CSE396 Lecture Thu. 2/4: Deterministic Finite Automata

We will give the dry formal definition before trying to liven it up in a few ways. Note I will have a few cosmetic differences from the text.

A **deterministic finite automaton** (**DFA**) is a 5-tuple $M = (Q, \Sigma, \delta, s, F)$ where:

- Q is a finite set of *states*.
- Σ is a finite alphabet.
- s, a member of Q, is the *start state*. [Text says q_0.]
- F, a subset of Q, is the set of *desired final states*, also called *accepting states*.
- δ is a function from $Q \times \Sigma$ to Q.

This "tuple" style of definition was introduced in the 1930s by French mathematicians writing under the fictional name Nicolas Bourbaki. A textbook by John Martin which we used before Mike Sipser's text came out made a joke that if you readily understand definitions like that, you muct be a mathematician. What I think it means, however, is that the Bourbakists were trying to do object-oriented programming before computers were invented. We can render the definition as:

```
class DFA {
   set<State> Q;
   set<char> Sigma;
   State s;                          //start state
   set<State> F;                     //accepting states
   State delta(State p, char c);     //is this sensible?
}
```

Indeed, in the *Turing Kit* software---written in Java by Mark Grimaldi while a student in this course in 1997---there is such a class. One change needed in "delta", however, motivates ways in which C# and Scala (among others) veered off from the original Java. As `delta`, it is a *class method* which makes it the same function for every DFA instance. It needs to be an *instance method*. In C++, one could do this "primitively" by making a pointer-to-member function field:

```
   State (*delta)(State p, char c);
```

Or, more cleanly (but also more fussily), one can define a separate *function-object* class, say `Delta`, with a method `apply(State p, char c)`, and have `Delta delta;` be the class field. However, I will favor a third way that will harmonize better with next week's definition of NFAs and that reflects the idea of a program being a set of *instructions*. The abstract fact is that every function f can be identified with the set of ordered pairs (a, b) such that $f(a) = b$. The `delta` function in this case has two arguments, so we get ordered triples instead of pairs. We can just treat these triples as instance data by writing:

```
   set<triple<State, char, State> > delta;
```

Every DFA instance will then automatically have its own set. Thus I prefer the definition of DFA to specify:

- δ, the set of *instructions*, aka. *tuples*, is a subset of $(Q \times \Sigma) \times Q$.
- In a DFA, for every $p \in Q$ and $c \in \Sigma$, there is a unique $q \in Q$ such that $(p, c, q) \in \delta$.

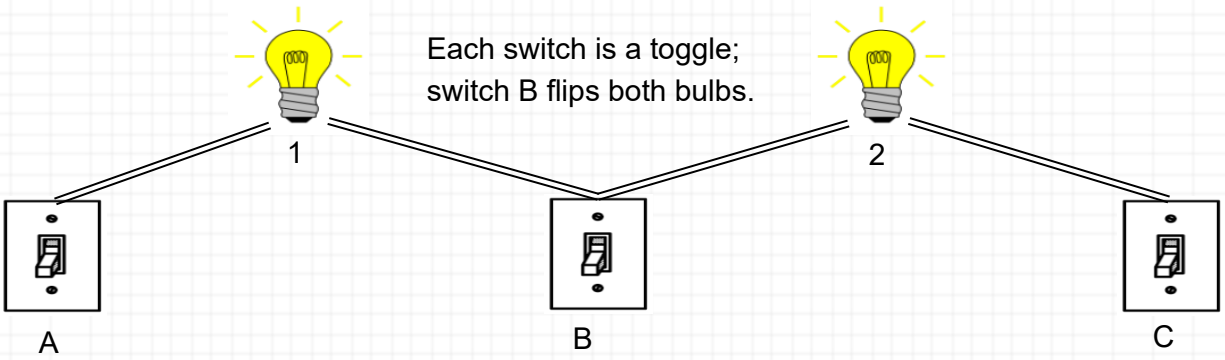
Relaxing the last clause will define an NFA ("without ϵ-arcs"). Another reason to think of instructions is how the machines look graphically:

There is a nice web applet for drawing DFAs, http://madebyevan.com/fsm/ by Evan Wallace, but it does not execute the machines you draw. That's where the *Turing Kit* comes in.

Before we go to the demo, one further remark about design principles. The definition says Q is a set of "states" without saying what those are, and my Java/C++ mockup code left `State` undefined. The text first exemplifies states as being observable conditions of a machine (an automatic door), but we will often want to think in terms of internal "states of mind" while processing a stream of data. Much more than any text I know, I want every "state" to have a comment or name signifying its purpose, much like commenting a line of code. The student who programmed the *Turing Kit* agreed wholeheartedly---its most overt difference from other machine apps one can find is the rich naming and tagging facility.

