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Plagiarism of any sort on this exam will result in you being given an F in the course. In addition, a request will be made to have you expelled from the university.

For each question, circle the correct answer.

1) Select the statement that is not a proposition.
   a) 5 + 4 = 8
   b) It will be sunny tomorrow.
   c) Take out the trash.
   d) Chocolate is the best flavor.

2) p = T, q = F, and r = F. Select the expression that evaluates to true.
   a) $p \wedge q$
   b) $\neg p$
   c) $q \vee r$
   d) $p \vee r$

3) p = F, q = T, and r = T. Select the expression that evaluates to false.
   a) $\neg q$
   b) $q \vee r$
   c) $q \wedge r$
   d) $p \vee r$

4) p = T, q = F, and r = T. Select the expression that evaluates to false.
   a) $p \vee \neg q$
   b) $p \vee q \vee r$
   c) $\neg(p \wedge \neg q)$
   d) $\neg(p \wedge q \wedge r)$

5) p = F, q = T, and r = T. Select the expression that evaluates to true.
   a) $\neg(q \vee r)$
   b) $(\neg p \wedge r) \vee q$
   c) $(\neg q \vee r) \wedge p$
   d) $p \vee \neg q \vee \neg r$

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6) Select the statement that is false.
   a) If 3 is a prime number, then 5 is a prime number.
   b) If 4 is a prime number, then 6 is a prime number.
   c) If 4 is a prime number, then 5 is a prime number.
   d) If 3 is a prime number, then 6 is a prime number.

7) p = T, q = F, and r = T. Select the expression that evaluates to false.
   a) $\neg(q \wedge r) \rightarrow p$
   b) $(p \wedge r) \rightarrow q$
   c) $(q \wedge r) \rightarrow p$
   d) $(q \wedge r) \rightarrow \neg p$

8) p = F, q = T, and r = T. Select the expression that evaluates to true.
   a) $\neg(q \vee p) \leftrightarrow r$
   b) $(\neg p \wedge r) \rightarrow q$
   c) $(q \vee \neg r) \rightarrow p$
   d) $q \leftrightarrow (p \wedge r)$

9) Select the proposition that is logically equivalent to $\neg p \rightarrow q$.
   a) $p \wedge \neg q$
   b) $p \vee q$
   c) $\neg p \vee q$
   d) $\neg p \wedge q$

10) Which law shows that the two propositions are logically equivalent?

$$\neg((w \vee p) \wedge (\neg q \wedge \neg w)) \quad \text{and} \quad \neg(w \vee p) \vee \neg(\neg q \wedge \neg w)$$

   a) DeMorgan's law
   b) Distributive law
   c) Associative law
   d) Complement law

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11) Which law shows that the two propositions are logically equivalent?

$r \wedge (p \vee q)$ and $r \wedge (q \vee p)$

a) DeMorgan's law
b) Distributive law
c) Associative law
d) Commutative law

12) The domain for variable x is the set of all integers. Select the statement that is false.

a) $\forall x\ (x^2 \neq 5)$
b) $\forall x\ (x^2 \geq x)$
c) $\forall x\ (x^2 > x)$
d) $\exists x\ (\sqrt{x} = x)$

13) The domain for variable x is the set of all integers. Select the statement that is true.

a) $\exists x\ (3x = 1)$
b) $\exists x\ (x^2 < 1)$
c) $\forall x\ (x^2 = 1)$
d) $\exists x\ (x^2 < 0)$

14) The predicate T is defined as: $T(x, y, z)$: $(x + y)^2 = z$. Select the proposition that is true.

a) T(4, 1, 5)
b) T(4, 1, 25)
c) T(1, 1, 1)
d) T(4, 0 2)

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15) The domain for variable x is the set {Ann, Ben, Cam, Dave}. The table below gives the values of predicates P and Q for every element in the domain.

Name	P(x)	Q(x)
Ann	F	F
Ben	T	F
Cam	T	T
Dave	T	T

Select the statement that is true.

a) $\forall x\,(Q(x) \rightarrow P(x))$

b) $\forall x\,(P(x) \rightarrow Q(x))$

c) $\forall x\,(P(x) \wedge Q(x))$

d) $\forall x\,(P(x) \vee Q(x))$

16) The domain for variable x is the set {Ann, Ben, Cam, Dave}. The table below gives the values of predicates P and Q for every element in the domain.

