PARALLEL IMPLEMENTATION OF DIJKSTRA'S ALGORITHM USING MPI LIBRARY ON A CLUSTER.

INSTRUCUTOR: DR RUSS MILLER

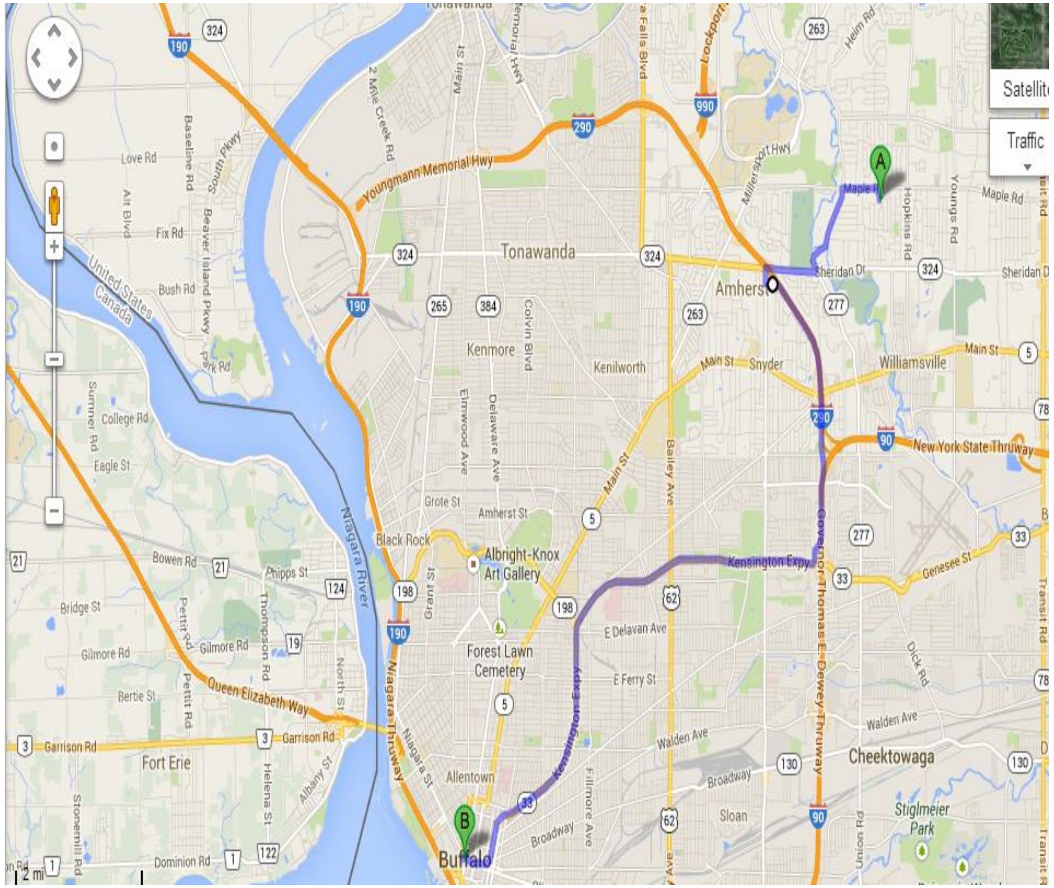
ADITYA PORE

THE PROBLEM AT HAND

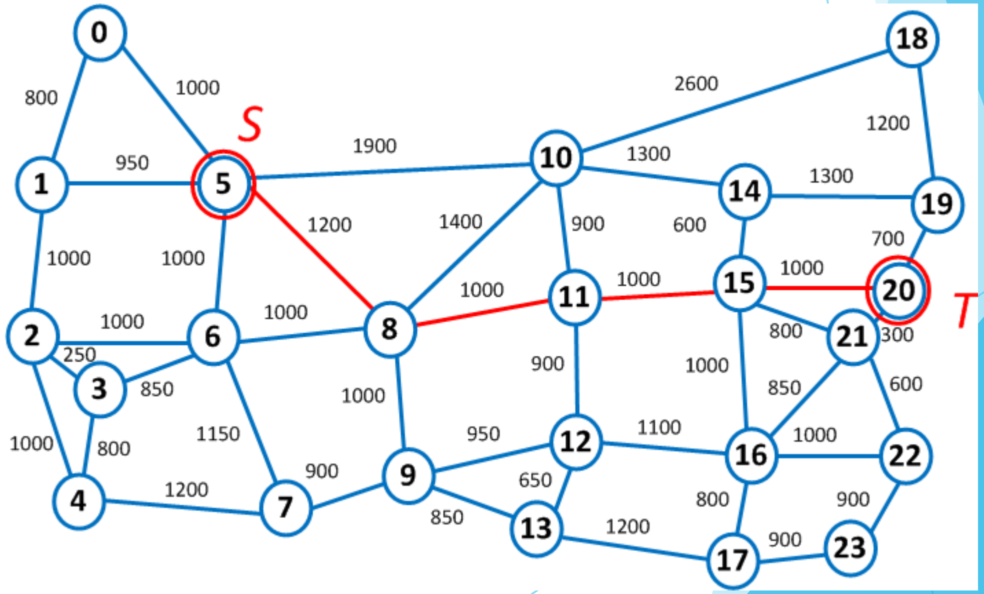
- Given : A directed graph ,G = (V, E) . Cardinalities |V| = n, |E| = m.
- S(Source): distinguished vertex of the graph.
- w : weight of each edge, typically, the distance between the two vertexes.
- **Single source shortest path:** The single source shortest path (SSSP) problem is that of computing, for a given source vertex s and a destination vertex t, the weight of a path that obtains the minimum weight among all the possible paths.

DIJKSTRA's ALGORITHM AT A GLANCE

- Dijkstra's algorithm is a graph search algorithm that solves single-source shortest path for a graph with nonnegative weights.
- Widely used in network routing protocol, e.g., Open Shortest Path First (OSPF) protocol



How to reach Downtown from Maple Road??



24 Node US-Mesh Network

LETS GET TO KNOW THE ALGORITHM WITH AN EXAMPLE

Dijkstra's Algorithm

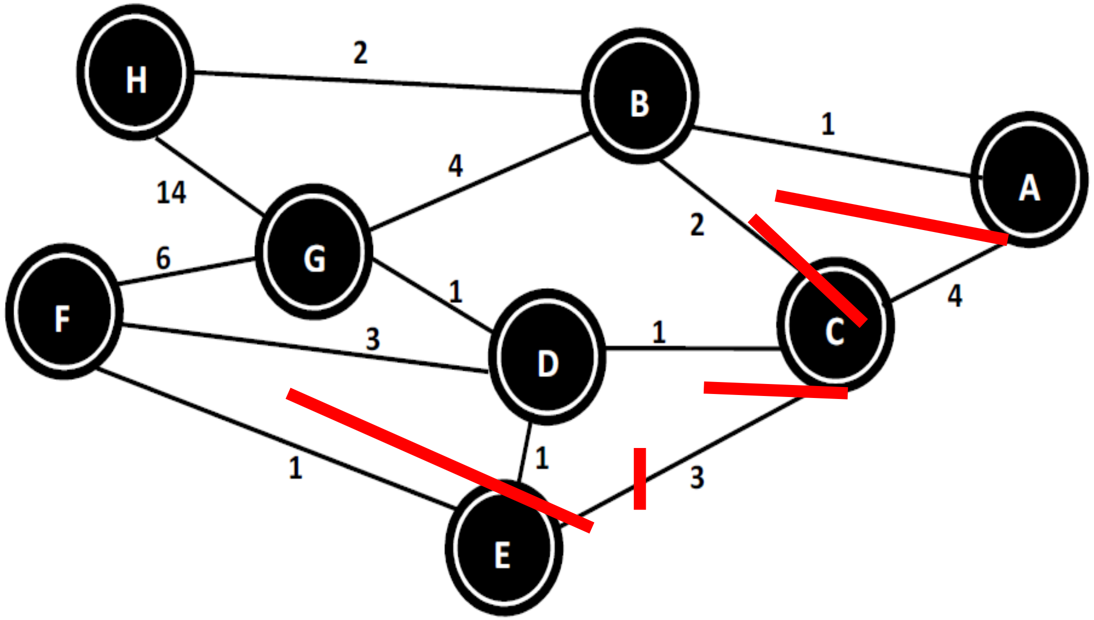


Fig. 2 8-node simple network

	(d, n)	(d, n)	(d, n)	(d, n)	(d, n)	(d, n)	(d, n)
cluster	B	C	D	E	F	G	H
A AB	1, A	4, A	∞	∞	∞	∞	∞
ABC		3, B	∞	∞	∞	5, B	3, B
ABCH			4, C	6, C	∞	5, B	3, B
ABCHD			4, C	6, C	∞	5, B	
ABCHDE				5, D	7, D	5, B	
ABCHDEG					6, E	5, B	
ABCHDEGF					6, E		

