CSE396, Spring 2021 Problem Set 6 Due Tue. 3/30, 11:59pm

Reading: On Tuesday we will finish section 2.3 and move on to chapter 3, so please read section 3.1 for Tuesday. Note that we will come back to section 2.2 after section 3.2.

Homework—part online (TopHat), part written, and all *individual work*:

(1) Using *TopHat*, the "Worksheet" titled *S21 HW6 Online Part* (10 Qs, 20 pts.)

The other two problems are to be submitted as PDFs using the *CSE Autograder* system.

(2) Let E be the language of all strings over $\Sigma = \{a, b\}$ that do not have the substring bb, and let G be the following context-free grammar:

$$\begin{aligned} S &\longrightarrow \epsilon \mid b \mid BS \mid SA \\ A &\longrightarrow aS \mid AA \\ B &\longrightarrow a \mid bAaB \end{aligned}$$

(a) Show that the string $babab$ is ambiguous in the grammar G, by giving two different parse trees. (6 pts.)

(b) Is any other variable besides S capable of deriving ϵ? Give one(s) if so. (3 pts.)

(c) Prove by structural induction that $L(G) \subseteq E$. *Hint:* Ask yourself what additional properties, besides not allowing a bb substring themselves, must the variables A and B maintain? (15 pts., for 24 total)

(3) Let $A = \{ a^n b^n : n \geq 1 \}$. Define E to be the language of strings that differ *in at most one place* from a string in A. An example of a string in E is $aaba$, since changing the last a to b gives a string in A. Note that E contains A, and that the strings in E have the same lengths as strings in A. Define G to be the context-free grammar $(\{ S, T, U \}, \{ a, b \}, R, S)$, where the rules in R are:

$$\begin{aligned} S &\to aSb \mid aTU \mid UTb \\ T &\to aTb \mid \epsilon \\ U &\to a \mid b. \end{aligned}$$

(a) For each of the following strings, say whether it belongs to E, and if so, give a leftmost derivation for it (6 pts. total): (i) ϵ, (ii) bb, (iii) $aaabb$, (iv) $aabbbb$.

(b) Find an ambiguous string and draw two different parse trees for it. (6 pts.)

(c) Prove by the structural induction technique that $L(G) \subseteq E$. (You may speak in terms of E "allowing up to one error." As usual, "reasonable proof shortcuts" are OK. 18 pts.)

(b) Is $L(G) = E$? Justify your answer briefly by referring to your parsing strategies in (a,b), but you need not give a formal proof. (6 pts., for 36 total on the problem, 80 on the set)