

# What Laws Act on the Mind?

Large data, regularities, and illusions

Kenneth W. Regan<sup>1</sup>

University at Buffalo (SUNY)

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<sup>1</sup>Joint work with Tamal Tanu Biswas and with grateful acknowledgment to UB's Center for Computational Research (CCR)

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- This excluded the first 8 moves in any game—“book” openings.

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If your actual score exceeds (falls short of) your expectation then your rating goes up (down).

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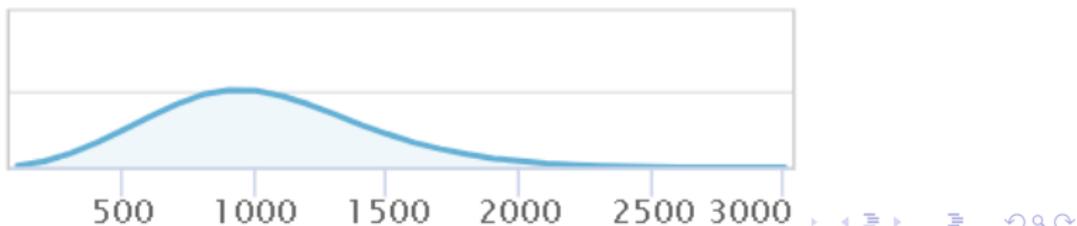
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- (Will discuss IPRs later; focus on values now.)

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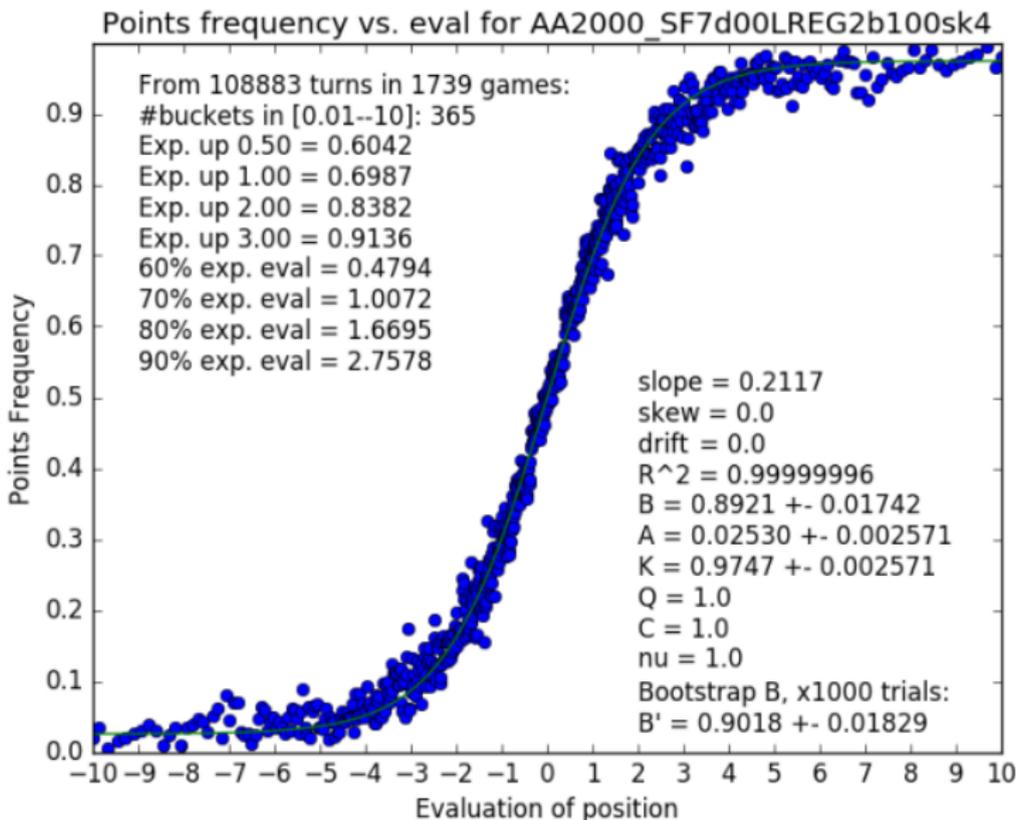
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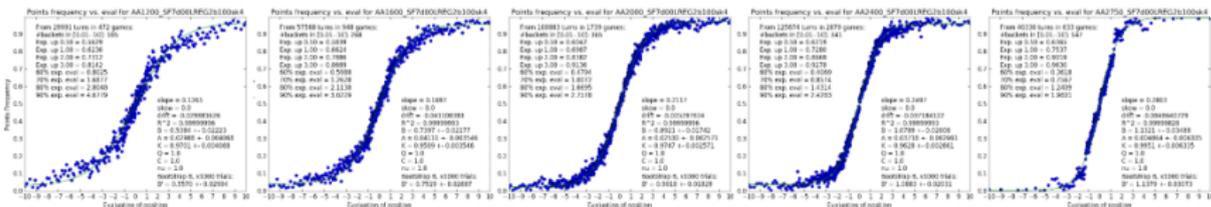
Refined to include small probability  $A$  of blundering away a “completely winning” game, giving a “generalized logistic” (Richards) curve:

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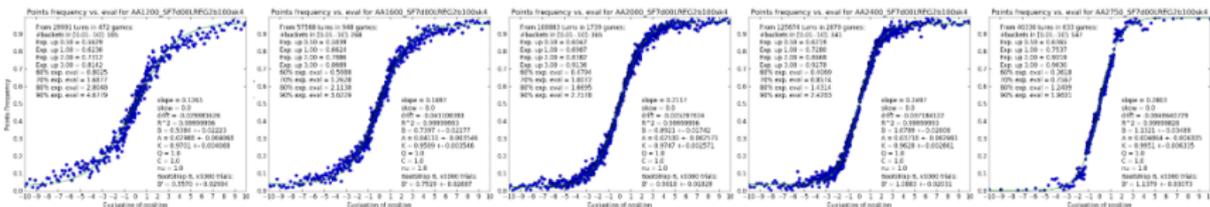
# Example For Elo 2000 Rating



# Slope as Rating Changes

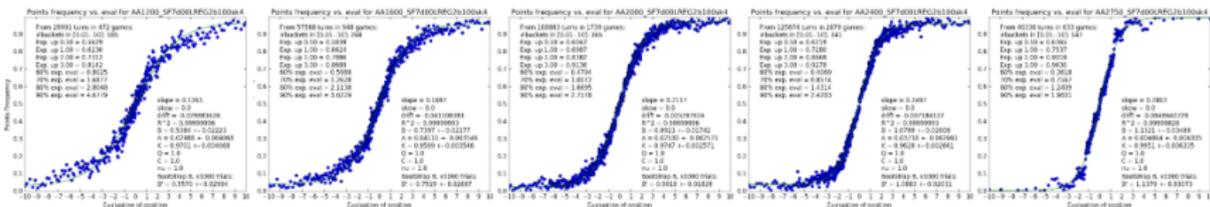


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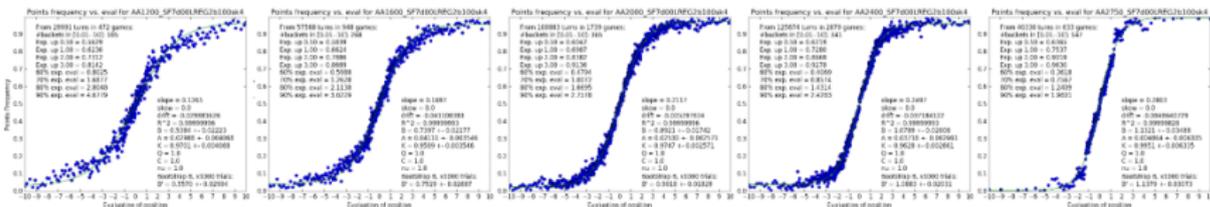
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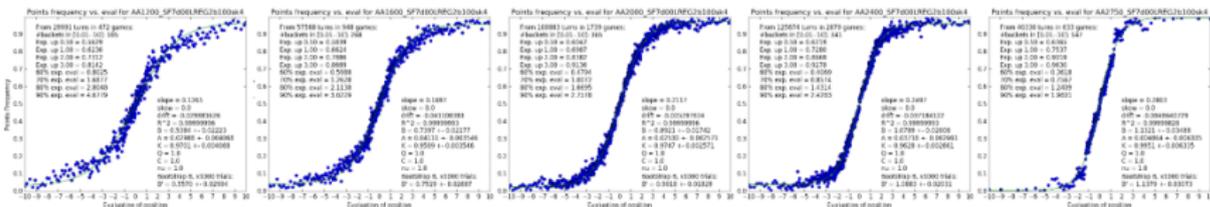
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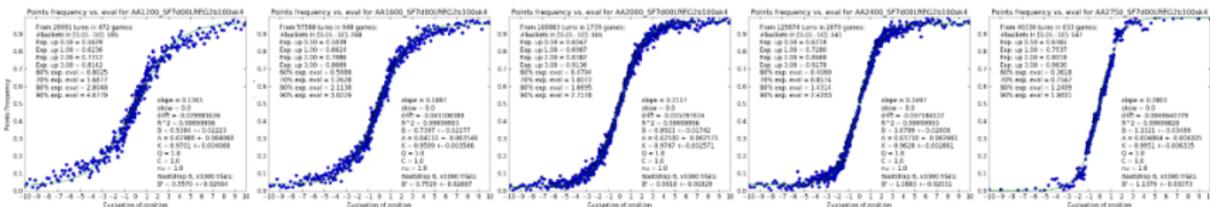
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- But I have to model human players of all levels  $R$  in my tests.