Table 1. The routing table for node A

Dijkstra’s algorithm 1st round

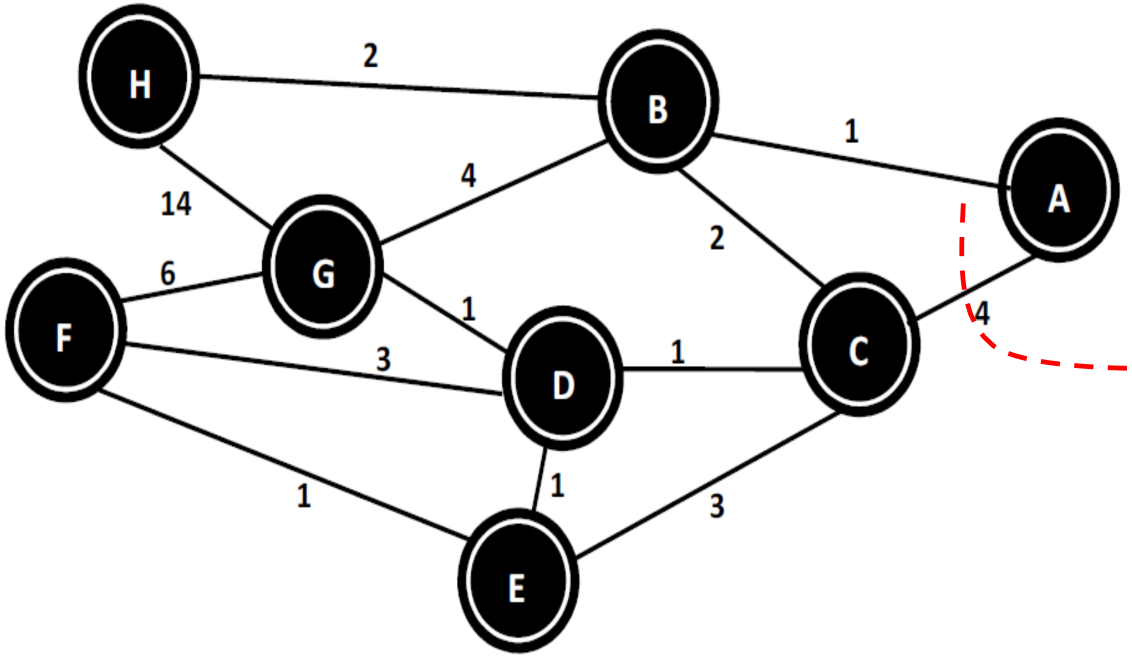


Fig. 2 8-node simple network

	(d, n)	(d, n)	(d, n)	(d, n)	(d, n)	(d, n)	(d, n)
cluster	B	C	D	E	F	G	H
A	1, A	4, A	∞	∞	∞	∞	∞
AB							

Table 1. The routing table for node A

Dijkstra's algorithm-2nd round

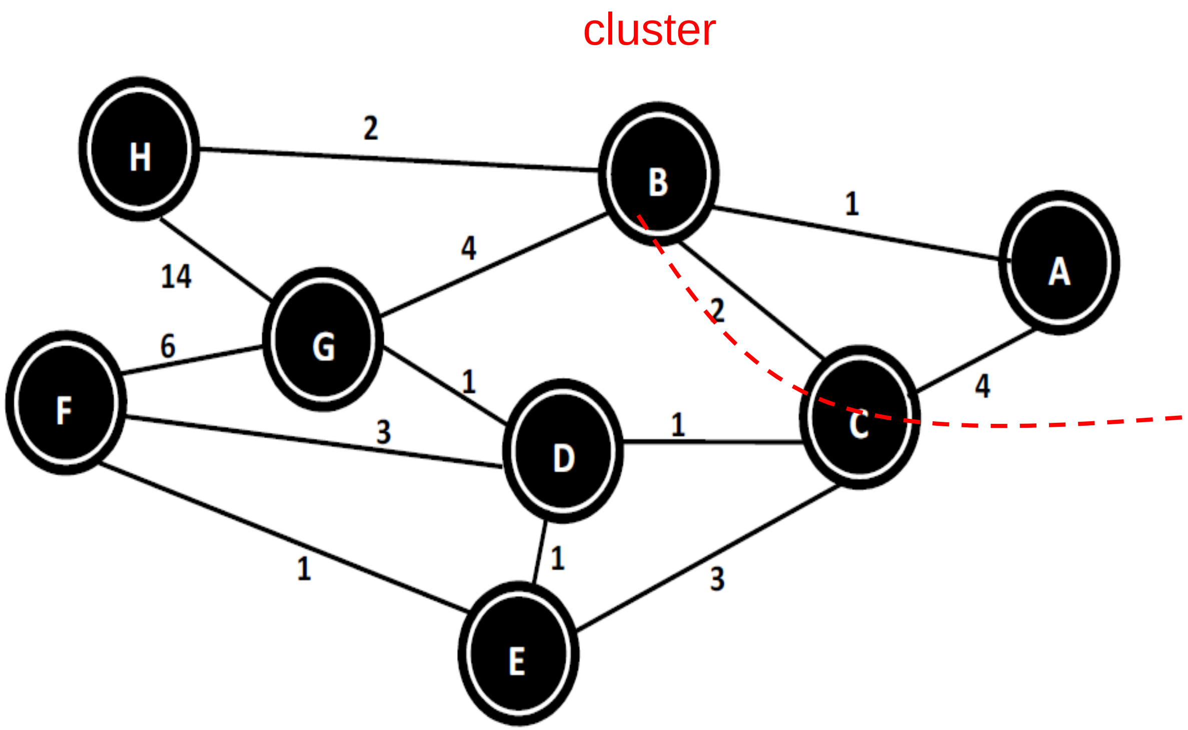


Fig. 2 8-node simple network

	(d, n)	(d, n)	(d, n)	(d, n)	(d, n)	(d, n)	(d, n)
cluster	B	C	D	E	F	G	H
A	1, A	4, A	∞	∞	∞	∞	∞
AB		3, B	∞	∞	∞	5, B	3, B
ABC							

Table 1. The routing table for node A

Dijkstra’s algorithm-3rd round

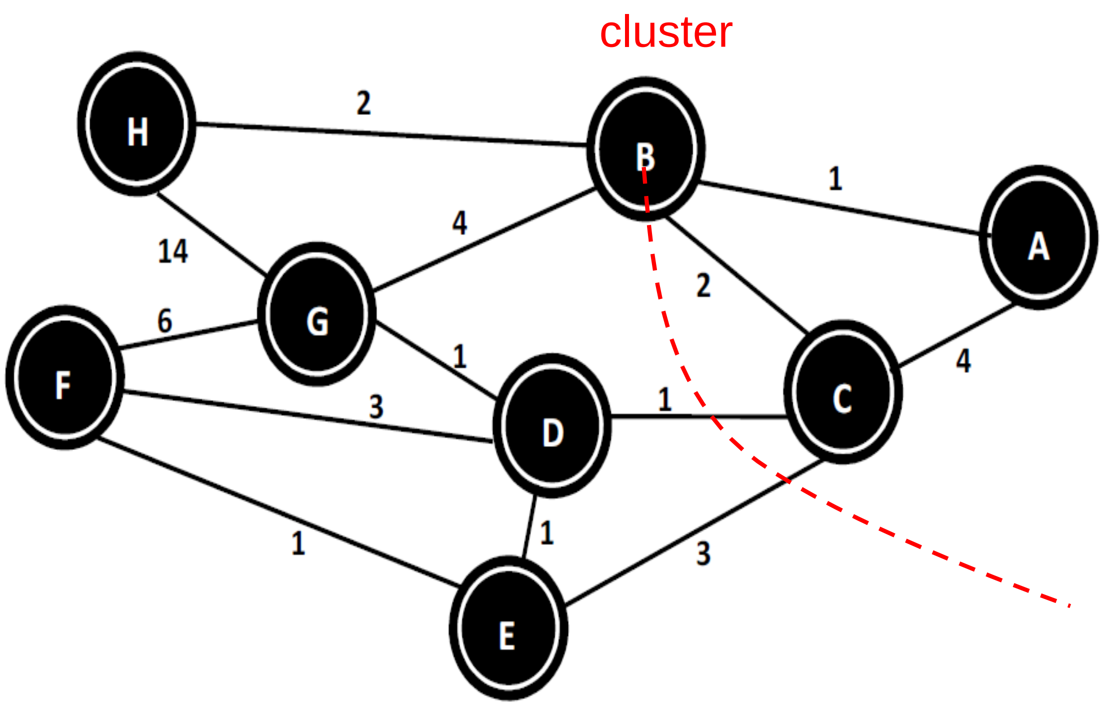


Fig. 2 8-node simple network

	(d, n)	(d, n)	(d, n)	(d, n)	(d, n)	(d, n)	(d, n)
cluster	B	C	D	E	F	G	H
A AB	1, A	4, A	∞	∞	∞	∞	∞
ABC		3, B	∞	∞	∞	5, B	3, B
ABCH			4, C	6, C	∞	5, B	3, B

