

This exam is closed book. Answer all questions in the blue books. *Unsupported work will receive no credit.*

1. (10%) Prove that $\sum_{k=1}^n \frac{1}{k} = \Theta(\log n)$.
2. (10%) Draw an 8-element **bitonic sort** unit.
3. Given a set of n **integer** values in the range of $[1,100]$, for each of the following architectures, give an efficient algorithm to sort these data items. Discuss the quality of each of your solutions in terms of time/space/processors.
 - a) (10%) RAM
 - b) (10%) PRAM
4. Define a *Reconfigurable Array of size n* to be a linear array augmented by a *reconfigurable bus*. The reconfigurable bus has the characteristic that every processor has the ability to split or connect the bus between itself and either (or both or neither) of its neighbors during *every step of an algorithm*. That is, the bus can be dynamically reconfigured on the fly. Further, on each bus segment, only 1 processor can perform a write and all processors can read the value written (i.e., the bus is CREW within each independent segment).
 - a) (10%) Suppose every processor contains either a 1 or a 0. Give an efficient algorithm to determine the LOGICAL-OR of the set of data. That is, determine whether or not there exists at least one "1".
 - b) (10%) Suppose every processor contains either a 1 or a 0. Give an efficient algorithm to determine the sum of these values.
 - c) (10%) Suppose every processor contains either a 1 or a 0. Give an efficient algorithm to label the connected components.
5. Given a set X of n points in the plane (i.e., 2-dimensional points), determine a subset of the points that are not *dominated* by any other point in X . A point $p = (p_x, p_y)$ is *dominated* by a point $q = (q_x, q_y)$ if and only if $p_x < q_x$ and $p_y < q_y$. That is, determine the subset of X for which no other point in X has both a larger x -value and a larger y -value. For each of the following architectures, give an efficient algorithm to solve this problem. Discuss the efficiency of each of your algorithms.
 - a) (15%) RAM
 - b) (15%) CREW PRAM