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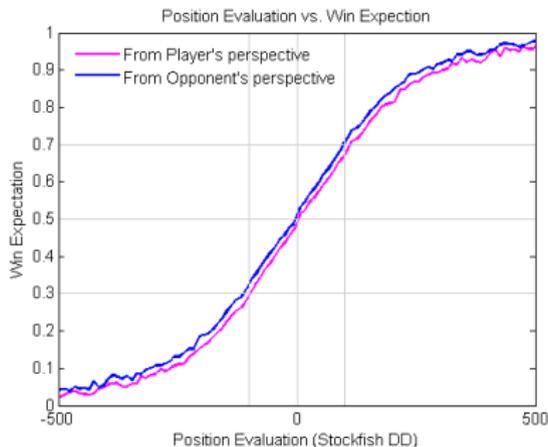
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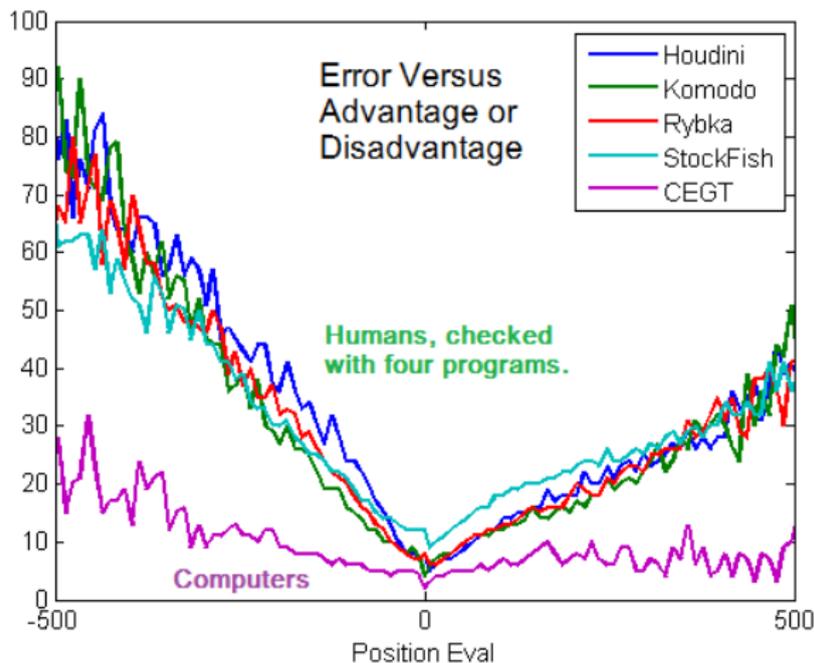
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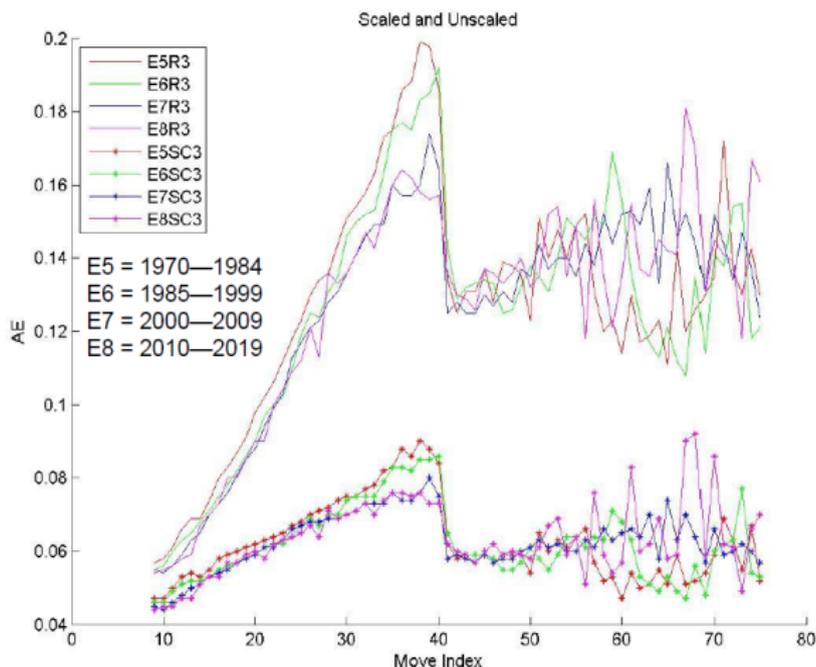
# Law of Relative Perceived Differences in Value



Values can be scaled to flatten this out and conform more to  $E$  scale.

# “Law” of Human Time Budgeting

## Error By Move Number in Games



Effect of time pressure approaching Move 40 is clear.

Moves 17—32 bridge between opening theory and worst of Zeitnot.

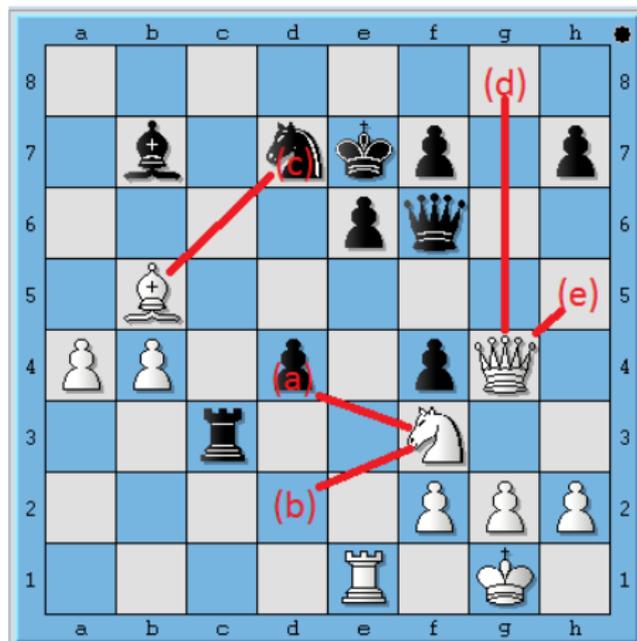
# Chess and Tests

The \_\_\_\_ of drug-resistant strains of bacteria and viruses has \_\_\_\_ researchers' hopes that permanent victories against many diseases have been achieved.