Table 1. The routing table for node A

Dijkstra’s algorithm-4th round

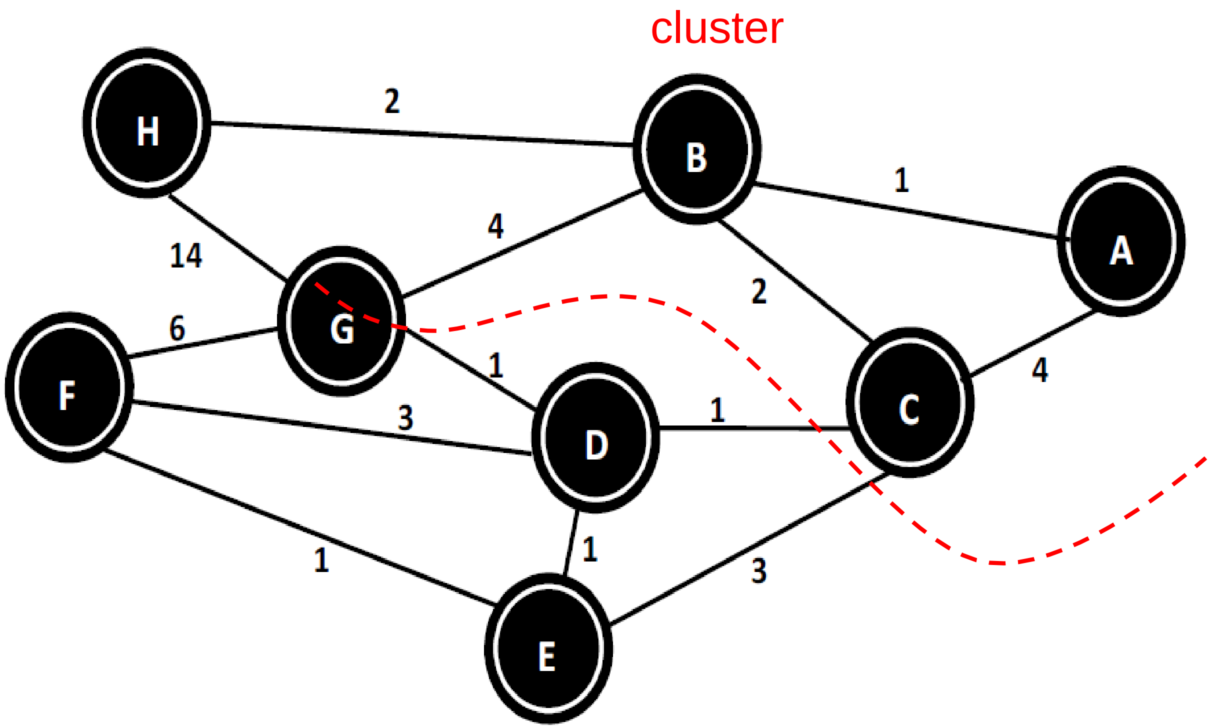


Fig. 2 8-node simple network

	(d, n)	(d, n)	(d, n)	(d, n)	(d, n)	(d, n)	(d, n)
cluster	B	C	D	E	F	G	H
A AB	1, A	4, A	∞	∞	∞	∞	∞
ABC		3, B	∞	∞	∞	5, B	3, B
ABCH			4, C	6, C	∞	5, B	3, B
ABCHD			4, C	6, C	∞	5, B	

Table 1. The routing table for node A

Dijkstra’s algorithm-5th round

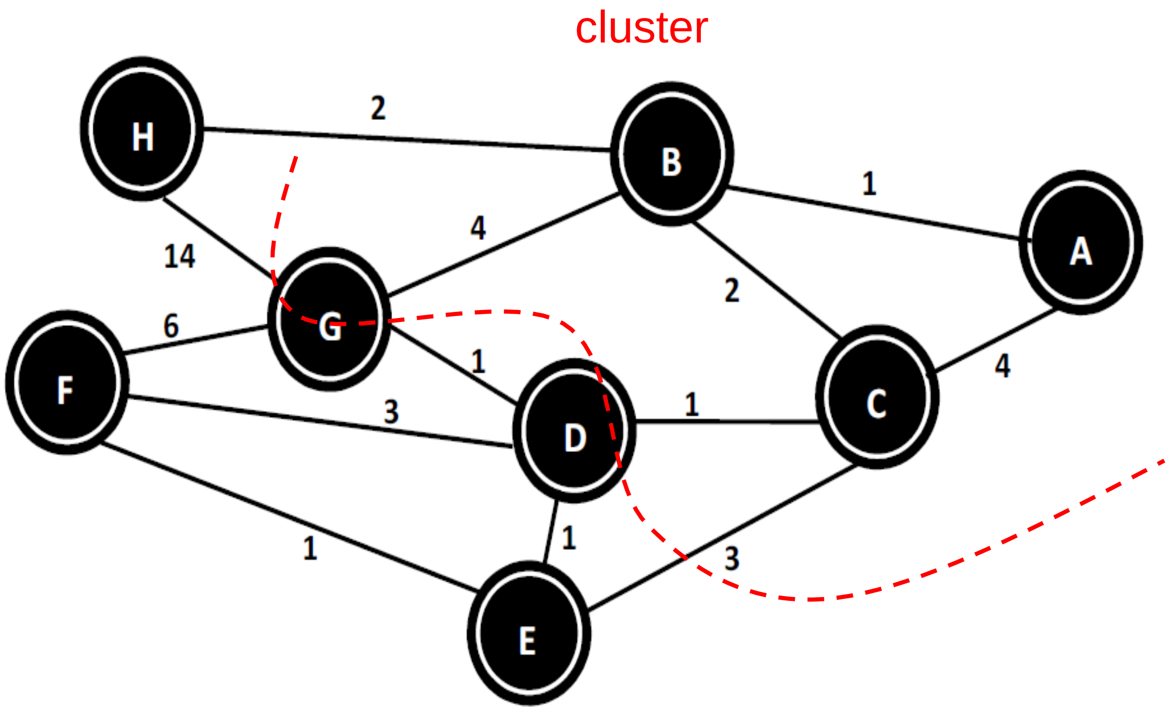


Fig. 2 8-node simple network

	(d, n)	(d, n)	(d, n)	(d, n)	(d, n)	(d, n)	(d, n)
cluster	B	C	D	E	F	G	H
A AB	1, A	4, A	∞	∞	∞	∞	∞
ABC		3, B	∞	∞	∞	5, B	3, B
ABCH			4, C	6, C	∞	5, B	3, B
ABCHD			4, C	6, C	∞	5, B	
ABCHDE				5, D	7, D	5, B	

