 10^3 + 4 \times 10^2 + 5 \times 10^1 + 6 \times 10^0$$

$$2) \quad 7,269_{10} = 7 \times 10^3 + 2 \times 10^2 + 6 \times 10^1 + 9 \times 10^0$$

$$3) \quad 3,720,452 = 3 \times 10^6 + 7 \times 10^5 + 2 \times 10^4 + 0 \times 10^3 + 4 \times 10^2 + 5 \times 10^1 + 2 \times 10^0$$

### Binary Numbers—Base 2

$$1) \quad 1101_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

$$2) \quad 11011_2 = 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$$

$$3) \quad 101011_2 = 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$$

Expand these binary numbers: (for answers see end of document)

$$a) \quad 11110_2 =$$

$$b) \quad 1010101_2 =$$

$$c) \quad 1110011_2 =$$

## Binary Arithmetic

### Binary to Decimal Conversion

To convert Binary numbers to their Decimal equivalent you need to be able to translate the powers of 2.

$$2^0 = 1$$

$$2^1 = 2$$

$$2^2 = 2 \times 2 = 4$$

$$2^3 = 2 \times 2 \times 2 = 8$$

$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$

$$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$$

[- there are 5 twos -]

$$2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$$

[- there are 6 twos -]

In general it is just easiest to remember at least the first five powers.

Working with our first expansion above:

$$1) \quad 1101_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

$$(8) + (4) + (0) + (1) = 13_{10}$$

$$\text{So, } 1101_2 = 13_{10}$$

$$2) \quad 11011_2 = 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$$

$$(16) + (8) + (0) + (2) + (1) = 27_{10}$$

$$\text{So, } 11011_2 = 27_{10}$$

$$3) \quad 101011_2 = 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$$

$$(32) + (0) + (8) + (0) + (2) + (1)$$

$$\text{So, } 101011_2 = 43_{10}$$

Now try these examples. Expand and convert these examples to decimal. (Answers found at end of document)

a)  $1101101_2 =$

b)  $11100_2 =$

c)  $1010101_2 =$

d)  $100001_2 =$



## Binary Arithmetic

Let's check this example. Expand the binary number and convert.

$$110100_2 = 1x2^5 + 1x2^4 + 0x2^3 + 1x2^2 + 0x2^1 + 0x2^0$$

$$(32) + (16) + (0) + (4) + (0) + (0)$$

$$110100_2 = 52_{10}$$

### 3) $44_{10} \rightarrow \text{Binary}$

$$\begin{array}{r} 2 \overline{) 44} \\ 2 \overline{) 22} \text{ R } 0 \\ 2 \overline{) 11} \text{ R } 0 \\ 2 \overline{) 5} \text{ R } 1 \\ 2 \overline{) 2} \text{ R } 1 \\ 1 \text{ R } 0 \end{array}$$

Recording from the bottom up

Recording the bits from left to right.



$$44_{10} = 101100_2$$

Check this example by expanding the binary and converting.

$$101100_2 = 1x2^5 + 0x2^4 + 1x2^3 + 1x2^2 + 0x2^1 + 0x2^0$$

$$(32) + (0) + (8) + (4) + (0) + (0) = 44_{10}$$

### 4) $103_{10} \rightarrow \text{Binary}$

$$\begin{array}{r} 2 \overline{) 103} \\ 2 \overline{) 51} \text{ R } 1 \\ 2 \overline{) 25} \text{ R } 1 \\ 2 \overline{) 12} \text{ R } 1 \\ 2 \overline{) 6} \text{ R } 0 \\ 2 \overline{) 3} \text{ R } 0 \\ 1 \text{ R } 1 \end{array}$$

Recording from the bottom up

Recording the bits from left to right.



$$103_{10} = 1100111_2$$

Check this example by expanding the binary and converting.

$$1100111_2 = 1x2^6 + 1x2^5 + 0x2^4 + 0x2^3 + 1x2^2 + 1x2^1 + 1x2^0$$

$$(64) + (32) + (0) + (0) + (4) + (2) + (1) = 103_{10}$$

## Binary Arithmetic

Try these examples:

a)  $33_{10} \rightarrow \text{binary}$

c)  $94_{10} \rightarrow \text{binary}$

b)  $76_{10} \rightarrow \text{binary}$

d)  $67_{10} \rightarrow \text{binary}$

## Binary Arithmetic

### Solutions :

Expanded binary numbers

$$a) 11110_2 = 1x2^4 + 1x2^3 + 1x2^2 + 1x2^1 + 0x2^0$$

$$b) 1010101_2 = 1x2^6 + 0x2^5 + 1x2^4 + 0x2^3 + 1x2^2 + 0x2^1 + 1x2^0$$

$$c) 1110011_2 = 1x2^6 + 1x2^5 + 1x2^4 + 0x2^3 + 0x2^2 + 1x2^1 + 1x2^0$$

Binary to Decimal Conversion

$$a) 1101101_2 = 109_{10}$$

Work:

$$1101101_2 = 1x2^6 + 1x2^5 + 0x2^4 + 1x2^3 + 1x2^2 + 0x2^1 + 1x2^0$$

$$64 + 32 + 0 + 8 + 4 + 0 + 1 = 109_{10}$$

$$b) 11100_2 = 28_{10}$$

Work:

$$11100_2 = 1x2^4 + 1x2^3 + 1x2^2 + 0x2^1 + 0x2^0$$

$$16 + 8 + 4 + 0 + 0 = 28_{10}$$

$$c) 1010101_2 = 85_{10}$$

Work:

$$1010101_2 = 1x2^6 + 0x2^5 + 1x2^4 + 0x2^3 + 1x2^2 + 0x2^1 + 1x2^0$$

$$64 + 0 + 16 + 0 + 4 + 0 + 1 = 85_{10}$$

$$d) 100001_2 = 33_{10}$$

Work:

$$100001_2 = 1x2^5 + 0x2^4 + 0x2^3 + 0x2^2 + 0x2^1 + 1x2^0$$

$$32 + 0 + 0 + 0 + 0 + 1 = 33_{10}$$

Decimal to Binary Conversion

$$a) 33_{10} \rightarrow \text{binary} = 100001_2$$

2	33	
2	16	R 1
2	8	R 0
2	4	R 0
2	2	R 0
1		R 0

Record bottom up

→

100001<sub>2</sub>

## Binary Arithmetic

b)  $76_{10} \rightarrow \text{binary} = 1001100_2$

$$\begin{array}{r}
 2 \overline{) 76} \\
 2 \overline{) 38} \text{ R } 0 \\
 2 \overline{) 19} \text{ R } 0 \\
 2 \overline{) 9} \text{ R } 1 \\
 2 \overline{) 4} \text{ R } 1 \\
 2 \overline{) 2} \text{ R } 0 \\
 1 \text{ R } 0
 \end{array}$$

Record bottom up



$1001100_2$

Let's check this answer

$$\begin{aligned}
 1001100_2 &= 1x2^6 + 0x2^5 + 0x2^4 + 1x2^3 + 1x2^2 + 0x2^1 + 0x2^0 \\
 &= 64 + 0 + 0 + 8 + 4 + 0 + 0 = 76_{10}
 \end{aligned}$$

c)  $94_{10} \rightarrow \text{binary} = 1011110_2$

$$\begin{array}{r}
 2 \overline{) 94} \\
 2 \overline{) 47} \text{ R } 0 \\
 2 \overline{) 23} \text{ R } 1 \\
 2 \overline{) 11} \text{ R } 1 \\
 2 \overline{) 5} \text{ R } 1 \\
 2 \overline{) 2} \text{ R } 1 \\
 1 \text{ R } 0
 \end{array}$$

Record bottom up



$1011110_2$

Check:

$$\begin{aligned}
 1011110_2 &= 1x2^6 + 0x2^5 + 1x2^4 + 1x2^3 + 1x2^2 + 1x2^1 + 0x2^0 \\
 &= 64 + 0 + 16 + 8 + 4 + 2 + 0 = 94_{10}
 \end{aligned}$$

d)  $67_{10} \rightarrow \text{binary} = 1000011_2$

$$\begin{array}{r}
 2 \overline{) 67} \\
 2 \overline{) 33} \text{ R } 1 \\
 2 \overline{) 16} \text{ R } 1 \\
 2 \overline{) 8} \text{ R } 0 \\
 2 \overline{) 4} \text{ R } 0 \\
 2 \overline{) 2} \text{ R } 0 \\
 1 \text{ R } 0
 \end{array}$$

Record bottom up



$1000011_2$

Check:

$$\begin{aligned}
 1000011_2 &= 1x2^6 + 0x2^5 + 0x2^4 + 0x2^3 + 0x2^2 + 1x2^1 + 1x2^0 \\
 &= 64 + 0 + 0 + 0 + 0 + 2 + 1 = 67_{10}
 \end{aligned}$$