- (a) vigor . . corroborated
- (b) feebleness . . dashed
- (c) proliferation . . blighted
- (d) destruction . . disputed
- (e) disappearance . . frustrated

(source: itunes.apple.com)

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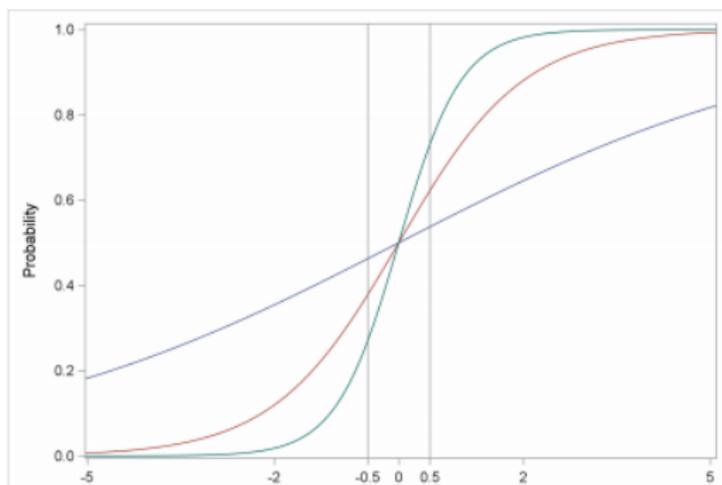
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- Students quantified by one aptitude parameter  $\theta$  (“the” grade).
- Each test question  $q$  determines a curve  $E_q(\theta) \equiv$  the likelihood of a person of skill  $\theta$  getting it right.

# Item-Response Theory

- Students quantified by one aptitude parameter  $\theta$  (“the” grade).
- Each test question  $q$  determines a curve  $E_q(\theta) \equiv$  the likelihood of a person of skill  $\theta$  getting it right.
- IRT posits this as always a Richards curve whose slope  $B$  is the sharpness of level that the question *discriminates*.

Figure 3 Item Characteristic Curves



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- Score = “Average Scaled Difference” (ASD).
- Also gives a *utility function* for possible moves.

## Obstacles to Directly Testing IRT in Chess

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- Otherwise, use my model’s MM% and ASD projections directly.

