Computational Complexity / Decision Making (at Chess)

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[1] Recent Students: Robert Surówka, Tamal Biswas, Michael Wehar, James Clay

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- The biggest cluster is the class of **NP-complete** problems.

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- We don't even know whether SAT can be solved in **linear** time!

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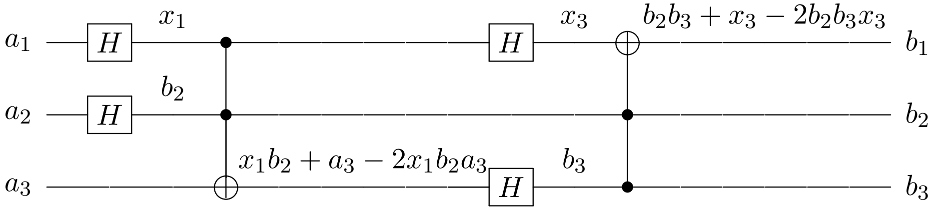

Decision Making in Chess...

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Decision Making in Chess... and Tests

The ____ of drug-resistant strains of bacteria and viruses has ____ researchers' hopes that permanent victories against many diseases have been achieved.

- (a) vigor . . corroborated
- (b) feebleness . . dashed
- (c) proliferation . . blighted
- (d) destruction . . disputed
- (e) disappearance . . frustrated

(source: itunes.apple.com)

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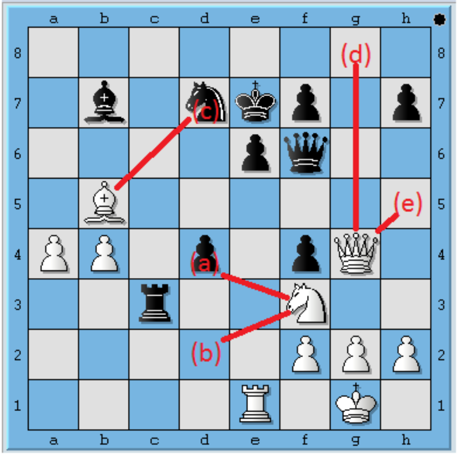

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8. **Discover new scientific regularities of human thought processes.**