Name	P(x)	Q(x)
Ann	F	F
Ben	T	F
Cam	T	T
Dave	T	T

Select the statement that is false.

a) $\exists x\,(P(x) \rightarrow Q(x))$

b) $\exists x\,(P(x) \wedge Q(x))$

c) $\exists x\,(\neg P(x) \wedge Q(x))$

d) $\exists x\,(P(x) \wedge \neg Q(x))$

17) Select the logical expression that is equivalent to: $\neg\exists x\,(P(x) \wedge Q(x))$

a) $\exists x\,(\neg P(x) \vee \neg Q(x))$

b) $\exists x\,(\neg P(x) \wedge \neg Q(x))$

c) $\forall x\,(\neg P(x) \vee \neg Q(x))$

d) $\forall x\,(\neg P(x) \wedge \neg Q(x))$

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18) Select the logical expression that is equivalent to: $\neg\forall x\ (\neg P(x) \vee Q(x))$

a) $\exists x\ (P(x) \wedge \neg Q(x))$

b) $\exists x\ (\neg P(x) \vee Q(x))$

c) $\forall x\ (P(x) \vee \neg Q(x))$

d) $\forall x\ (\neg P(x) \wedge Q(x))$

19) Select the set that is equal to: {3, 5, 7, 9, 11, 13}

a) $\{x \in Z: 3 < x < 14\}$

b) $\{x \in R: 3 \leq x < 14\}$

c) $\{x \in Z: x \text{ is odd and } 3 \leq x \leq 14\}$

d) $\{x \in Z: x \text{ is prime and } 3 \leq x < 14\}$

20) A = {1, 2, {3, 4}, {5, 6, 7}}. Select the statement that is true.

a) $\{3\} \in A$

b) $\{3, 4\} \subseteq A$

c) $\{1, 2\} \subseteq A$

d) $\{1, 2\} \in A$

21) A = {1, 2, {3, 4}, {5, 6, 7}}. Select the correct value for |A|.

a) 4

b) 5

c) 6

d) 7

22) A = $\{x \in Z: x \text{ is a prime number}\}$. B = {4, 7, 9, 11, 13, 14}. Select the set corresponding to $A \cap B$.

a) $\varnothing$

b) {7, 11, 13}

c) {7, 9, 11, 13}

d) {4, 7, 9, 11, 13, 14}

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23) A = {x ∈ Z : x is a prime number}. B = {4, 7, 9, 11, 13, 14}. C = {x ∈ Z: 3 ≤ x ≤ 10}. Select the set corresponding to (A ∪ B) ∩ C.

a) {3, 5, 7}
b) {3, 4, 7, 9}
c) {3, 4, 5, 7, 9}
d) {3, 4, 5, 7, 9, 11, 13}

24) Select the set that is equivalent to (B ∩ C) ∪ ∅.

a) ∅
b) B
c) C
d) B ∩ C

25) Select the set that is equivalent to C ∪ (C ∩ B).

a) ∅
b) C
c) C ∪ B
d) B ∩ C

26) A = {a, b, c, d}. X = {1, 2, 3, 4}.

The function f:A → X is defined as f = {(a, 4), (b, 1), (c, 4), (d, 4)}

Select the set corresponding to the range of f.

a) {∅}
b) {1}
c) {1, 4}
d) {1, 2, 3, 4}

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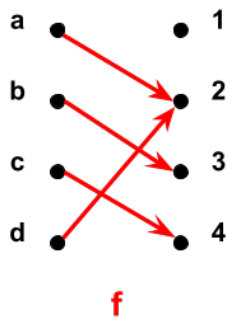
27) A = {a, b, c, d}. X = {1, 2, 3, 4}.

The function f: A → X is defined by the arrow diagram below.