# The MM% Projection, 1600-to-2700 Levels

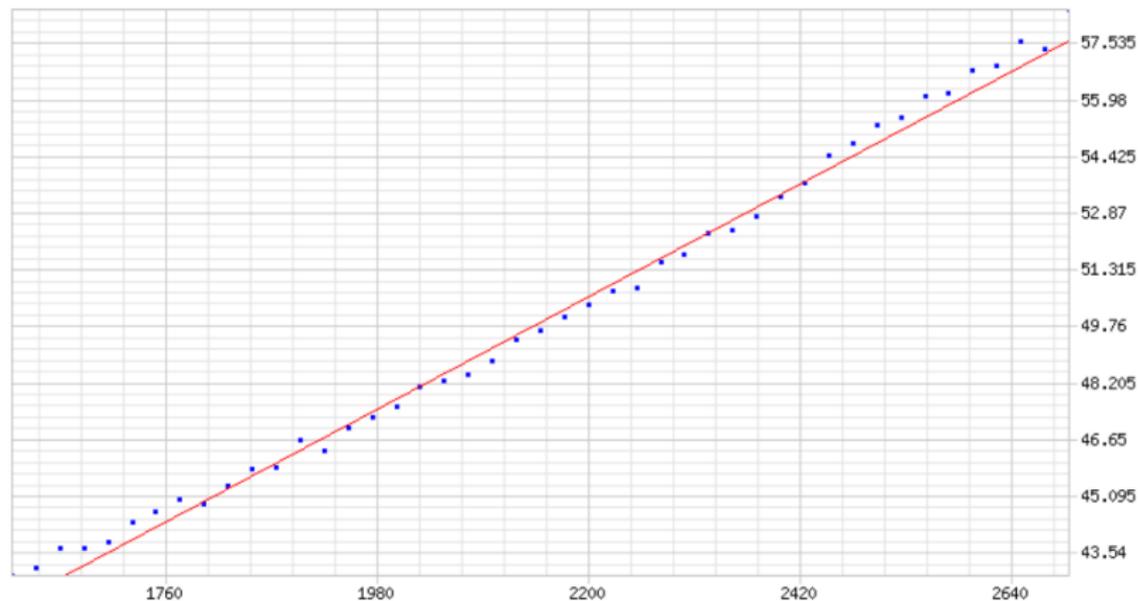
## Function

$$f(x) = 19.654619721630443 + 0.014057033867393376x$$

## R-Squared

$$R^2 = 0.99303212012685$$

## Graph



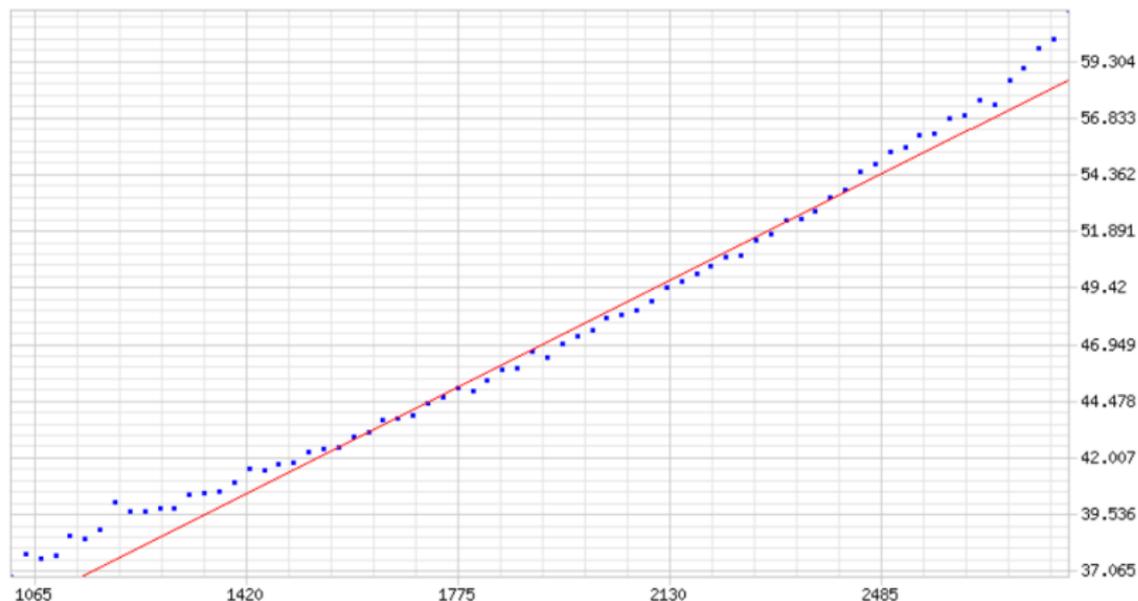
## Now Including 1025–1600, 2725–2800:

**Function**

$$f(x) = 21.86511755244366 + 0.013085915894893769x$$

**R-Squared**

$$R^2 = 0.97835646846452$$

**Graph**

# Quadratic Not Linear Law?

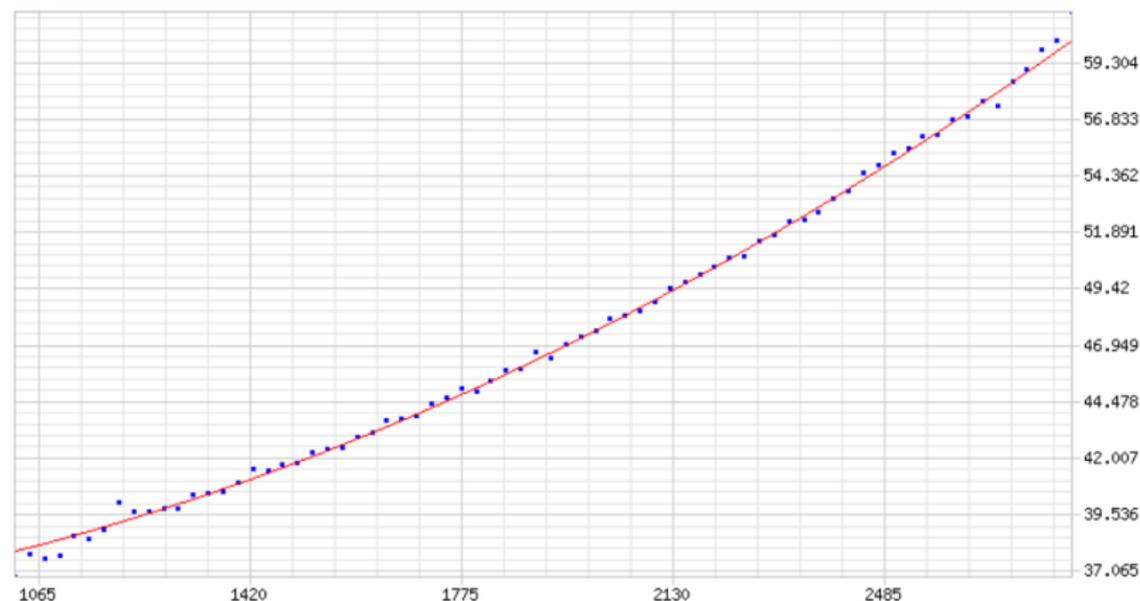
## Function

$$f(x) = 34.66026963709357 - 0.00024349241455471368x + 0.0000033522002997568x^2$$

## R-Squared

$$R^2 = 0.99779719205296$$

## Graph



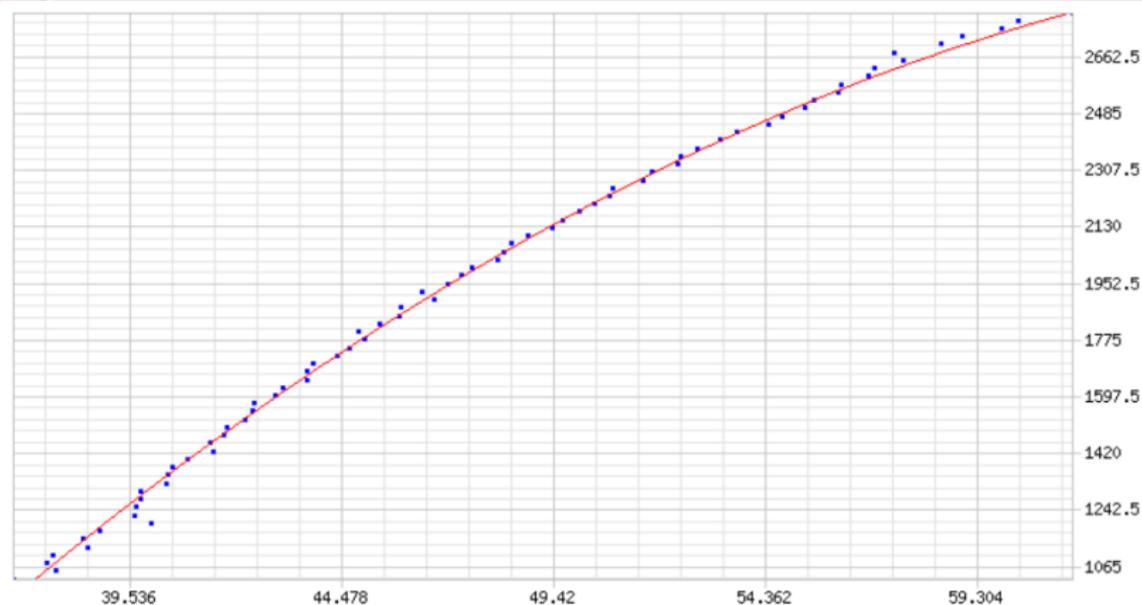
## Same With X,Y Axes Flipped...