Table 1. The routing table for node A

Dijkstra's algorithm-6th round

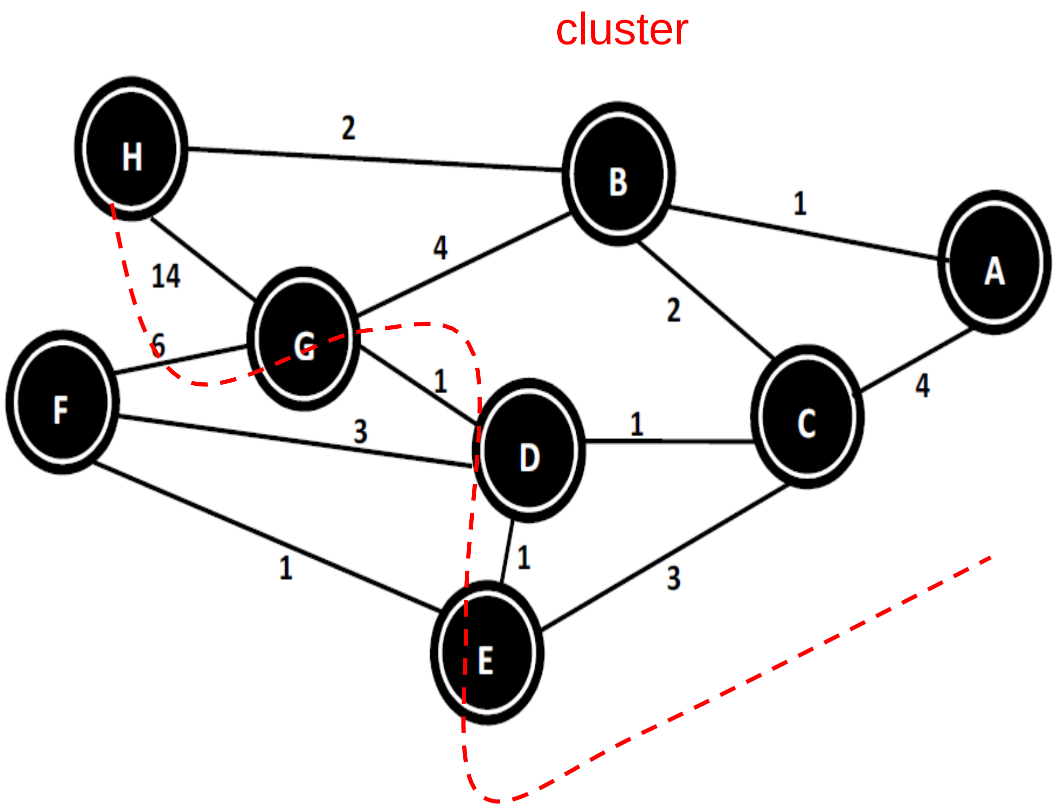


Fig. 2 8-node simple network

	(d, n)	(d, n)	(d, n)	(d, n)	(d, n)	(d, n)	(d, n)
cluster	B	C	D	E	F	G	H
A AB	1, A	4, A	∞	∞	∞	∞	∞
ABC		3, B	∞	∞	∞	5, B	3, B
ABCH			4, C	6, C	∞	5, B	3, B
ABCHD			4, C	6, C	∞	5, B	
ABCHDE				5, D	7, D	5, B	
ABCHDEG					6, E	5, B	
					6, E		

Table 1. The routing table for node A

Dijkstra's algorithm-6th round

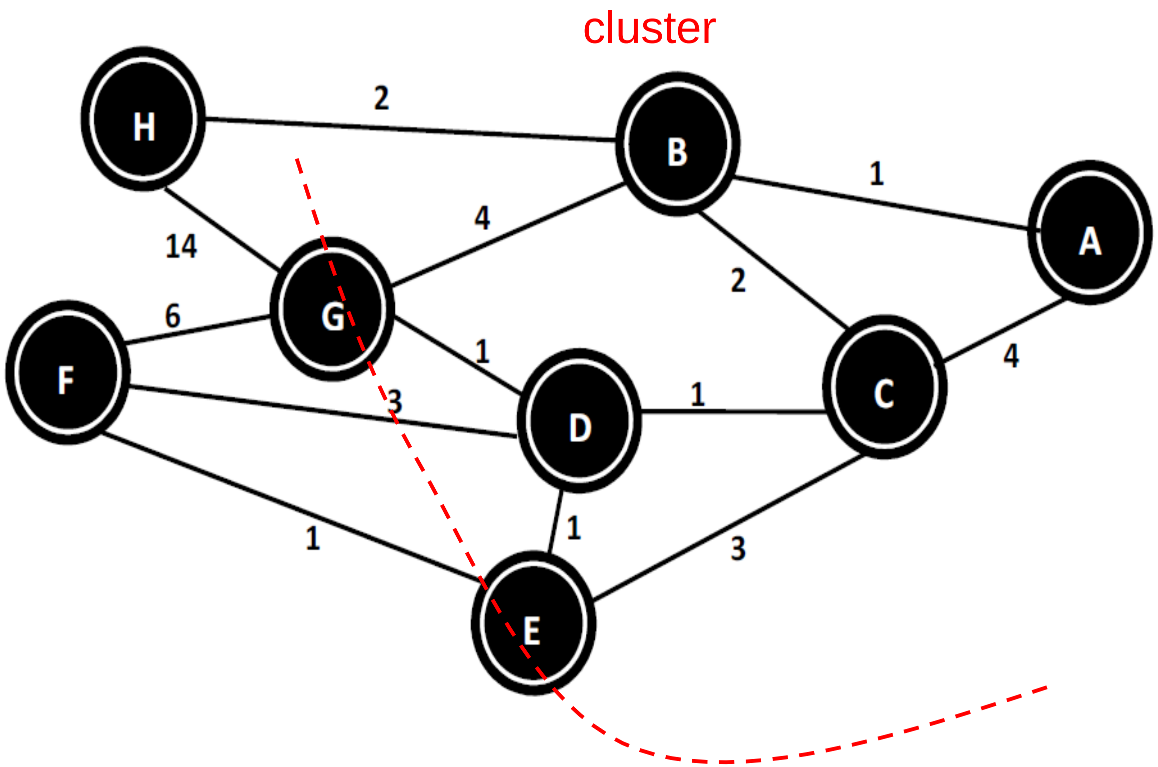


Fig. 2 8-node simple network

	(d, n)	(d, n)	(d, n)	(d, n)	(d, n)	(d, n)	(d, n)
cluster	B	C	D	E	F	G	H
A AB	1, A	4, A	∞	∞	∞	∞	∞
ABC		3, B	∞	∞	∞	5, B	3, B
ABCH			4, C	6, C	∞	5, B	3, B
ABCHD			4, C	6, C	∞	5, B	
ABCHDE				5, D	7, D	5, B	
ABCHDEG					6, E	5, B	
ABCHDEGF					6, E		

Table 1. The routing table for node A

SEQUENTIAL DIJKSTRA'S ALGORITHM

Create a cluster *cl[V]*
Given a source vertex *s*
While (there exist a vertex that is not in the cluster *cl[V]*)
{
FOR (all the vertices outside the cluster)
Calculate the distance from non-member vertex to *s* through the cluster
END
***** O(V) *****
Select the vertex with the shortest path and add it to the cluster
***** O(V) *****
}

ANALOGY

```
DIJKSTRA(G, w, s)
INITIALIZE-SINGLE-SOURCE(G, s)
S = ∅
Q = G.V
while Q ≠ ∅
    u = EXTRACT-MIN(Q)
    S = S ∪ {u}
    for each vertex v ∈ G.Adj[u]
        RELAX(u, v, w)
```

DIJKSTRA'S ALGORITHM

- **Running time $O(V^2)$**
- In order to obtain the routing table, we need O(V) rounds iterations (until all the vertices are included in the cluster).
- In each round, we will update the value for O(V) vertices and select the closest vertex, so the running time in each round is O(V).
- So, the total running time is $O(V^2)$

- **Disadvantages:**
- If the scale of the network is too large, then it will cost a long time to obtain the result.
- For some time-sensitive app or real-time services, we need to reduce the running time.

PARALLEL DIJKSTRA’S ALGORITHM

- Each core identifies its closest vertex to the source vertex;
- Perform a parallel prefix to select the globally closest vertex;
- Broadcast the result to all the cores;
- Each core updates its cluster list.

THE ACTUAL ALGORITHM AT WORK

Parallel Dijkstra’s algorithm

- Step 1: find the closest node in my subgroup.
- Step 2: use parallel prefix to find the global closest.

	(d, n)	(d, n)	(d, n)	(d, n)	(d, n)	(d, n)	(d, n)
cluster	B	C	D	E	F	G	H
A AB	1, A	4, A	∞	∞	∞	∞	∞
ABC		3, B	∞	∞	∞	5, B	3, B
ABCH			4, C	6, C	∞	5, B	3, B
ABCHD			4, C	6, C	∞	5, B	
ABCHDE				5, D	7, D	5, B	
ABCHDEG					6, E	5, B	
ABCHDEGF					6, E		

PARALLEL DIJKSTRA'S ALGORITHM

Create a cluster *cl[V]*
Given a source vertex *s*
Each core handles a subgroup of *V/P* vertices
While (there exist a vertex that is not in the cluster *cl[V]*)
{

FOR (vertices in my subgroup but outside the cluster)

Calculate the distance from non-member vertex to *s* through the cluster;

Select the vertex with the shortest path as the local closest vertex;

END

**** Each processor work in parallel *O(V/P)* ****

MPI _MINLOC operation??

Use the parallel prefix to find the global closest vertex among all the local closest vertices from each core.

**** Parallel prefix *log(P)* ****

}

PARALLEL DIJKSTRA'S ALGORITHM

RUNNING TIME : $O(V^2/P + V \log(P))$

2

- P is the number of cores used.
- In order to obtain the routing table, we need O(V) rounds iteration (until all the vertices are included in the cluster).
- In each round, we will update the value for O(V) vertices using P cores running independently, and use the parallel prefix to select the global closest vertex, so the running time in each round is $O(V/P)+O(\log(P))$.
- So, **the total running time is $O(V /P + V^2 \log(P))$**

RESULTS AND ANALYSIS

- Implemented using MPI : Stats Averaged over 10 rounds of Computation.
- Establish trade-off between running times as a function of number of cores deployed.
- Evaluate speed up and efficiency!!!!

- EXPERIMENT A: (More Graphs and Analysis)
  - Compute for fixed size input:10000
  - Run Routines for :1 32-core node,3 12-core node,16 dual-core

- EXPERIMENT B: (Achieved Desired Results)
  - Compute for different input size: Typically 625,2500,10000
  - Run Routine on 1 32-core Node.

