PARALLELIZING THE FLOYD-WARSHALL ALGORITHM

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CSE 633: Parallel Algorithms
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Contents

• Introducing Floyd-Warshall Algorithm
• Why parallelizing makes sense?
• Serial Implementation
• Parallel Implementation
• Unequal Row Distribution
• Negative Cycle detection
• Results and Visualizations
• Planning for Block Based Floyd-Warshall
• Progress Report
• What’s next?
• References
What is Floyd-Warshall Algorithm?

An algorithm for finding the shortest path between all pair of nodes in a directed graph.

Some points to keep in mind –

1) While algorithms like Dijkstra's and Bellman Ford also find shortest path between source and destination nodes, Floyd-Warshall takes it a step further and calculates shortest distance between every pair of node unlike the former two.

2) The algorithm has many applications ranging from Network Routing, Transportation Planning to Social Networks and Robotics.
Why do we care about parallelizing it?

- Time Complexity of Floyd-Warshall is $O(n^3)$ where $n$ is the number of nodes in the graph.
- This is because for each pair of vertices, the algorithm considers all possible intermediate vertices and computes the shortest path between them. Therefore, the running time of the algorithm is proportional to the cube of the number of vertices in the graph.

$$d_{ij}^{(k)} \leftarrow \min(d_{ij}^{(k-1)}, d_{ik}^{(k-1)} + d_{kj}^{(k-1)})$$

- This means that –

<table>
<thead>
<tr>
<th>Computations</th>
<th>Nodes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>10</td>
</tr>
<tr>
<td>1,000,000</td>
<td>100</td>
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<tr>
<td>1,000,000,000</td>
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</tbody>
</table>
Okay, but how does it work, Serially?

The way Floyd-Warshall algorithm works is by traversing over every intermediate node and finding a shortest path from Source S, to destination D, going through Intermediate node I.

Here pointer k is traversing over the intermediate nodes, i is traversing over all the sources and j over all the destinations possible,

\[
n = \text{cardinality}(V);
\]

\[
\text{for } k = 1 \text{ to } n \text{ do}
\]

\[
\text{for } i = 1 \text{ to } n \text{ do}
\]

\[
\text{for } j = 1 \text{ to } n \text{ do}
\]

\[
\text{if } \text{distance}[i][j] > \text{distance}[i][k] + \text{distance}[k][j] \text{ then}
\]

\[
\text{distance}[i][j] \leftarrow \text{distance}[i][k] + \text{distance}[k][j];
\]

\[
\text{end}
\]

\[
\text{end}
\]

\[
\text{end}
\]
Now, how do you parallelize it?

Can be parallelized using different ways, but two approaches that I considered were -

**BLOCK BASED PARALLELIZATION**

In this approach, the adjacency matrix is divided into blocks with each block being assigned to one of the processes. Each process is responsible for calculating the shortest paths within its block.

**WHEN** – More efficient when matrix is large, and processes are less.

**ROW BASED PARALLELIZATION**

In this approach, the adjacency matrix is divided into rows with each row being assigned to one of the processes. Each process is responsible for calculating the shortest paths within the rows assigned to it.

**WHEN** – More efficient when matrix is small, and processes are more.
What and how?

To get things up and moving, I decided to proceed with *Row based parallelization*.

- For each matrix of size n x n, and p processes, each process receives a matrix of size [n/p][n].
- As we can see on the right, to fill in the value of arr[i][j], we need arr[i][k] and arr[k][j].
- Thanks to the row-based approach, process P can locally access arr[i][k], but for arr[k][j], the process that was assigned kth row, has to somehow broadcast all the elements of the kth row to all the processes.
Okay, what about Other Processes?

So, we updated the distances for one processes, but other processes also need these updated distances to calculate distances for their sub-matrix.