**Function**

$$f(x) = -5224.3797654152 + 224.51739158320626x - 1.5285546730040955x^2$$

**R-Squared**

$$R^2 = 0.99814244490643$$

**Graph**

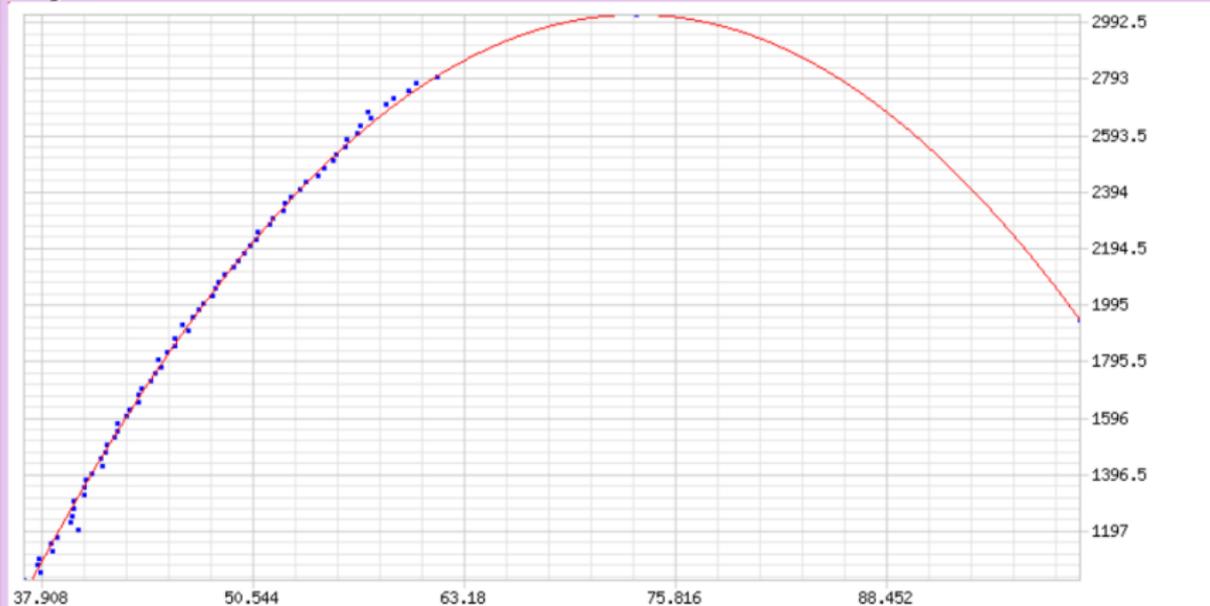
## ...And Extended...

**Function**

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$$R^2 = 0.99825130391887$$

**Graph**

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- Analogy to catching particles with a river sieve.