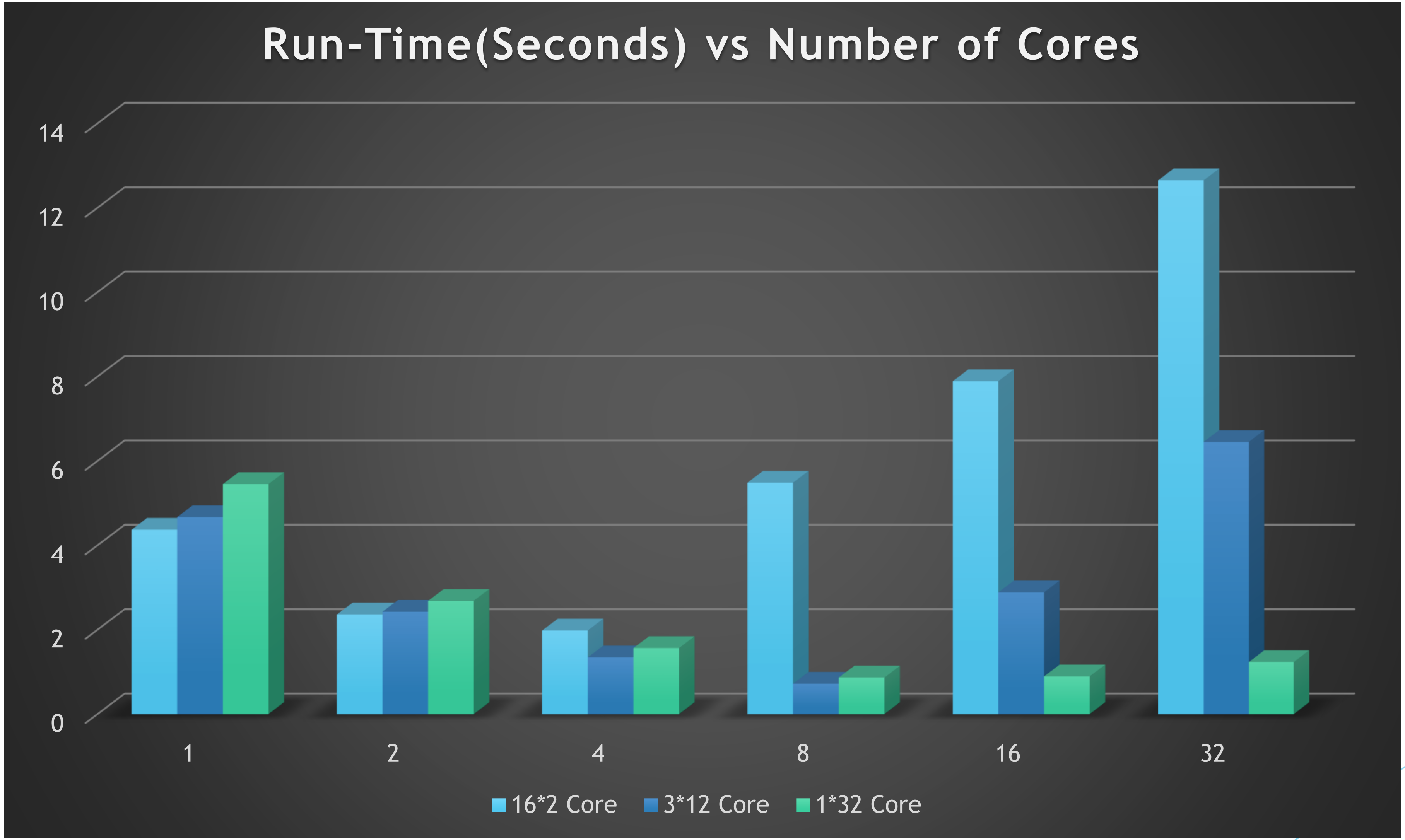
EXPERIMENT A : RUN TIME

Tabulation of Results :
Relationship Observed : Number of Cores Versus The Running Time(seconds)
Conclusions:

(a) Run Time is Inversely proportional to number of cores :Cores belong to the same node in cluster
(b) Significant Increase observed for two configurations out of three, namely 16*2 Core and 3*12 Core.

Number of Cores	1	2	4	8	16	32
Configurations		RUNTIMES	I	N	SECONDS	
1)16*2 Core	4.37263	2.36273	1.98442	5.48834	7.89371	12.65342
2)3*12 Core	4.67321	2.42865	1.34567	0.72341	2.88764	6.45321
3)1*32 Core	5.45321	2.68753	1.56782	0.86754	0.89654	1.23609

EXPERIMENT A : RUN TIME
GRAPHICAL DESCRIPTION OF RUN TIME ANALYSIS



EXPERIMENT A : SPEED UP

Tabulation of Results :

Relationship Observed : Number of Cores Versus The Speed-Up

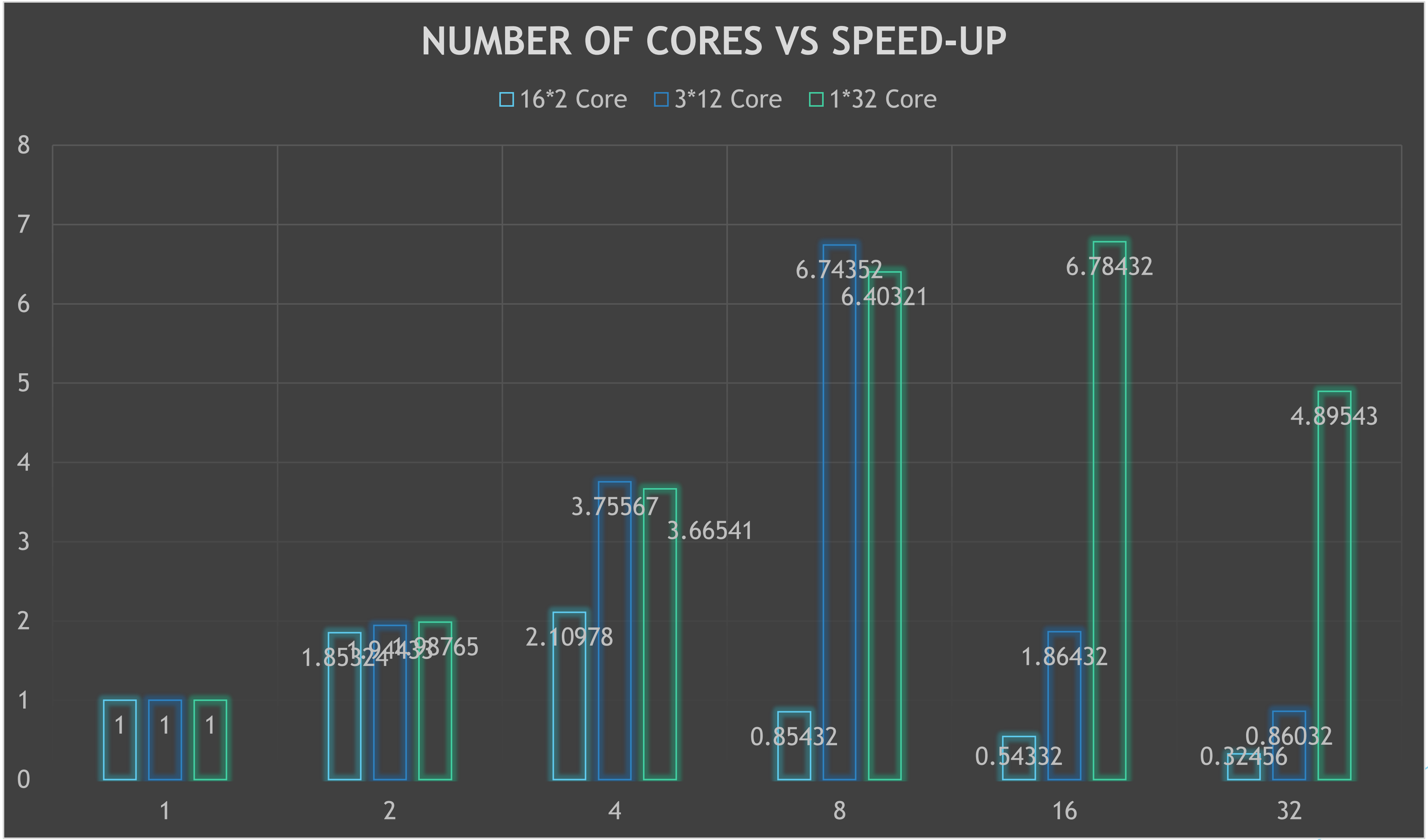
Conclusions:

(a) Speed-Up is Directly proportional to number of cores :Cores belong to the same node in cluster

(b) Significant Decrease observed for two configurations out of three, namely 16*2 Core and 3*12 Core.