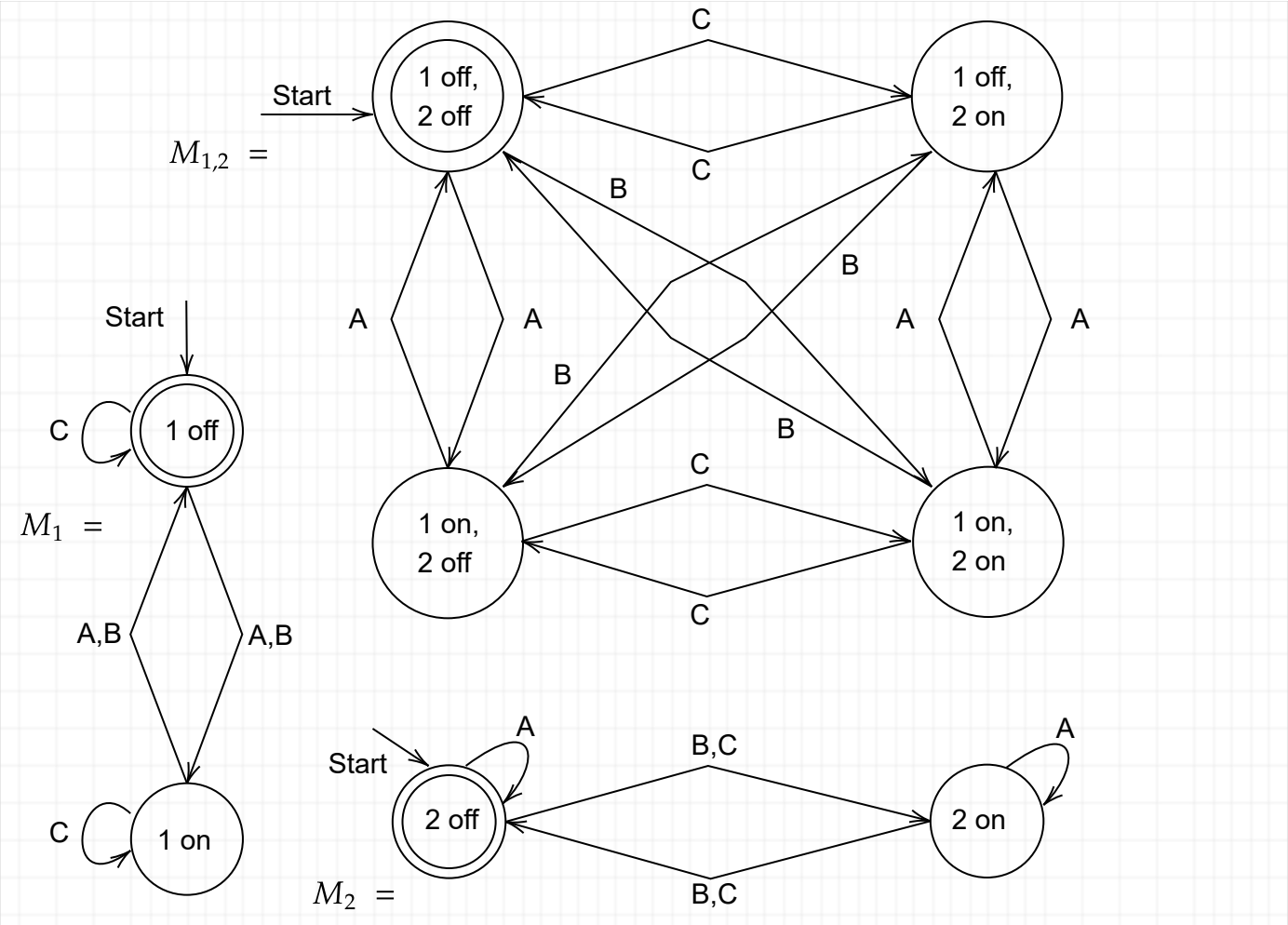
[Much of the rest of the lecture will be a demo of the *Turing Kit* software. On a Windows PC it now seems that no setup is required at all: just unzip the .jar file linked from the course webpage (no need to bother with the setup instructions link now), navigate your comamnd line to it, and enter the one command `java -cp TKIT70.jar Main` (then load `DragonSL.tmt` to see the first demo machine and do `View→Auto Resize` to work around a window-sizing bug). Its use will be optional; it even supports printing but its Postscript job handling has been wonky in the past.]

(Light bulbs and switches from http://clipart-library.com/free/light-bulb-clipart-transparent.html)



We want L to be the set of streams of actions over the day that, assuming both lights were initially off, leave them both off. What should be the alphabet for these actions?

What should the states of the system be? Include 8 switch combos or just the lights?



The DFA M_{12} is the "Cartesian Product for AND" of the DFA M_1 tracking light bulb 1 and the DFA M_2 tracking light bulb 2. The set of ordered pairs {(1off,2off), (1off,2on),(1on,2off),(1on,2on)} is the ordinary Cartesian product of the set {1off,1on} of states of bulb 1 and the set {2off,2on} for bulb 2.