Modeling and Predictivity (at Chess)

How "Beating the Bookie" may help in fraud detection

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[1] Joint work with Tamal Tanu Biswas and with grateful acknowledgment to UB's Center for Computational Research (CCR)

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- Predictive Analytics: Inferring the probabilities p_j of various events j:
  - Risk or damage events.
  - Voter j choosing candidate i.
  - Student i choosing answer j.
  - Player choosing move m_j at chess.

Chess and Tests: Prediction ≈ Grading

The ____ of drug-resistant strains of bacteria and viruses has ____ researchers' hopes that permanent victories against many diseases have been achieved.

- (a) vigor . . corroborated
- (b) feebleness . . dashed
- (c) proliferation . . blighted
- (d) destruction . . disputed
- (e) disappearance . . frustrated

(source: itunes.apple.com)

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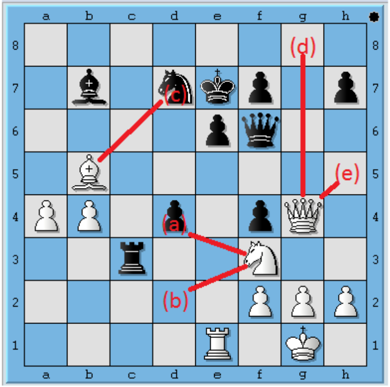

Multinomial Logit Model

Given options $m_1, \ldots, m_J$ and information $X = X_1, \ldots, X_J$ about all of them, and characteristics S of a person choosing among them, we want to project the probabilities p_j of m_j being chosen.

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Finally obtain β by fitting; e^α becomes a constant of proportionality so that the p_j sum to 1.

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The β can be absorbed as $(\frac{1}{s})^c$ even when $c \neq 1$ so my nonlinearized utility still fits the setting. Then abstractly:

$$\begin{aligned} \frac{\log(1/p_j)}{\log(1/p_1)} &= \exp(\beta U_j) =_{def} L_j \\ \log(1/p_j) &= \log(1/p_1) L_j \\ \log(p_j) &= \log(p_1) L_j \\ p_j &= p_1^{L_j}. \end{aligned}$$

Analogy to power decay, Zipf's Law... *Proceed to demo.*