Number of Cores	1	2	4	8	16	32
Configurations	SPEED UP: GIVES A MEASURE OF SCALABILITY OF THE SYSTEM					
16*2 Core	1	1.85324	2.10978	0.85432	0.54332	0.32456
3*12 Core	1	1.94433	3.75567	6.74352	1.86432	0.86032
1*32 Core	1	1.98765	3.66541	6.40321	6.78432	4.89543

EXPERIMENT A : SPEED-UP
GRAPHICAL DESCRIPTION OF SPEED-UP ANALYSIS



EXPERIMENT A : EVALUATING EFFICIENCY VIA SPEED-UP

Tabulation of Results :

Relationship Observed : Number of Cores Versus The Efficiency

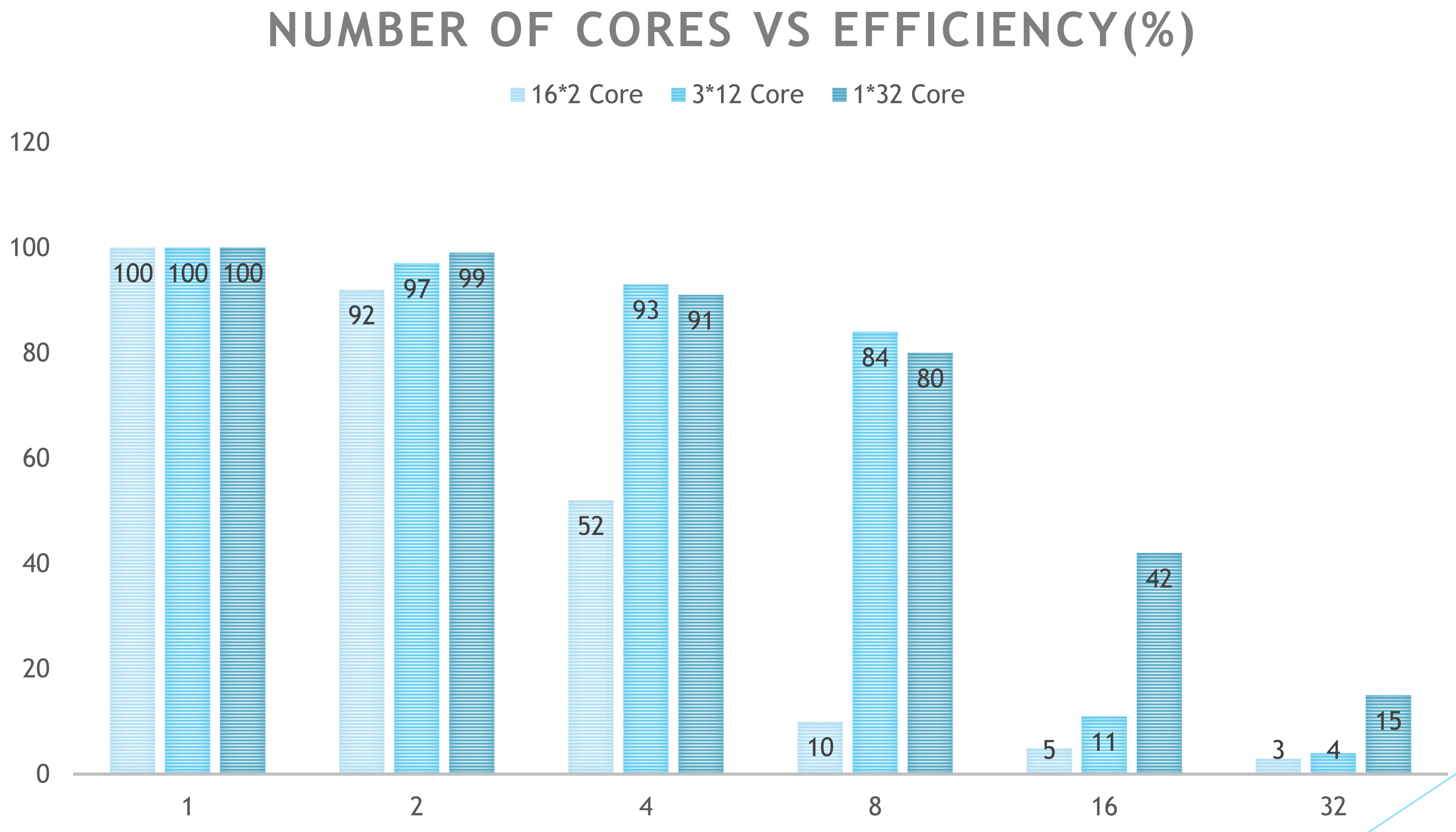
Conclusions:

(a) Efficiency varies inversely with number of cores .

(b) Significant Decrease observed for two configurations out of three, namely 16*2 Core and 3*12 Core

Number of Cores	1	2	4	8	16	32
Configurations	EFFICIENCY: Gives a measure of fraction of time utilized by processors (Cores) for particular Computation.					
16*2 Core	1	0.92662	0.52745	0.10679	0.05395	0.03014
3*12 Core	1	0.97216	0.93891	0.84294	0.11652	0.04688
1*32 Core	1	0.99383	0.91630	0.80040	0.42402	0.15298

EXPERIMENT A : EFFICIENCY
GRAPHICAL DESCRIPTION OF ANALYSIS



EXPERIMENT A : COST

- Tabulation of Results :
- Relationship Observed : Number of Cores Versus Cost of Computation
- Conclusions:

(a) Run Time is Inversely proportional to number of cores

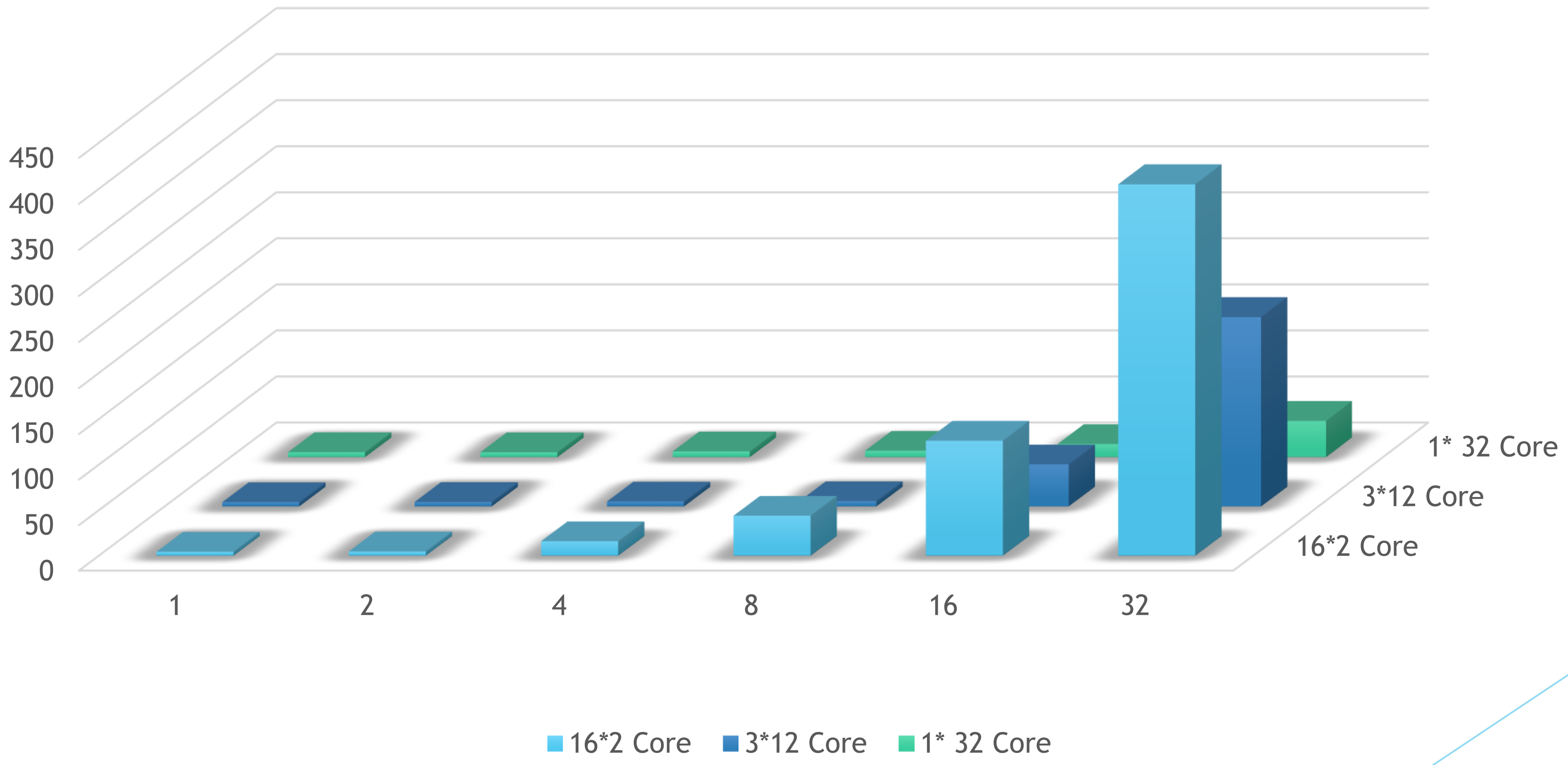
(b) Significant Increase observed for 16*2 Core configuration.