Before updating kth row, we send it to all the other processes. This enables them to proceed immediately to do their work.
Pseudo code for Parallel Approach

MPI Init
n ← size of rows
pid ← id of process
pN ← number of processes
$D^{(0)} ←$ input distance matrix
for $k ← 1$ to $n$
    do for $i ← \frac{\text{pid} \times n}{pN}$ to $n$
        do for $j ← 1$ to $n$
            do $d^{(k)}_{ij} ← \min (d^{(k-1)}_{ij}, d^{(k-1)}_{ik} + d^{(k-1)}_{kj})$
            send $i$'st row to another processes
            receive updated rows from another processes
        return $D^{(n)}$
MPI Finalize

$T_{Floyd} = \frac{N^3}{P} + \text{TIME TO COMMUNICATE}$

For $i=2$, $j=2$ and $k=0$
Unequal Row Distribution

• Worked on unequal row distribution.
• Earlier, the program where number of rows where not divisible by the number of processes would give incorrect results.
• Calculated sendCounts and displacements to be used for Scatterv and Gatherv.
• Scatterv and Gatherv unequally distributed the rows to different processes.
Detect Negative Cycles

• We know that minimum distance from one vertex to itself is always zero.

• However, if we see that the distance is negative, we can imply that there must be a negative cycle as the distance after traversing got negative.

```cpp
for (int i = 0; i < rowsPerProcess; i++)
{
    if (localArr[i * n + i] < 0)
    {
        cout << "Error: Negative Cycle Found\n";
        MPI_Abort(MPI_COMM_WORLD, 1000);
    }
}
```
Performance

With [2,000 x 2,000] matrix
4 Million elements

With [10,000 x 10,000] matrix
100 Million elements

Communication overtakes Computation
Scaling Processes with Minimal Nodes
With $[4,000 \times 4,000]$ matrix

<table>
<thead>
<tr>
<th>Processes</th>
<th>Nodes</th>
<th>Processes /Node</th>
<th>Time (in seconds)</th>
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<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>57.203</td>
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Communication overtakes Computation
### Scaling Nodes with Minimal Processes

With $[4,000 \times 4,000]$ matrix

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**Scaling Processes with Minimal Nodes**

With [4,000 x 4,000] matrix – Comparing Multiple Nodes vs Multiple Processes

**Note:** More time taken by “Minimal Nodes, Multiple Processes” owing to Context Switching by CPU and/or Resource Contention at Nodes.
Contrasting Serial vs Parallel Running Times

• Varied the vertices of the Graph keeping the number of nodes constant (40).

• With 6000 vertices, Sequential process takes around 312s whereas Parallel takes around 11.90s.
Speed Up

• Calculated using $T_{seq} / T_{parallel}$.

• For 2000 vertices, the max speedup of 13 comes around when 20 processes work in parallel.

• For 4000 vertices, speedup of 23 comes around with 40 processes. After which communication overtakes computation and the speedup decreases.

• For 10,000 vertices, max speedup might come later (couldn’t compute due to limited computational resources).
Cost

- Shows effect of cubic-algorithm complexity.
- Calculated using

\[ \text{Number of Processes} \times T_{\text{parallel}} \]
Planning for Block Based Floyd-Warshall

**Row Based**

Taking Intermediate node as 1

\[
\begin{align*}
\begin{bmatrix}
0 & 1 & 2 & 3 \\
0 & 2 & \infty & \infty \\
1 & \infty & 0 & \infty \\
2 & \infty & \infty & 0 \\
3 & \infty & \infty & \infty
\end{bmatrix}
\end{align*}
\]

**Block Based**

\[
\begin{align*}
[2][0] &= [2][1] + [1][0] \\
[2][1] &= [2][1] + [1][1] \\
[2][2] &= [2][2] + [1][2] \\
[2][3] &= [2][1] + [1][3]
\end{align*}
\]
1. Looking into - sbatch: error: Batch job submission failed: Requested node configuration is not available for requesting more than 140 nodes – **Infiniband has max spread of 140 nodes. Increased Processes thereafter. ✔**

2. Draw Inferences and Calculate Speed Up – **Done ✔**

3. Trying to make the algorithm work for cases where number of rows is not completely divisible by number of processes. Exploring MPI_Scatterv and MPI_Gatherv for that – **Done. ✔**

4. Next, I intend to work upon negative cycle detection – **Done. ✔**

5. Block-based Floyd-Warshall to find out if it performs better than the row-based parallelization? – 🕒
What’s next?

1. Implement Block-based Floyd-Warshall
2. Find out if Block Based parallelization performs better than the row-based parallelization?
3. Have an Open MP implementation
4. Use Open MP and MPI collectively to implement Floyd-Warshall and make observations
References


4. Dr. Matthew D. Jones. *Lectures on MPI & CCR*


Thank You