

CSE396, Spring 2026 Problem Set 8 Due Tue. 4/14, 11:59pm

This homework is **now** due on **Tuesday**, April 14, not on Friday—and extended one day from Monday. Prelim II is still set for **Tuesday, April 21**, in class period. Once again, the last part of this homework arises from the previous Tuesday lecture.

Reading

For Thursday, please continue reading Chapter 4 to the end. For next week, please read section 5.1, skim/skip section 5.2, and read section 5.3. Then go back and re-read section 5.1 with the idea of employing the formal version of the reducibility concept. It strikes me as odd that the titular concept of the chapter is formalized only at the end—the text puts the intuition first, but my teaching style is more to regard intuition as guided by formal structure.

The dividing line between what is covered for Prelim II is basically: decidable languages *yes* (cumulatively regular languages through CFLs and the non-CFLs we’ve built efficient Turing machines for), methods for showing undecidability *no*. Thus up through section 4.1 in the text, but not 4.2 or chapter 5. Not uncountability, not diagonalization. The one exception is that in Thursday’s lecture I may tell which problems involving grammars are undecidable as facts to know, without yet giving their proofs.

——**Homework**——part online and all *individual work*——due **Tuen. 4/14, 11:59pm**——

(1) Using *TopHat*, the “Worksheet” titled **Spr 2026 HW8 Online Part**. Ten questions, each worth 2 points, for 20 total. The TopHat deadline is now the 14th but still strict.)

(2) Say that a context-free grammar $G = (V, \Sigma, \mathcal{R}, S)$ is in “short form” if for each $A \in V$, the rules for the variable A *collectively* have no more than two occurrences of variables *total* on *all their right-hand sides*. This form allows $A \rightarrow \epsilon$ freely and allows unit rules $A \rightarrow B$ (but at most two of them for each A) unlike Chomsky normal form. But if you have a rule $A \rightarrow BC$, or even $A \rightarrow BB$ or $A \rightarrow AA$, then no other rules for A may involve variables. For argument’s sake we’ll fix $\Sigma = \{0, 1\}$ for this problem.

The goal is to show that for every regular language L there is a CFG G in short form such that $L = L(G)$. Prove this by structural induction on the following grammar G_{REG} that generates regular expressions R :

$$E \rightarrow \emptyset \mid \epsilon \mid 0 \mid 1 \mid (E \cup E) \mid (E \cdot E) \mid (E^*)$$

Put another way, describe and verify an algorithm to convert any regular expression R into a short-form CFG G such that $L(G) = L(R)$, so that the algorithm works by recursion on sub-expressions. That way your answer should follow the same structure as the conversion from regexps to equivalent NFAs shown in class.

(3) Give a decision procedure for the following computational problem:

INSTANCE: A DFA M .

QUESTION: Does $L(M)$ equal its reversal?

Note that the Thu. 3/3 lecture led off with how to convert a DFA M into an NFA N such that $L(N) = L(M)^R$. You do not need to repeat fine details of this conversion but can use it as a line of your procedure the way the text does in section 4.1. Ultimately your algorithm should produce a DFA M' such that deciding the E_{DFA} problem on M' tells you the answer about M —it will flow into the text’s Theorem 4.5 but you should show all the steps. (12 pts.)

(4) Consider the following context-free grammar G :

$$\begin{aligned} S &\rightarrow AC \mid DC \\ A &\rightarrow aS \mid BA \\ B &\rightarrow \epsilon \mid SCS \\ C &\rightarrow BD \mid AS \\ D &\rightarrow BB \mid b \end{aligned}$$

- (a) Find the whole set N of nullable variables. Then carry out the step that adds rules skipping any subset of *occurrences* of nullable variables to get a new grammar G_1 . Note that if S is nullable then you get $L(G_1) = L(G) \setminus \{\epsilon\}$, else you get $L(G_1) = L(G)$. (Do **not** do the text’s initial step of adding a new start variable S_0 . 6 + 6 = 12 pts.)
- (b) Your grammar G_1 will have several unit rules—but don’t include “self-rules” like $A \rightarrow A$. Draw a directed graph whose nodes are the five variables and which has an edge (A, B) if $A \rightarrow B$ is a unit rule. Then take the transitive closure of the graph, which will tell you all pairs (A, B) such that $A \implies^* B$. Here we still *ignore* self-loops; that is, we only consider $B \neq A$. (6 + 6 = 12 pts., for 24 on the problem and 83 on the set)