(c) Parallel computing is cost effective for modest speedups.

Number of Cores	1	2	4	8	16	32
Configurations	Cost: Product of number of cores(resources) used times execution time					
16*2 Core	4.37263	4.72546	15.93768	43.90672	126.29936	404.9094
3*12 Core	4.67321	4.85730	5.38268	5.78728	46.20224	206.5027
1*32 Core	5.45321	5.37506	6.27128	6.94032	14.34464	39.55488

EXPERIMENT A : COST
GRAPHICAL DESCRIPTION

Number of Cores VS Cost of Computation



EXPERIMENT B : RUN TIME

- Tabulation of Results :
- Relationship Observed : Input-Size VS Running-Time
- Conclusions:

(a) Run Time varies Inversely with the number of Cores.

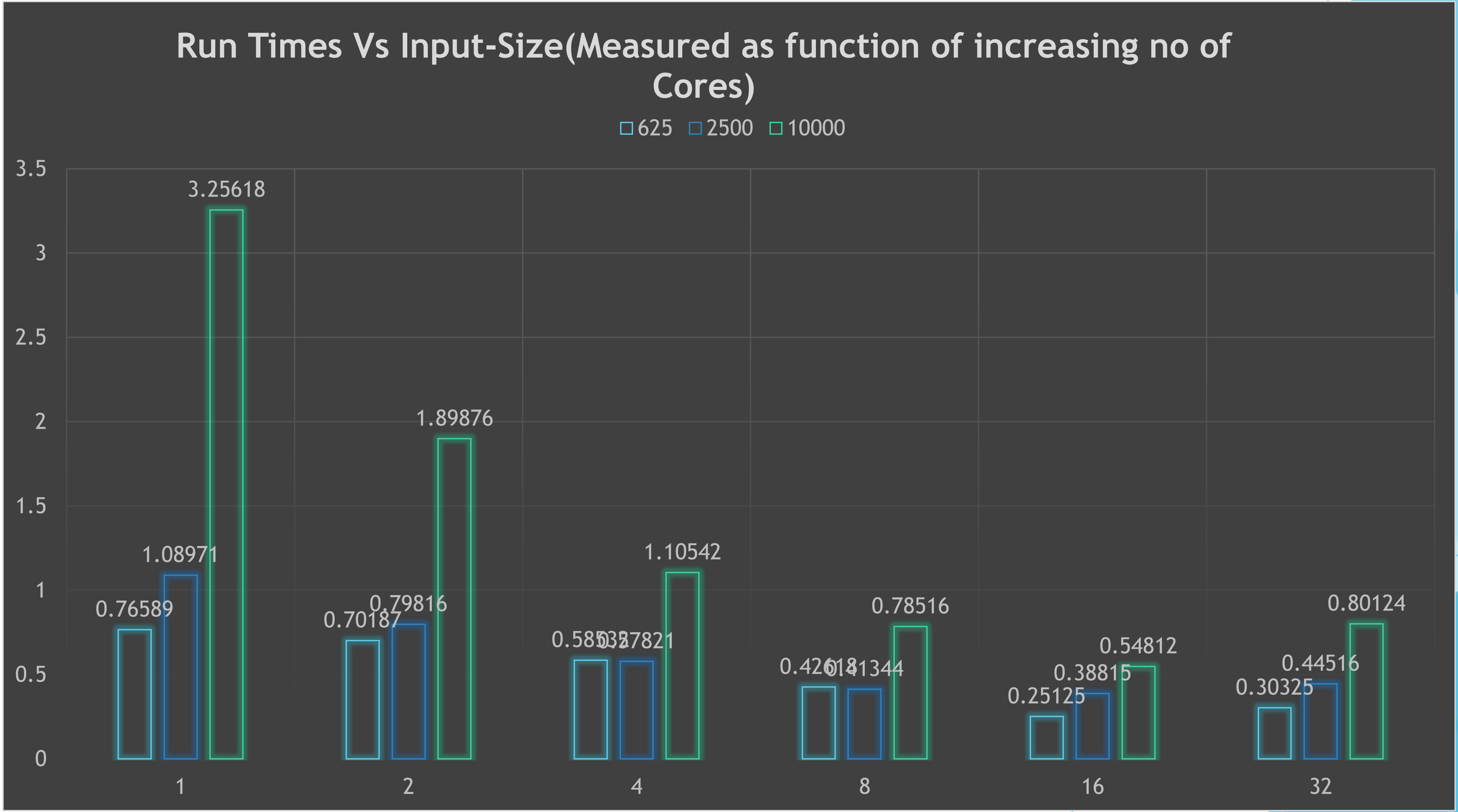
(b) Algorithm found to be most-effective performance-wise for 16 Core configuration.

(c) 32-Cores: Run time increases Slightly as communication overhead defeats the purpose of using more number of cores for computation.

Number of Cores	1	2	4	8	16	32
Input-Size	RUNTIME		IN		SECONDS	
625	0.76589	0.70187	0.58532	0.42618	0.25125	0.30325
2500	1.08971	0.79816	0.57821	0.41344	0.38815	0.44516
10000	3.25618	1.89876	1.10542	0.78516	0.54812	0.80124

EXPERIMENT B: RUN TIME

GRAPHICAL DESCRIPTION OF RUN TIME ANALYSIS



EXPERIMENT B : SPEED UP

Tabulation of Results :

Relationship Observed : Input-Size(with increasing number of nodes) Versus The Speed-Up

Conclusions:

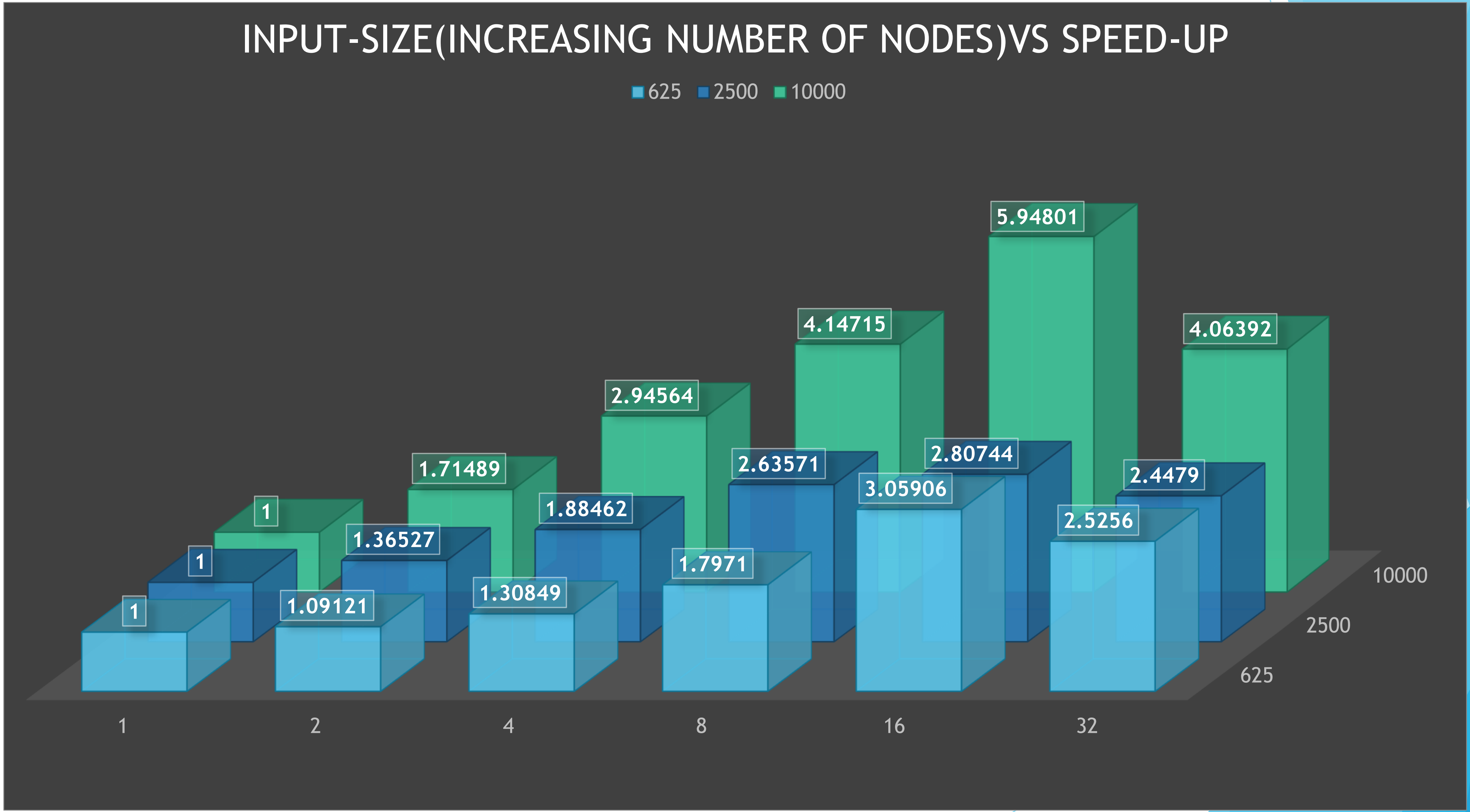
(a) Speed-Up is Directly proportional to number of cores.