# Linear Law For ASD Looks Good...But...

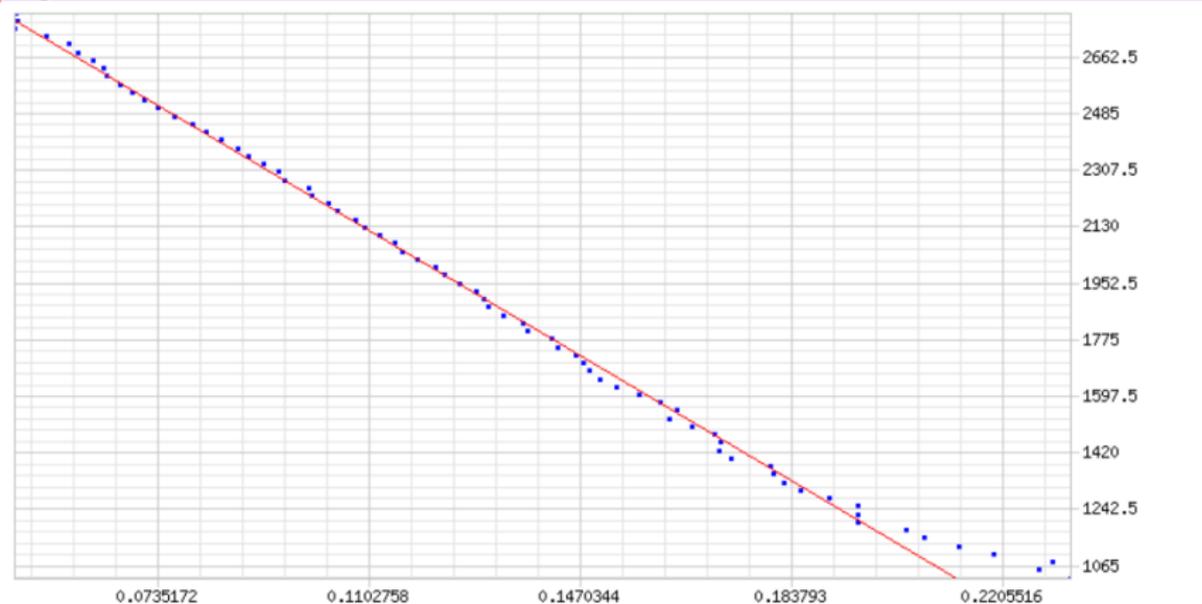
**Function**

$$f(x) = 3298.02376454243 - 10688.627382908597x$$

**R-Squared**

$$R^2 = 0.99037759880581$$

**Graph**



# Quadratic Law Has Higher “Rating of Perfection”

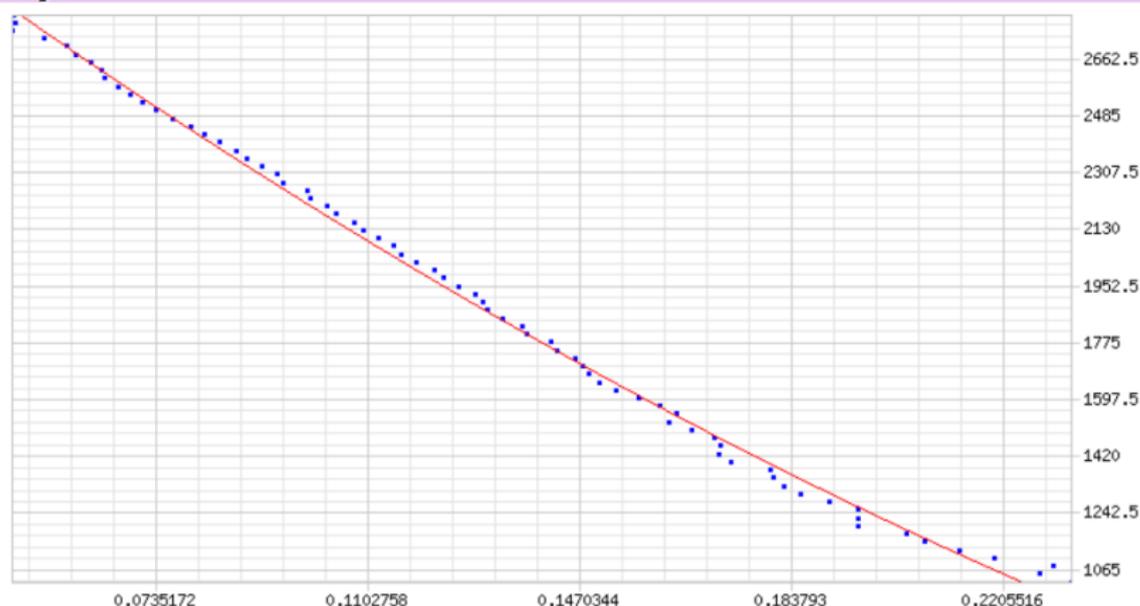
## Function

$$f(x) = 3462.663010383108 - 13884.604914850042x + 13415.403252920698x^2$$

## R-Squared

$$R^2 = 0.99676481397797$$

## Graph



# Multiplying By $4pq$ Recovers Good Linear Fit

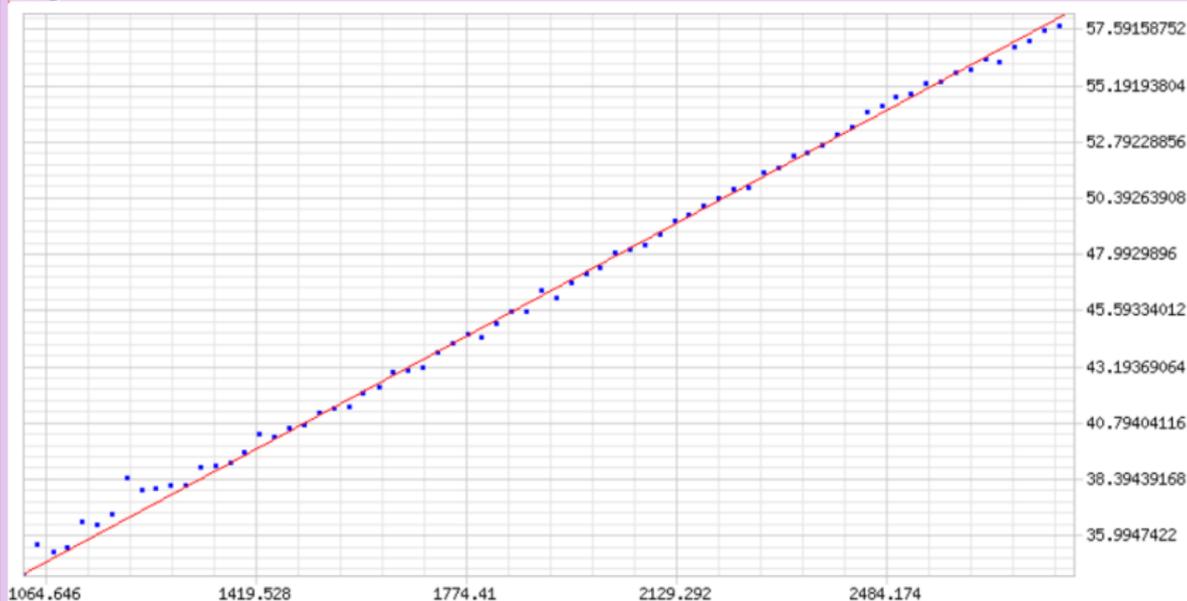
## Function

$$f(x) = 20.42277725109287 + 0.013578631028477313x$$

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$$R^2 = 0.99732175601628$$

## Graph



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- Can we reward *depth-of-thinking* directly?

