Introduction

Dijkstra's algorithm is a method for finding the shortest paths between nodes in a graph, by iteratively selecting the node with the lowest known distance from the start node and updating the distances to its neighbors.
Introduction

Dijkstra’s algorithm was conceived by computer scientist Edsger W. Dijkstra in 1956.

This algorithm finds the shortest distance from a source node to all the other nodes in a given weighted graph.

It is ideal for solving the single source shortest path problem.
Common uses

- IP routing to find Open shortest Path First.
- Google maps for navigation
- Delivery route optimization
- Warehouse robot pickers
- Modeling biological network pathways
Dijkstra’s Algorithm Sequential

This algorithm finds the shortest path from a source node to other nodes in a graph by sequentially going over temporary distances to each node, picking the nearest node and then updating the distance to its neighbors. This step is repeated till all nodes are visited.
## Adjacency Matrix

```
<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
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<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
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<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
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<tr>
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<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
```
Sequential Pseudocode

function Dijkstra(Graph, source):
    dist[] := array of distances initialized to infinity for all nodes
    prev[] := array of predecessors initialized to NULL for all nodes
    visited[] := array initialized to false for all nodes
    dist[source] := 0

    while there are unvisited nodes:
        u := node with the minimum distance in dist[] among unvisited nodes
        mark u as visited

        for each neighbor v of u:
            if v is not visited and dist[u] + weight(u, v) < dist[v]:
                dist[v] := dist[u] + weight(u, v)
                prev[v] := u

    return dist[], prev[]
Example:

```
Example:

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```

```graph
c d
<table>
<thead>
<tr>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>2</td>
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<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>b</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>a</td>
</tr>
<tr>
<td>7</td>
</tr>
</tbody>
</table>

c d
<table>
<thead>
<tr>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>c</td>
</tr>
</tbody>
</table>

```

```
<table>
<thead>
<tr>
<th>nodes</th>
<th>Shortest</th>
<th>prev</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>c</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>d</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>e</td>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>

```

Graphs at Pace
Example:

\[
\text{visited} = (a) \\
\text{unvisited} = (b, c, d, e)
\]
Example:

visited = (a, c)
unvisited = (b, d, e)
Example:

visited = (a, c, b)
unvisited = (d, e)
Example:

```
visited = (a, c, b, d)
unvisited = (e)
```

<table>
<thead>
<tr>
<th>nodes</th>
<th>Shortest</th>
<th>prev</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0</td>
<td>c</td>
</tr>
<tr>
<td>b</td>
<td>1</td>
<td>a</td>
</tr>
<tr>
<td>c</td>
<td>3</td>
<td>a</td>
</tr>
<tr>
<td>d</td>
<td>5</td>
<td>c</td>
</tr>
<tr>
<td>e</td>
<td></td>
<td>d</td>
</tr>
</tbody>
</table>

Graphs at Pace
Dijkstra's algorithm

Parallel Dijkstra’s approaches

Two approaches:
- **Single source (Vertex Centric)**
  - Multiple sources across each processor
  - Less communication required (Not the objective of this course)
  - Better for multiple source path computation
- **Shared source (Edge Centric)**
  - A single source used by all processors
  - More communication required to find global minimum.
Parallel Dijkstra’s

1. Initialize MPI environment.
2. Read the number of vertices 'n' from command line argument.
3. Broadcast 'n' to all processes.
4. Calculate the local size of matrix.
5. Allocate memory for local matrices, distances, and predecessors.
7. Initialize random weight matrix on process 0 and broadcast it.
8. Perform Dijkstra's initialization locally:
   - Set the known status of the source node to 1.
   - For all other nodes, set known status to 0.
   - Initialize distances from source node and predecessors.
9. Execute Dijkstra's algorithm:
   - Find the node with the minimum tentative distance among unvisited nodes.
   - Share the minimum distance globally.
   - Update local distances and predecessors based on global minimum distances.
10. Gather local distances and predecessors to process 0.
11. Print global distances and paths.
12. Finalize MPI environment.
Overall performance with 40000 vertices

For Reference:

\# of tasks = \# of Cores per node

Total \# of cores = \# of nodes * \# of tasks per node
Overall performance with 40000 vertices

For Reference:

- \# of tasks = \# of Cores per node
- Total \# of cores = \# of nodes * \# of tasks per node
- Each line represents a constant number of nodes
Overall performance with 40000 vertices but for x>20

Cropped graph, with a zoomed in perspective for a better understanding.

Key for number of tasks per node
Time taken for 1 Task per node with 40000 vertices
Time taken for 10 Task per node with 40000 vertices
Time taken for 26 Task per node with 40000 vertices
Time taken for 32 Task per node with 40000 vertices
Time taken for 40 Task per node with 40000 vertices
Time taken for 1 Node with 40000 vertices
Speedup for 1 Node with 40000 vertices

![Graph showing speedup vs number of cores](image)
Cost for 1 Node with 40000 vertices

Cost vs Number of Processors

Cost

0 10 20 30 40 50
Number of Processors

data_1
Time taken for 2 Nodes with 40000 vertices
Speedup for 2 Nodes with 40000 vertices

![Graph showing speedup vs number of cores for 2 nodes with 40000 vertices]
Cost for 2 Nodes with 40000 vertices

Cost vs Number of Processors

Cost

Number of Processors
Time taken for 4 Nodes with 40000 vertices
Speedup for 4 Nodes with 40000 vertices

Speedup vs Number of Cores (4 Nodes)

- Speedup on the y-axis
- Number of Cores on the x-axis

Graph showing the speedup with respect to the number of cores for 4 nodes and 40000 vertices.
Cost for 4 Nodes with 40000 vertices
Time taken for 8 Nodes with 40000 vertices
Speedup for 8 Nodes with 40000 vertices
Cost for 8 Nodes with 40000 vertices

Graphs at Pace
Time taken for 16 Nodes with 40000 vertices

![Graph showing time taken vs. number of tasks per node for 16 nodes with 40000 vertices.](image-url)
Speedup for 16 Nodes with 40000 vertices

![Graph showing speedup vs number of cores for 16 nodes with 40000 vertices](image)
Cost for 16 Nodes with 40000 vertices
Time taken for 32 Nodes with 40000 vertices
Speedup for 32 Nodes with 40000 vertices
Cost for 32 Nodes with 40000 vertices
Time taken for 40 Nodes with 40000 vertices
Speedup for 40 Nodes with 40000 vertices

![Graph showing speedup vs number of cores for 40 nodes]
Cost for 40 Nodes with 40000 vertices
Time taken for 44 Nodes with 40000 vertices

Graphs at Pace
Speedup for 44 Nodes with 40000 vertices

Graphs at Pace
Cost for 44 Nodes with 40000 vertices

Cost vs Number of Processors

Cost vs Number of Processors

Cost

Number of Processors

0 200 400 600 800 1000 1200 1400 1600

0 20 40 60 80 100 120 140 160

data_44
Time taken for 46 Nodes with 40000 vertices
Speedup for 46 Nodes with 40000 vertices

![Speedup vs Number of Cores (46 Nodes)]
Cost for 46 Nodes with 40000 vertices

Cost vs Number of Processors

Cost

Number of Processors

0 250 500 750 1000 1250 1500 1750

0 20 40 60 80 100 120 140 160

data_46
Time taken for 50 Nodes with 40000 vertices
Cost for 50 Nodes with 40000 vertices
Time taken for 60 Nodes with 40000 vertices
Speedup for 60 Nodes with 40000 vertices
Cost for 60 Nodes with 40000 vertices

Graph vs Number of Processors

Cost vs Number of Processors

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Graphs at Pace
Time taken for 80 Nodes with 40000 vertices
Speedup for 80 Nodes with 40000 vertices

![Graph showing speedup vs number of cores for 80 nodes with 40000 vertices. The graph demonstrates a decrease in speedup as the number of cores increases, with a sudden drop before stabilizing at around 0.3.]
Cost for 80 Nodes with 40000 vertices
Time Taken for 1 nodes with increasing vertices

1 core => 625 vertices
2 cores => 1250 vertices
4 cores => 2500 vertices
8 cores => 5000 vertices
16 cores => 10000 vertices
32 cores => 20000 vertices
Speedup for 1 nodes with increasing vertices

1 core => 625 vertices
2 cores => 1250 vertices
4 cores => 2500 vertices
8 cores => 5000 vertices
16 cores => 10000 vertices
32 cores => 20000 vertices
Time Taken for 2 nodes with increasing vertices

- 1 core => 625 vertices
- 2 cores => 1250 vertices
- 4 cores => 2500 vertices
- 8 cores => 5000 vertices
- 16 cores => 10000 vertices
- 32 cores => 20000 vertices
Speedup for 2 nodes with increasing vertices

Speedup vs Number of Cores (2 Nodes)

Number of Cores

Speedup
Conclusions

- We could see that when increasing the scaling, initially we notice a constant drop in the time taken from parallelization.
- After a certain point, especially with over 20 nodes, and around 40 tasks per node, we see a gradual increase in time taken for most scenarios.
- This tells us that Parallelization is beneficial to us, to a certain extent, but when the overhead of inter process communication, and extent of parallelization is really high, the cons outweigh the pros of parallelization.
References

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Thank you

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