(b) Significant Decrease observed , after a certain point for all three input sizes owing to communication latency.

(c) As the input size increases, the number of cores used to achieve maximum speed up increases.

Number of Cores	1	2	4	8	16	32
Input-Size	SPEED UP: most obvious benefit of using a parallel computer is the reduction in the running time of the code.					
625	1	1.09121	1.30849	1.79710	3.05906	2.52560
2500	1	1.36527	1.88462	2.63571	2.80744	2.44790
10000	1	1.71489	2.94564	4.14715	5.94801	4.06392

EXPERIMENT B : SPEED-UP
GRAPHICAL DESCRIPTION OF SPEED-UP ANALYSIS



EXPERIMENT B : EVALUATING EFFICIENCY VIA SPEED-UP

Tabulation of Results :

Relationship Observed : Input-Size(Increasing number of cores)Versus The Efficiency

Conclusions:

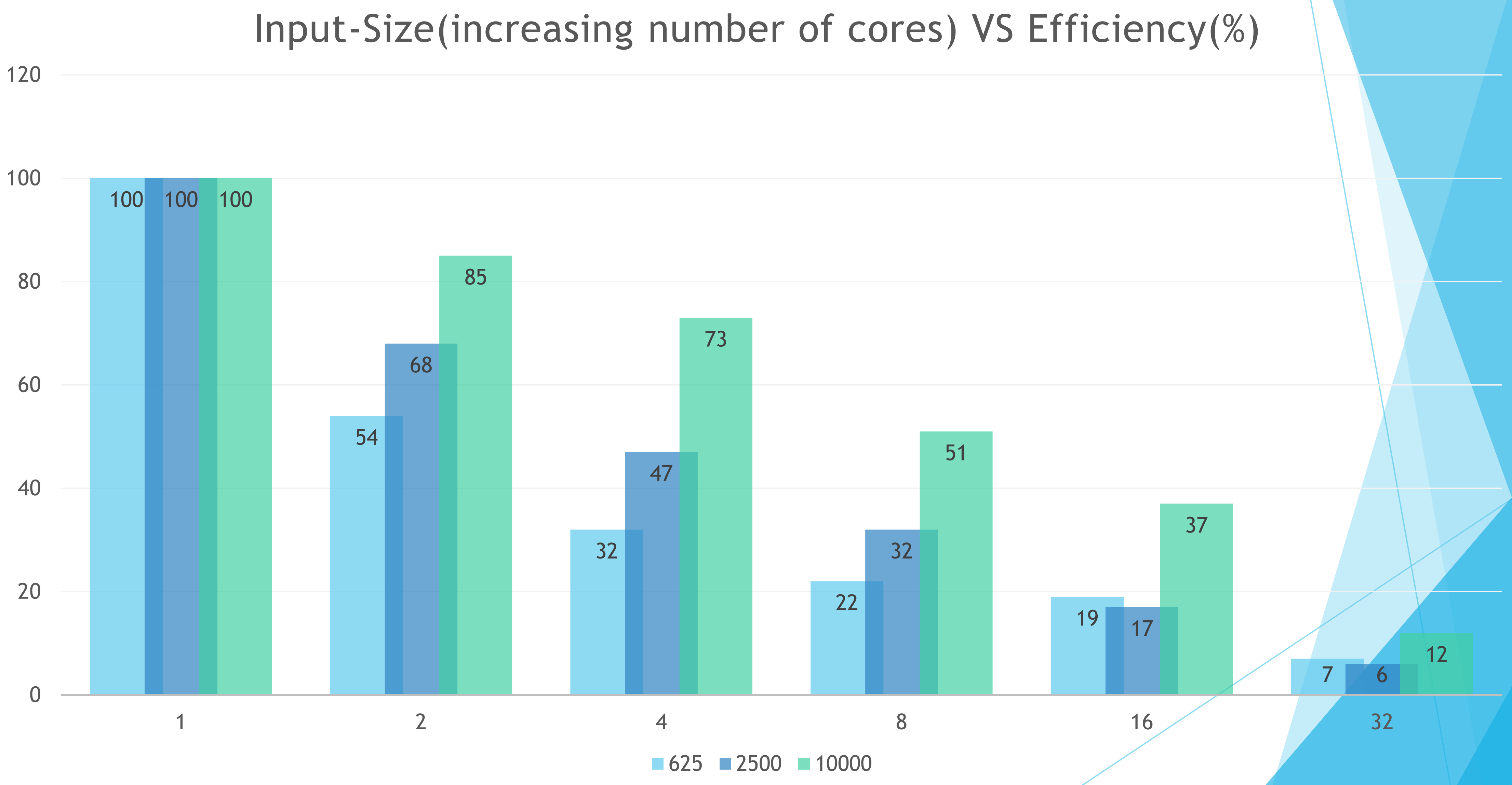
(a) Efficiency varies inversely with number of cores .

(b) Significant Decrease observed as number of cores increases

(c) Gives an indication that benefit of reduced running time cannot outperform cost of operation.

Number of Cores	1	2	4	8	16	32
Input-Size	Efficiency: For example, if E = 50%, the processors are being used half of the time to perform the actual computation					
625	1	0.54560	0.32712	0.22463	0.19119	0.07893
2500	1	0.68263	0.47115	0.32946	0.17546	0.07649
10000	1	0.85744	0.73641	0.51834	0.37175	0.12699

EXPERIMENT B : EFFICIENCY
GRAPHICAL DESCRIPTION OF ANALYSIS



A QUICK LOOK UP AT THE AMDAHL'S LAW

- The maximum speed up that can be achieved by using N resources is :

  1/(F+(1-F)/N).

- As an example, if *F* is only 10%, the problem can be sped up by only a maximum of a factor of 10, no matter how large the value of *N* used.

- A great part of the craft of parallel programming consists of attempting to reduce *F* to the smallest possible value.

SUMMARY OF ACCOMPLISHMENTS

- Parallel Implementation using MPI library routines and CCR .
- Intel implementation of the Message Passing Interface
- Multi-network support :TCP/IP, Infiniband, Myrinet- by default the best network is tried first.
- GNU Compiler Wrapper
- Used simplified startup **mpirun**
- Launch combines mpd daemons and mpiexec.
- Detailed Understanding of MPI APIs()
- MPI Init() and MPI Finalize()
- MPI Comm size() and MPI Comm rank()
- MPI_Reduce() MPI_Bcast()
- MPI_Gather()

REFERENCES

- Dijkstra, E. W. (1959). "A note on two problems in connection with graphs,". Numerische Mathematik 1: 269–271. doi:10.1007/BF01386390.
- Cormen, Thomas H.; Leiserson, Charles E.; Rivest, Ronald L.; Stein, Clifford (2001). "Section 24.3: Dijkstra's algorithm". Introduction to Algorithms (Second ed.). MIT Press and McGraw-Hill. pp. 595–601. ISBN 0-262-03293-7.
- A. Crauser, K. Mehlhorn, U. Meyer, P. Sanders, “A parallelization of Dijkstra’s shortest path algorithm”, in Proc. of MFCS’98, pp. 722-731, 1998.
- Y. Tang, Y. Zhang, H. Chen, “A Parallel Shortest Path Algorithm Based on Graph-Partitioning and Iterative Correcting”, in Proc. of IEEE HPCC’08, pp. 155-161, 2008.
- Stefano, A. Petricola, C. Zaroliagis, “On the implementation of parallel shortest path algorithms on a supercomputer”, in Proc. of ISPA’06, pp. 406-417, 2006.
- http://www.cse.buffalo.edu/faculty/miller/Courses/CSE633/Ye-Fall-2012-CSE633.pdf

THANK-YOU

ANY QUESTIONS??