(Semi-)Deep Learning, Information Complexity, and Hard-to-Refute Competence

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Outline

I. Brainstorm of ingredients, motivations, and examples on whiteboard.

II. Define concepts and present relevant facts:

- Information (Kolmogorov) Complexity.
- Boolean and Threshold Circuits.
- The class $TC^0 = \bigcup_d TC^0_d$ and frontiers of lower bounds.
- Approximation and Refutation Problems

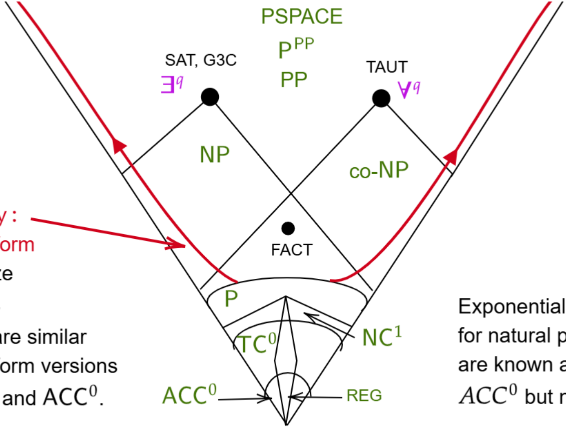
III. Discussion, maybe some demos, as time allows.

Kolmogorov Complexity

- Let us fix a universal "interpreter" I.
- (Technically any fixed universal Turing machine will do.)
- $K(x) = \min\{|s| : I(s) = x\}$.
- For the MNIST dataset, can distinguish $K(g)$ for its correct labeling function $g(x)$, from the KC of the dataset itslf.
- The *time* for $I(s)$ to output x is classically not considered...
- ...which makes the function $K(x)$ **uncomputable**.
- Various notions of *resource-bounded* information complexity...we will not specify.
- Important test case: Given an algorithm A that tries to compute function $g(n)$, is $s_n =$ "the lexicographically first string x_n of length n such that $A(x) \neq g(x)$" a meaningful short seed for x_n?

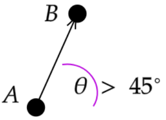
Threshold Circuits

Computational Complexity Classes



P / poly : nonuniform poly-size circuits.

There are similar nonuniform versions of TC^0 and ACC^0.



means $A \leq_m^{tc^0} B$

Exponential lower bounds for natural problems (in NEXP) are known against nonuniform ACC^0 but not against TC^0.

Some regular languages are complete for NC^1 via even finer reductions than tc^0 reductions, and so belong to TC^0 if and only if $TC^0 = NC^1$ (which I believe true).

The "Chasm At Depth 4"

- Actually pertains to arithmetic circuits, not threshold circuits.
- But relevant since basic arithmetical operations belong to TC_3^0, even division.
- Agrawal-Vinay 2008, Koiran 2012: **Theorem**: Any multlinear polynomial $p(x_1, \dots, x_n)$ that is computable by arithmetic circuits of size $2^{o(n)}$ has depth-4 arithmetic circuits of size $2^{o(n)}$.

The Complexity Class Neighborhood...