## Decision Model: Linear or Log-Linear or ...

- A “classical” *decision model* predicts the likelihood  $\ell_i$  of a decision outcome  $m_i$ , which becomes its forecast probability  $p_i$  after normalization, in terms of its *utility*  $u_i$  to the decider.

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- So which law holds in chess: linear or log-linear?

# Evidence for Neither: Needs “LogLogRadical” Model

Log-log-linear equation:

$$\log \log(1/p_i) - \log \log(1/p_1) = \beta u_i$$

yields

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My deployed model inverts  $\beta$  as  $1/s$  where  $s$  stands for *sensitivity*, and makes utility nonlinear with a second parameter  $c$  (for *consistency*):

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Triple-decker exponentiation. *Is it a natural law? Or an unnatural law?*

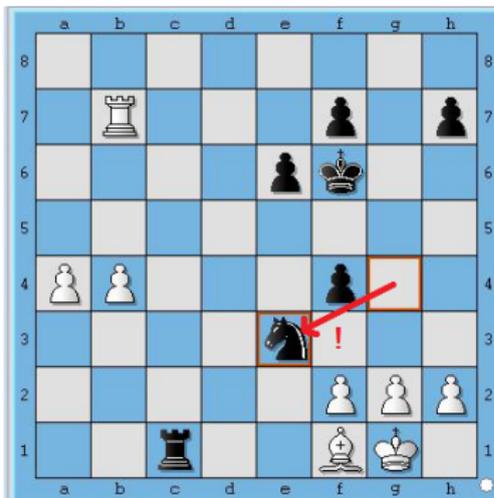
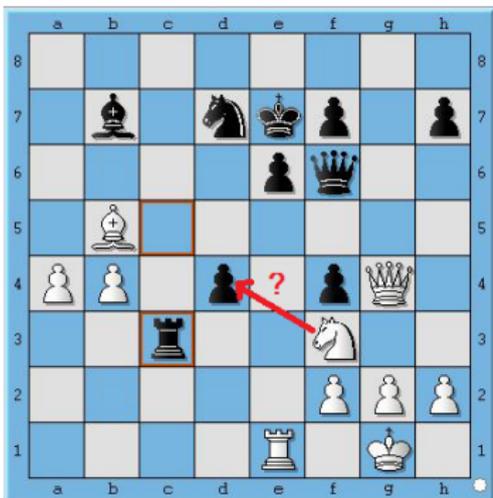
## Check of Log-Linear Model: London 1883 Tmt

Rk	ProjVal	Actual	Proj%	Actual%	z-score
1	4870.99	4871.00	47.34%	47.34%	z = +0.00
2	1123.22	1729.00	10.94%	16.85%	z = +19.88
3	633.30	951.00	6.21%	9.32%	z = +13.27
4	459.83	593.00	4.56%	5.88%	z = +6.44
5	370.58	410.00	3.72%	4.11%	z = +2.11
6	311.98	295.00	3.16%	2.99%	z = -0.99
7	270.56	247.00	2.75%	2.51%	z = -1.46
8	239.36	197.00	2.44%	2.01%	z = -2.79
9	214.30	169.00	2.19%	1.73%	z = -3.15
10	193.93	104.00	1.99%	1.07%	z = -6.57

## With LogLog-Radical Model (first line is MM%)

Rk	ProjVal	Sigma	Actual	Proj%	Actual%	z-score
1	4871.02	47.02	4871.00	47.34%	47.34%	z = -0.00
2	1786.89	37.32	1729.00	17.41%	16.85%	z = -1.55
3	929.87	28.60	951.00	9.11%	9.32%	z = +0.74
4	589.93	23.29	593.00	5.85%	5.88%	z = +0.13
5	419.35	19.84	410.00	4.21%	4.11%	z = -0.47
6	315.24	17.32	295.00	3.19%	2.99%	z = -1.17
7	246.68	15.39	247.00	2.51%	2.51%	z = +0.02
8	198.71	13.85	197.00	2.03%	2.01%	z = -0.12
9	161.54	12.52	169.00	1.65%	1.73%	z = +0.60
10	134.18	11.43	104.00	1.38%	1.07%	z = -2.64

# The Deepest Mental Influence?



Values by depth of search:

Move	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Nd2	103	093	087	093	027	028	000	000	056	-007	039	028	037	020	014	017	000	006	000
Bxd7	048	034	-033	-033	-013	-042	-039	-050	-025	-010	001	000	-009	-027	-018	000	000	000	000
Qg8	114	114	-037	-037	-014	-014	-022	-068	-008	-056	-042	-004	-032	000	-014	-025	-045	-045	-050
...			...			...			...		...			...				...	
Nxd4	-056	-056	-113	-071	-071	-145	-020	-006	077	052	066	040	050	051	-181	-181	-181	-213	-213

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- New model parameter  $h$  (for nautical “heave”) multiplies  $\rho$ .

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- Currently trying to have  $s, c$  touch components of  $\rho$  directly and add parameters that preserve the “canyon” shape.

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