Select the set of pairs that defines a function that is equal to f.

a) f = {(a, 2), (b, 3), (d, 2)}
b) f = {(a, 2), (b, 3), (c, 4), (d, 2)}
c) f = {(a, 2), (b, 3), (c, 4), (d, 4)}
d) f = {(a, 1), (b, 3), (c, 4), (d, 4)}

28) Select the value of ⌊4.2⌋

a) 0
b) 4
c) 4.2
d) 5

29) Select the value of ⌈-5.8⌉

a) -5
b) -6
c) 5
d) 6

30) f: Z → Z. f(x) = x + 3. Select the correct description of the function f.

a) One-to-one and onto
b) One-to-one but not onto
c) Onto but not one-to-one
d) Neither one-to-one nor onto

31) f: Z^+ → Z^+. f(x) = x + 3. Select the correct description of the function f.

a) One-to-one and onto
b) One-to-one but not onto
c) Onto but not one-to-one
d) Neither one-to-one nor onto

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32) A = {a, b, c, d}. X = {1, 2, 3, 4}. Each choice defines a function whose domain is A and whose target is X. Select the function that has a well-defined inverse.
   a) f = {(a, 3), (b, 4), (c, 3), (d, 4)}
   b) f = {(a, 3), (b, 3), (c, 3), (d, 3)}
   c) f = {(a, 3), (b, 4), (c, 2), (d, 1)}
   d) f = {(a, 3), (b, 4), (c, 2), (d, 4)}

33) Select the expression that is equal to $(3^{k+1})^2$
   a) 3^{k+2}
   b) 3^{k+3}
   c) 3^{2k+1}
   d) 3^{2k+2}

34) Select the value that is equal to $\lfloor \log_2 29 \rfloor$
   a) 2
   b) 3
   c) 4
   d) 5

35) What is the common ratio of the following geometric sequence? 27, 9, 3, 1, ...
   a) 27
   b) 9
   c) 3
   d) 1/3

36) A sequence $\{a_n\}$ is defined as follows: $a_0 = 2$, $a_1 = 1$, and for $n \geq 2$, $a_n = 3 \cdot a_{n-1} - n \cdot a_{n-2} + 1$. What is a_3?
   a) -2
   b) -1
   c) 1
   d) 2

37) A sequence is defined by the recurrence relation $f_n = n \cdot f_{n-1} - f_{n-3}$. How many initial values are required so that the sequence is well defined for all $n \geq 0$?
   a) 0
   b) 1
   c) 2
   d) 3

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38) A population of mice increases by 10% every year. Define g_n to be the number of mice after n years. Select the recurrence relation that describes the sequence $\{g_n\}$.

a) $g_n = (1.01) \cdot g_{n-1}$
b) $g_n = (1.1) \cdot g_{n-1}$
c) $g_n = (.01) \cdot g_{n-1} + g_{n-2}$
d) $g_n = (.1) \cdot g_{n-1} + g_{n-2}$

39) Q(n) is a statement parameterized by a positive integer n. The following theorem is proven by induction:

For any positive integer n, Q(n) is true.

What must be proven in the inductive step?

a) For any integer $k \geq 1$, Q(k-1) implies Q(k).
b) For any integer $k \geq 1$, Q(k) implies Q(n).
c) For any integer $k \geq 1$, Q(k).
d) For any integer $k \geq 1$, Q(k) implies Q(k+1).

40) $\sum_{i=1}^{n} i =$

a) n^2
b) $\frac{n(n+1)}{2}$
c) $\frac{(n-1)(n+1)}{2}$
d) n^3

41) $\sum_{j=0}^{n} 2^j =$

a) $2^j + \sum_{j=0}^{n-1} 2^j$
b) $2^{n-1} + \sum_{j=0}^{n-1} 2^j$
c) $2^n + \sum_{j=0}^{n-1} 2^j$
d) $2^n + \sum_{j=0}^{n} 2^j$
e) $1 + \sum_{j=1}^{n-1} 2^j$

42) (bonus)Prof. Miller works in which area?

a) Algorithms
b) Big Data
c) Logic
d) AI

43) (bonus)Prof. Miller was founding director of which center?

a) National Center for Supercomputing
b) New York Center for Computational Science
c) Center for Computational Research
d) San Diego Supercomputing Center
e) None of the above